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TWO-STAGE PROBLEMS OF OPTIMAL LOCATION AND DISTRIBUTION OF THE HUMANITARIAN LOGISTICS SYSTEM'S STRUCTURAL SUBDIVISIONS

Purpose. To ensure the rational organization of the evacuation of people from a region affected by an emergency by developing a mathematical and algorithmic toolkit that will allow for the early distribution of transport and material resources, maximizing coverage of the affected areas while minimizing evacuation time.

Methodology. System analysis of evacuation processes; mathematical modeling, the theory of continuous problems of optimal partitioning of sets, non-differentiable optimization.

Findings. The object of the study is the two-stage evacuation logistic processes that occur when serving the population of areas affected by emergencies of a natural or technogenic nature. The research considers the possibility of optimally distributing human flows within the transportation system, the structural subdivisions of which are first-stage centers (first aid stations that carry out the reception of citizens from areas affected by the disaster) and second-stage centers (specialized units of the emergency aid system that provide further services to the evacuated population). The proposed mathematical model deals with the problem of optimally partitioning a continuous set with the placement of subset centers and additional connections. Methods for its solution have been described. We demonstrate the versatility of these models, as they can be used to describe logistic evacuation processes, organize assembly points, intermediate locations, evacuation reception points, and those providing primary assistance to the affected population. We calculate the appropriate number of essential products and deliver them from existing warehouses through distribution centers to the affected areas.

Originality. As preventive measures to increase the level of population safety during an emergency, we consider the optimal placement of rescue facilities and the zoning of the territory to distribute evacuation traffic. We also address the problem of the optimal distribution of human flows in the transport and logistics system.

Practical value. The presented models, methods, and algorithms enable the solution of many practical problems related to the development of preventive measures and the planning of rescue operations to ensure the population's safety in case of emergencies. The theoretical results obtained are translated into specific recommendations that can be utilized when addressing logistical problems related to the organization of primary evacuation of the population from affected areas and their subsequent transportation to safer locations for further assistance.

Keywords: *humanitarian logistics, two-stage evacuation, territorial distribution, mathematical modeling*

Introduction. According to the results of emergency situations (ES) monitoring in 2019, as presented in the Reports of the State Emergency Service of Ukraine [1], more than half of them are natural and social ones, while the rest are technogenic ones. Whereas in 2016–2017, there was a tendency of decreasing the total number of ES, in 2019, compared to 2018, the total number of ES increased by 14.1 %. In 2021, compared to 2020, this number increased by 6.9 % [2].

Simultaneously, the number of technogenic ES increased by 12.8 % (resulting from fires and explosions, accidents in life support systems, and sudden building breakdowns), and the number of natural emergencies increased by 1.6 %. The reduction in the number of deaths due to natural disasters in 2016–2018 provides grounds for concluding that certain measures were effective in preventing and mitigating the consequences of emergencies.

One of the primary methods for safeguarding the population during a large-scale technogenic or natural emergency is evacuating them and relocating them to prepared safe areas outside the affected zones and the sources of the emergency. Evacuation planning is a crucial and intricate component of emergency management due to the high level of uncertainty and the involvement of numerous players and agencies. Naturally, the selection of specific measures is determined by evaluating all threats, the required speed of response, geographical location, and the infrastructure's specific characteristics in the potential disaster zone. The combination of all actions to evacuate the population can be either preventive or responsive in nature. If the possibility of extraordinary circumstances is anticipated but not certain, population evacuation is executed as a preventive measure. In the event of an actual emergency, people and objects must be evacuated as swiftly as possible.

To reduce the number and scale of disasters through prevention, early warning, and minimizing total losses in case of a specific emergency, it is necessary to determine the optimal number of territorially distributed structures in the civil defense system. These structures should be equipped with trained personnel, specialized machinery and equipment, medicines, and life support resources, among others [3]. Planning evacuation is a time-consuming process due to the objective nature of conditions formalization. It involves considering available resources and the multifaceted possibilities of their utilization.

Utilizing mathematical methods and models that accurately describe the emerging problems allows for a quantitative assessment, not only of the evacuation process itself but also of the associated costs. This approach aids in developing effective management solutions aimed at cost minimization, reduction in evacuation time, and other factors, considering the available resources. Therefore, the problem of the rational territorial distribution of elements within the civil protection system remains highly relevant.

Literature review. Studies on evacuation processes using mathematical programming models and methods are extensively documented in [4, 5]. Among them, there are location-allocation models for developing evacuation plans during hurricanes or earthquakes [6, 7], the problem of identifying optimal evacuation routes and shelters that arise in urban emergencies [8], as well as deliberate actions or natural disasters in confined spaces such as stadiums, museums, conference halls, or shopping centers [9].

One of the most significant directions in modern scientific research in the field of life safety is humanitarian logistics (HL). Its purpose is to study problems related to possible disasters or emergencies and to develop measures for mitigating these problems and managing the situation. In [10], a systematic review of scientific papers published between 2000 and 2020 is provided. In the review, issues of HL, challenges in the stages before and after a disaster, and potential human and economic losses are examined from various perspectives. Additionally, several mathematical models and algorithms are proposed to enhance the efficiency of logistics operations. The authors classify optimization problems into three groups based on the main problem under investigation: 1) object location and optimal coverage; 2) network models and transportation and distribution issues; 3) mass population evacuation, which, in turn, is studied from the perspectives of both governmental and private organizations.

Many models are inherently stochastic. In [11], a two-stage stochastic model for evacuation planning is discussed. The model is employed for the optimal placement of shelters and the assignment of evacuees to the nearest shelters while minimizing the expected total evacuation time. In [12], the authors propose an optimization model for the shelter location problem under conditions of uncertain demand. Using the central limit theorem, they constructed a nonlinear deterministic equivalent model, which was subsequently reformulated by approximating the nonlinear components with linear functions. Thus, a mixed-integer linear programming model with a maximin quality criterion was derived. This model can be solved using standard methods. Having obtained various solutions to optimization problems for different parameter combinations, the authors emphasize the importance of considering aspects such as housing utilization ratio, level of service provision, and demand.

In general, scientific research related to emergency management can be categorized based on the planning stage being considered.

During the preparation stage for a natural disaster, primary attention is directed towards planning actions for emergency situations. This includes reinforcing buildings and infrastructure [13, 14], establishing aid centers, and setting up evacuation shelters early on [15]. In [16], the stochastic problem of determining a set of shelter locations during the preparation stage for natural disasters was investigated. To address this problem, a genetic algorithm was proposed as a solution, particularly in cases with large dimensions.

Many of the existing emergency logistics models typically assume an evacuation process based on fixed and predetermined destinations from a strategic perspective. However, the unpredictable and turbulent nature of a disaster can disrupt these predictions. Furthermore, the primary objective in emergency situations is to move people out of the affected zone to a safe location, regardless of where that may be.

In [17], a mathematical model is presented that combines decisions on shelter locations with the maximum flow problem to choose safe destinations and maximize the number of people sent to them. In this paper, the authors develop a mixed-integer linear programming model for selecting one or more destinations in a capacitive network. The solution methods are based on existing algorithms for finding the maximum flow, incorporating heuristics that utilize the concept of adding a super receiver to the network for quick upper limit estimation. The problem of determining the destination is considered across five levels of natural disaster severity.

At the post-disaster stage, the focus shifts to the process of evacuating the population to protective structures or safe zones, distributing aid, and transporting the wounded to shelters [18]. The proposed mathematical models are either linear or involve problems of mixed-integer linear programming, or they are considered dynamic network flows [19, 17]. Optimization criteria include total time or evacuation distance, evacuee waiting time, or the cost of traffic flow [20].

To ensure timely evacuation from affected areas, decisions regarding shelter location and route destination should be considered simultaneously. In the context of emergency management, the problems of shelter location and evacuation routes have been investigated by numerous optimization researchers, both individually and collaboratively. Two-level programming, in which decisions about shelter locations and route assignments are provided in the upper and lower layers, respectively, is one of the most common approaches to solving these two problems separately at an early stage.

For example, in [21], a bilevel multiobjective optimization problem for determining the evacuation location is proposed. The model focuses on two categories of decision-makers – city planners and evacuees. It involves determining the optimal locations for placing shelters in such a way that the traffic distribution in the existing network is optimized for emergency routing. One of the optimization criteria expresses the monetary equivalent of housing construction costs, while the second describes the system of routes and represents the overall capacity of shelters. Thus, both the pre-disaster phase and the post-disaster phase are considered simultaneously.

In the upper-level decision-making, the simulated objectives include minimizing construction costs and maximizing the coverage of residential areas by minimizing the total transportation time in the evacuation process. At the lower level, the goal is to minimize individual evacuation time in the transport network while maintaining balance conditions in the system, preventing evacuees from further reducing their efforts to move. The authors claim that the proposed model can be effectively used to make decisions during the pre-crisis evacuation planning stages, but the study has several limitations. For example, background traffic in the network is not taken into account. Additionally, the nature of the disaster is considered generic, although the type of emergency may impact evacuee behavior.

The paper [22] presents a two-stage mathematical model for the improvement of post-earthquake conditions. In the first stage, they investigate the locations of shelters for the initial placement of people, the placement of first aid kits, as well as the distances that people travel from crisis areas to shelters in the event of an earthquake. In the second stage, the reduction and coverage of needs after people are placed in shelters are studied.

Emergency evacuation is typically dynamic, as the intensity and consequences of an emergency may change over time. Furthermore, traffic behavior during emergency situations can be unpredictable, leading to the necessity to manage dynamic situations such as panic, roadblocks, or a failure to respond to an evacuation warning, among others. Planning alternative evacuation strategies in advance is crucial.

In [23], they investigated the problem of dynamic route planning during evacuation in a confined space, specifically addressing the multi-objective dynamic planning of a route network. The authors modeled evacuation from multiple sources to various locations within a limited space, aiming to minimize overall delay and maximize evacuation efficiency. The problem was examined in 3D scenarios, offering an intuitive visualization of the geographical space and contributing to the development and implementation of an evacuation plan. Utilizing auxiliary graph transformation, they proposed a heuristic algorithm that relates to the classical problem of minimum weighted set covering.

Mass emergency evacuation is inherently a complex process that can sometimes lead to chaotic situations and unforeseen consequences. In many emergency scenarios, mass evacuation becomes necessary to address serious public threats within tight space-time constraints. To gain a better understanding of complex phenomena like mass evacuation and to analyze potential outcomes, agent-based models simulating individual behavior have been developed. However, their implementation, especially when applied to large geographical areas or complex behavior patterns, poses significant computational challenges.

The primary strategy for addressing such computational tasks involves dividing transport networks into smaller regions and reducing associated computational costs through the utilization of modern cyberinfrastructure and cyberGIS. In [24], a new algorithm for network division was developed to enhance the scalability of agent-based simulations for mass evacuation. This algorithm is founded on modern computing infrastructure employing CyberGIS, which models spatial movement during an emergency evacuation.

Specifically, the algorithm, referred to as Voronoi clustering algorithm based on target shift (ViCTS), is constructed using Voronoi network diagrams. It is designed to tackle computational scalability issues arising from the unique characteristics of evacuation traffic. As noted by the authors, results from computational experiments demonstrate that ViCTS outperforms known network distribution algorithms when modeling microscopic traffic. This improvement is achieved by balancing computing loads and reducing data exchange between high-performance parallel computing resources.

Models were developed to determine the optimal spatial distribution of emergency evacuation centers, such as schools, colleges, hospitals, and fire stations, to enhance flood emergency planning in the Sylhet region of northeastern Bangladesh [25]. In the first step, flood susceptibility maps were generated using machine learning models, including the Levenberg-Marquardt Neural Network (LM-NN), Neural Network and Decision Trees (DT), and the Multi-Criteria Decision-Making Method (MCDM). Mathematical approaches in GIS were proposed for addressing four well-known problems that impact emergency rescue time: maximum coverage, maximum attendance, p-median location, and set coverage.

In contrast to most of the considered problems, which are typically formulated as integer linear programming problems, the proposed mathematical model for the problem of placing the emergency evacuation system's subdivisions in case of an emergency has a continuous nature. The formulation of such multi-stage transport and logistics problems is suitable when the population is very large, and residents continuously occupy the territory.

The purpose of research. Utilizing mathematical methods and models that accurately describe the problems that arise allows for a quantitative assessment, not only of the characteristics of the evacuation process itself but also of the associated costs. This approach facilitates the development of effective management solutions aimed at minimizing costs, evacuation time, and other factors, considering the available resources. Therefore, the issue of the rational territorial distribution of elements within the civil protection system remains undoubtedly relevant.

Hence, *the research's purpose* is to enhance the level of population safety during emergencies by developing and substantiating preventive measures through the optimal placement of rescue facilities and territorial zoning to manage evacuation traffic efficiently. Mathematical modeling is conducted with the utilization of the theory and methods for solving two-stage problems of optimal set partitioning as described in [26, 27].

The object of the research is two-stage evacuation logistic processes that occur while assisting the population in areas affected by emergency situations of a natural or technogenic nature.

The subject of the research is mathematical models for solving problems related to the placement of emergency evacuation system subdivisions in case of an emergency.

To achieve this goal, the following tasks need to be addressed:
- analyzing the evacuation logistic processes using a systems approach;
- developing a mathematical model for multi-stage transport and logistics problems;
- demonstrating the feasibility of using the proposed approach for the development of preventive measures and the planning of rescue operations to ensure the safety of the population in case of emergencies through solving model problems.

Materials and methods. Imagine an emergency developing (or potentially developing) in a specific territory, a situation that could endanger people's health. The task at hand is to determine suitable safe locations for establishing primary evacuation centers (collection points) for the residents of the affected area. These centers serve as staging points for their subsequent transportation to designated population reception centers, such as hospitals or shelters, in the shortest possible time. Simultaneously, it is desirable to divide the affected territory into zones and allocate each zone to the appropriate collection point to ensure the swift transportation of evacuated residents from hazardous areas. Thus, the evacuation of the population is planned to occur in two stages: the first stage involves moving individuals from the affected zone to the corresponding primary collection point, and the second stage entails transporting them from these locations to specially designated emergency aid centers (refer to Fig. 1). Of course, the

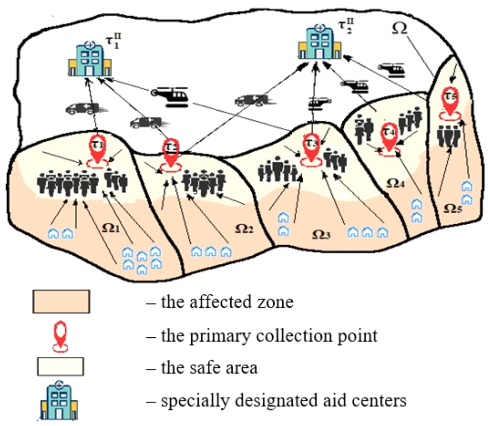


Fig. 1. Scheme of the two-stage evacuation of residents from area Ω affected by the natural disaster

allocation of the affected population to centers should take into account the capacity of these centers.

Mathematical model. We will use the following notations: Ω is the territory of some region that is (maybe) damaged as a result of an emergency, m^2; $\widehat{\Omega} \subseteq \Omega$ is a safe area where centers of the first stage (primary collection points for the affected population) can be accommodated, m^2; $\rho(x)$ is a function that describes the distribution of residents at point x of the set Ω, people/m^2; N is the number of centers of the first stage; M is the number of centers of the second stage; S is total population in the given territory, people; τ_i^r are coordinates of the i^{th} center of the r^{th} stage; b_i^r is a capacity of the i^{th} center of the r^{th} stage, $r = \mathrm{I}, \mathrm{II}$, people; $c_i^{\mathrm{I}}(x, \tau_i^{\mathrm{I}})$ is evacuation time of a resident from point $x \in \Omega$ to the center τ_i^{I}, which will be considered proportional to the distance between two points, hour/person; $c_{ij}^{\mathrm{II}}(\tau_i^{\mathrm{I}}, \tau_j^{\mathrm{II}})$ is a cost of transporting of an evacuee from center τ_i^{I} to center τ_j^{II}, UAH/person; a_i^{I} is a cost of setting up the primary population collection point at τ_i^{I}, calculated for one evacuated person, UAH/person; a_j^{II} – is fixed organizational cost of center τ_j^{II}, calculated for one person, UAH/person; ν_{ij} is the number of evacuees transported from the first-stage center τ_i^{I} to the second-stage center τ_j^{II}, people, $i = \overline{1, N}$; $j = \overline{1, M}$;

$$\Sigma_\Omega^N = \left\{ \overline{\omega} = \{\Omega_1, \ldots, \Omega_N\} : \bigcup_{i=1}^{N} \Omega_i = \Omega, \Omega_i \cap \Omega_j = \varnothing, i \neq j; i, j = \overline{1, N} \right\}$$

is a class of all possible partitions of set Ω into N subsets; $\overline{\omega} = \{\Omega_1, \Omega_2, \ldots, \Omega_N\}$ is an element of class Σ_Ω^N.

We need to find such a partition of set Ω into N subsets $\overline{\omega} = \{\Omega_1, \Omega_2, \ldots, \Omega_N\}$ (some of them may be empty), determine the placement coordinates of the subsets' centers $\tau_1^{\mathrm{I}}, \ldots, \tau_N^{\mathrm{I}}$ on $\widehat{\Omega}$ and such transportation volumes $\nu_{11}, \ldots, \nu_{NM}$, that provide

$$\min_{\overline{\omega} \in \Sigma_\Omega^N,\ \nu \in R_{NM}^+, \tau^{\mathrm{I}} \in \widehat{\Omega}^N} F(\overline{\omega}, \tau^{\mathrm{I}}, \nu), \tag{1}$$

where

$$F(\overline{\omega}, \tau^{\mathrm{I}}, \nu) = \beta_1 \sum_{i=1}^{N} \int_{\Omega_i} (c_i^{\mathrm{I}}(x, \tau_i) + a_i^{\mathrm{I}}) \rho(x) dx + \\ + \beta_2 \sum_{i=1}^{N} \sum_{j=1}^{M} (c_{ij}^{\mathrm{II}}(\tau_i^{\mathrm{I}}, \tau_j^{\mathrm{II}}) + a_j^{\mathrm{II}}) \nu_{ij}$$

and the following constraints are met

$$\int_{\Omega_i} \rho(x) dx = \sum_{j=1}^{M} \nu_{ij}, \quad i = \overline{1, N}; \tag{2}$$

$$\sum_{i=1}^{N} \nu_{ij} = b_j^{\mathrm{II}}, \quad j = \overline{1, M}, \tag{3}$$

where $\beta_1, \beta_2 \geq 0, \beta_1^2 + \beta_2^2 \neq 0$ are given coefficients that determine the priority of terms and take into account their normalization and non-dimensionality.

When solving the problem (1–3) we can estimate the capacity of the first-stage centers (the number of people that this center can accommodate and ensure their further transportation). Let us denote these values $b_i^{\mathrm{I}}, i = \overline{1, N}$. If $\overline{\omega}^* = \{\Omega_1^*, \Omega_2^*, \ldots, \Omega_N^*\}$ is an optimal partition of the problem, then $b_i^{\mathrm{I}} = \int_{\Omega_i^*} \rho(x) dx, i = \overline{1, N}$.

Universality of the model (1–3). One of the tasks of civil defense and the Unified State System of Prevention and Elimination of Emergencies is the primary life support of the population affected by an emergency situation (ES) of a natural or technogenic nature. Life support of the population (LS) during an ES encompasses a set of activities aimed at creating and maintaining conditions that are minimally necessary to save lives and support the health of people in emergency zones, along their evacuation routes, and in places where evacuees are accommodated. These conditions adhere to norms and standards established for emergency situations, developed and approved through the established procedure (Code of Civil Protection of Ukraine, dated 02.10.2012 No. 5403-VI, Regulations on the Unified State System of Civil Protection, PKMU dated 09.01.2014 No. 11). Emergency life support activities encompass various aspects, including medical care, the provision of water, food, housing, communal services, essential supplies, transportation, as well as psychological and informational support.

To ensure the protection of the population, the environment, and economic assets from emergencies of natural and technogenic origins, a state reserve of material and technical resources, food, medical supplies, and other necessities is pre-established.

The development of action plans for first-priority life support during an emergency is the responsibility of regional and territorial management authorities and should be carried out daily. These plans must take into consideration forecasts of potential emergency situations in the region, such as natural disasters and accidents (as per the confirmation of the Plan for responding to situations of national-level significance, PKMU dated March 14, 2018, No. 223). The unpredictability of emergencies, particularly earthquakes and technogenic catastrophes, the absence of methods and tools for short-term forecasting of their occurrence time, the extensive affected areas, and the likelihood of mass population losses necessitate a high level of preparedness of regional services to mitigate their consequences and organize first-priority life support (Regulations on the Unified State System of Civil Protection, PKMU dated 01/09/2014, No. 11).

The proposed two-stage placement-distribution model (equations 1 to 3) is versatile, as it can be applied to mathematically describe the optimal location of resource warehouses, which serve as points of concentration for material goods within emergency zones, storage facilities for emergency supplies, and distribution centers for essential items and personal protective equipment stations (Fig. 2). In this scenario, the resource flow follows a different path, from second-stage centers (state reserves, warehouses) to first-stage centers (personal protective equipment and essential items stations), and from there, it reaches the residents of affected territories, even those in the most remote areas, in the shortest possible time. Conditions (2) and (3) set the capacity limitations for first- and second-stage centers while allowing for the potential satisfaction (to a certain extent) of the needs of the entire population within the affected area.

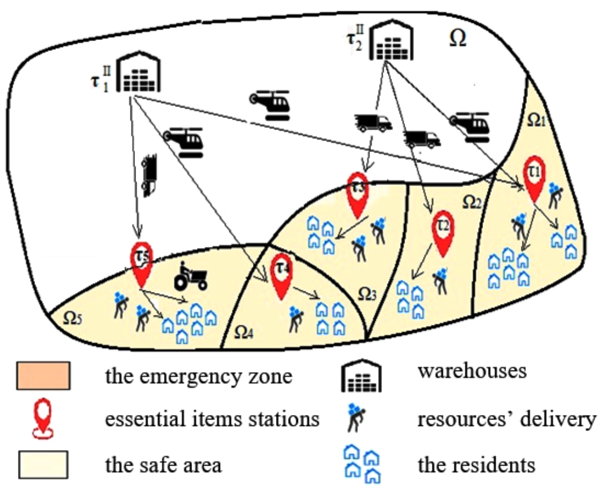


Fig. 2. Scheme of two-stage distribution of material and technical resources in emergency zones

Considering the versatility of the model (equations 1 to 3), we will proceed with the assumption that in the subsequent theoretical descriptions of specific cases of the problems formulated above and similar practical scenarios, the region where the population is distributed will be referred to as a set; intermediate or reception stations – first-stage centers; areas for collection (service)of the population, which correspond to τ_i^{I}, – subsets; the capacity of the centers of any stage – their capacity, and the values of the flow of residents (or products) $v_{11}, \dots, v_{NM}$ – the transportation volume. In addition, when describing practical problems, we will consider the words "territory", "region", and "area" as synonyms.

By specifying the initial conditions, it is possible to derive various distinct variations of the problem (equations 1 to 3). The presence or absence of additional constraints on the centers' capacity or locations is determined by various formulations [26], such as: the problem of optimal partitioning of sets with additional connections (OPSAC) with fixed centers and no restrictions on the capacity of the first-stage centers; the OPSAC problem with fixed centers and constraints on the capacity of the first-stage centers; the OPSAC problem involving the placement of centers and constraints on the capacity of the first-stage centers, among others.

We will explore the method for solving the problem (equations 1 to 3) in two variants – one with the placement of the first-stage centers on a continuous set and the other with their fixed coordinates.

Methodology. First, we will focus on a specific case of the problem (equations 1 to 3), which involves fixed first-stage centers. In this scenario, the centers are predetermined, and their capacities are known in advance. In this case, condition (2) should be replaced with the following constraints

$$\int_{\Omega_i} \rho(x)dx = b_i^{\mathrm{I}}, i = \overline{1,N}; \tag{4}$$

$$\sum_{j=1}^{M} \nu_{ij} = b_i^{\mathrm{I}}, i = \overline{1,N}. \tag{5}$$

In the context of the population evacuation problem, constraints (4) consider that the total number of residents in the zone corresponding to the i^{th} center of the first stage equals its capacity. Condition (5) implies that the number of residents evacuated from the i^{th} first-stage center equals the capacity of that center. According to conditions (3), the j^{th} second-stage center can accommodate the entire population evacuated and transported to that center. Each intermediate evacuation center has its own service zone, which does not intersect with others and covers the entire territory affected by the emergency.

A necessary and sufficient condition for solving problem (1, 3–5) is the following dual equality [26]

$$\sum_{i=1}^{N} b_i^{\mathrm{I}} = \int_{\Omega} \rho(x)dx = \sum_{j=1}^{M} b_j^{\mathrm{II}}. \tag{6}$$

The separability of the target function and the fulfillment of conditions allow narrowing the solution of the original problem (1, 3–5) down to the sequential solution of two problems.

Problem A. (optimal partitioning of a set with fixed centers and constraints in form of equations)

$$F_1(\overline{\omega}) \to \min, \overline{\omega} \in \Sigma_{\Omega}^{N}, \text{ subject to (4);}$$

$$F_1(\overline{\omega}) = \sum_{i=1}^{N} \int_{\Omega_i} (c_i^{\mathrm{I}}(x,\tau_i) + a_i^{\mathrm{I}})\rho(x)dx.$$

Problem B. (linear programming of the transport type) subject to (3–5)

$$F_2(\nu) \to \min, \nu \in R_{NM}^{+};$$

$$F_2(\nu) = \sum_{i=1}^{N}\sum_{j=1}^{M} (c_{ij}^{\mathrm{II}}(\tau_i^{\mathrm{I}},\tau_j^{\mathrm{II}}) + a_j^{\mathrm{II}})\nu_{ij}.$$

Problem A can be solved using the appropriate method from [28]. To solve problem B, the potential method, for example, can be applied. Condition (6) ensures the possibility of solving both the transport and the OSP problems.

Next, we will consider problem (1–3) with unknown centers of the first stage without restrictions on their capacity under the assumption that the following condition is met

$$\int_{\Omega} \rho(x)dx = \sum_{j=1}^{M} b_j^{\mathrm{II}}.$$

Let us introduce the vector function $\lambda(\cdot)$, the components of which are the characteristic functions of the subsets forming the partition of Ω

$$\lambda_i(x) = \begin{cases} 1, x \in \Omega_i \\ 0, x \in \Omega \backslash \Omega_i, i = \overline{1,N} \end{cases}.$$

Now, we can rewrite problem (1–3) as following

Problem C.

$$I_1(\lambda(\cdot),\tau^{I},\nu) \to \min_{(\lambda(\cdot),\tau^{\mathrm{I}},\nu)\in\Gamma\times\hat{\Omega}^{N}\times R_{NM}^{+}};$$

$$I_1(\lambda(\cdot),\tau^{\mathrm{I}},\nu) = \beta_1 \sum_{i=1}^{N}\int_{\Omega} (c_i^{\mathrm{I}}(x,\tau_i) + a_i^{\mathrm{I}})\rho(x)\lambda_i(x)dx +$$
$$+ \beta_2 \sum_{i=1}^{N}\sum_{j=1}^{M} (c_{ij}^{\mathrm{II}}(\tau_i^{\mathrm{I}},\tau_j^{\mathrm{II}}) + a_j^{\mathrm{II}})\nu_{ij},$$

subject to

$$\sum_{i=1}^{N} \nu_{ij} = b_j^{\mathrm{II}}, \quad j = \overline{1,M};$$

$$\sum_{i=1}^{N} \nu_{ij} = \int \rho(x)\lambda_i(x)dx, \quad i = \overline{1,N};$$

$$\Gamma = \{\lambda(\cdot) = (\lambda_1(\cdot),\dots,\lambda_N(\cdot)) : \lambda_i(x) = 0 \vee 1;$$
$$i = \overline{1,N}, \quad \sum_{i=1}^{N} \lambda_i(x) = 1 \quad a.e. \text{ for } x \in \Omega\},$$

and then relax the vector function $\lambda(\cdot)$, assuming that its components can be changed to range from 0 to 1. Applying the apparatus of duality theory via necessary and sufficient conditions of the existence of the saddle point of Lagrange functional we can obtain the solution $\{\lambda^*(\cdot), \tau^{\mathrm{I}*}, \nu^*\}$ of the relaxed problem C in the following form: when function $\rho(x) \geq 0$ almost everywhere for $x \in \Omega$, and condition (6) is met for $i = \overline{1,N}$ and almost all $x \in \Omega$

$$\lambda_i^*(x) = \begin{cases} 1, \beta_1(c_i^{\mathrm{I}}(x,\tau_i^{\mathrm{I}*}) + a_i^{\mathrm{I}}) + \psi_i^* \leq \beta_1(c_k^{\mathrm{I}}(x,\tau_k^{\mathrm{I}*}) + a_k^{\mathrm{I}}) + \psi_k^* \\ \quad i \neq k, e.a. \text{ for } x \in \Omega, k = \overline{1,N}, \text{ then } x \in \Omega_i^*, \\ 0 \quad \text{otherwise} \end{cases}$$

where

$\tau_i^{\mathrm{I}*}$, $i = \overline{1,N}$, $\psi_1^*, \dots, \psi_N^*$ and $\eta_1^*, \dots, \eta_M^*$ is the optimal solution of the next problem

$$G(\psi, \eta) \to \max, \quad \psi \in R^N, \quad \eta \in R^M; \tag{7}$$

$$G(\psi,\eta) = \min_{\{\tau^{\mathrm{I}},\nu\}\in\hat{\Omega}^{N}\times R_{NM}^{+}} P(\{\tau^{\mathrm{I}},\nu\},\psi,\eta);$$

$$P(\{\tau^{\mathrm{I}},\nu\},\psi,\eta) = \int_{\Omega} \underline{\min}_{k=\overline{1,N}} (\beta_1(c_k^{\mathrm{I}}(x,\tau_k^{\mathrm{I}}) + a_k^{\mathrm{I}}) + \psi_k)\rho(x)dx +$$
$$+ \sum_{i=1}^{N}\sum_{j=1}^{M} (\beta_2(c_j^{\mathrm{II}}(\tau_i^{\mathrm{I}},\tau_j^{\mathrm{II}}) + a_j^{\mathrm{II}}) - \eta_j - \psi_i)\nu_{ij} + \sum_{j=1}^{M} \eta_j b_j^{\mathrm{II}};$$

$$\beta_2(c_j^{\mathrm{II}}(\tau_i^{\mathrm{I}},\tau_j^{\mathrm{II}}) + a_j^{\mathrm{II}}) - \eta_j - \psi_i \geq 0, \quad i = \overline{1,N}, \quad j = \overline{1,M}. \tag{8}$$

The conditions (8) for $\tau_i^{\mathrm{I}*}$, ν_{ij}^*, $i = \overline{1,N}$, $j = \overline{1,M}$, $\psi_1^*, \dots, \psi_N^*$ and $\eta_1^*, \dots, \eta_M^*$ can be rewritten as following

$$\beta_2(c_j^{II}(\tau_i^{I*},\tau_j^{II})+a_j^{II})-\eta_j^*-\psi_i^*>0 \quad \text{if } \nu_{ij}^*=0;$$

$$\beta_2(c_j^{II}(\tau_i^{I*},\tau_j^{II})+a_j^{II})-\eta_j^*-\psi_i^*=0 \quad \text{if } \nu_{ij}^*>0, i=\overline{1,N}, j=\overline{1,M}.$$

Using them we can rewrite the function $P(\{\tau^I, \nu\}, \psi, \eta)$ in the following way

$$\breve{P}(\tau^I,\psi)=\int_\Omega \min_{k=\overline{1,N}}(\beta_1(c_k^I(x,\tau_k^I)+a_k^I)+\psi_k)\rho(x)dx+$$
$$+\sum_{j=1}^M b_j \max_{i=\overline{1,N}}(\psi_i-\beta_2(c_j^{II}(\tau_i^I,\tau_j^{II})+a_j^{II})).$$

Iterative algorithms for solving emergency logistics problems. Algorithm 1 (for the problem (1–3) with fixed first-stage centers).

Initialization. We will specify the number of first-stage and second-stage centers N and M, its placement τ_i^I,τ_j^{II}, the values a_i^I,a_j^{II},b_j^{II} $i=\overline{1,N}$, $j=\overline{1,M}$, the constants of priority β_1, $\beta_2\geq 0$, the function $\rho(x)\geq 0$ for $x\in\Omega$; $\varepsilon>0$. Region Ω is covered with a rectangular grid $\bar{\Omega}$. We will specify the initial vector-function $\lambda^{(0)}(x), \forall x\in\bar{\Omega}$.

1. We calculate the value of the $b_i^{I(0)}$ by the following formula

$$b_i^{I(0)}=\int_\Omega \rho(x)\lambda_i^{(0)}(x)dx, \quad i=\overline{1,N}.$$

2. Calculate $\nu_{ij}^{(0)}$, $i=\overline{1,N}, j=\overline{1,M}$, and potentials $\psi_i^{(0)}$, $i=\overline{1,N}$, and $\eta_j^{(0)}, j=\overline{1,M}$, by solving the next transport problem

$$\sum_{i=1}^N\sum_{j=1}^M(c_{ij}^{II}(\tau_i^I,\tau_j^{II})+a_j^{II})\nu_{ij}\to\min; \tag{9}$$

$$\sum_{j=1}^M \nu_{ij}=b_i^I, \quad i=\overline{1,N}; \tag{10}$$

$$\sum_{i=1}^N \nu_{ij}=b_j^{II}, \quad j=\overline{1,M}; \tag{11}$$

$$\nu_{ij}\geq 0, \quad i=\overline{1,N}, \quad j=\overline{1,M}, \tag{12}$$

with $b_i^I=b_i^{I(0)}, i=\overline{1,N}$.

We calculate the value of the objective function $I(\lambda(\cdot), \tau^I, \nu)$ according to the given centers by the formula and $\lambda(\cdot)=\lambda^{(0)}(\cdot)$, $v=v^{(0)}$.

After k = 1, 2, 3,... steps we got some vector-functions $\lambda_i^{k+1}(x)$ $i=\overline{1,N}$; the values $b_i^{I(k)},\psi_i^{(k)}$ $i=\overline{1,N}$ and $\eta_j^{(k)}, j=\overline{1,M}$ as a result of the algorithm. Let us describe the $(k+1)^{th}$ step of the algorithm.

The $(m+1)^{th}$ step.

1. Calculate $\lambda_i^{k+1}(x)$ by the formula

$$\lambda_i^{(k+1)}(x)=\begin{cases}1, & \beta_1(c_i^I(x,\tau_i^I)+a_i^I)+\psi_i^{(k+1)}= \\ & =\min\limits_{s=\overline{1,N}}(\beta_1(c_s^I(x,\tau_s^I)+a_s^I)+\psi_s^{(k+1)}). \\ 0 & \text{otherwise}, \quad i=\overline{1,N}, \forall x\in\Omega\end{cases}$$

2. Calculate the values $b_i^{I(k+1)}$ with the formula

$$b_i^{I(k+1)}=\int_\Omega \rho(x)\lambda_i^{(k+1)}(x)dx, \quad i=\overline{1,N}.$$

3. Find the values $\nu_{ij}^{(k+1)}$, $i=\overline{1,N}$, $j=\overline{1,M}$, $\psi_i^{(k+1)}, i=\overline{1,N}$, and $\eta_j^{(k+1)}, j=\overline{1,M}$, solving the problem (9–12) with $b_i^I=b_i^{I(k+1)}$, $i=\overline{1,N}$.

4. Calculate the value of the objective function $I(\lambda(\cdot),\tau^I,\nu)$ with $\lambda_i(x)=\lambda_i^{(k+1)}(x)$, $\nu_{ij}=\nu_{ij}^{(k+1)}$, $i=\overline{1,N}$, $j=\overline{1,M}$ (τ^I – the fixed vector).

5. If a condition

$$\left|I(\lambda^{(k+1)}(\cdot),\tau^I,\nu^{(k+1)})-I(\lambda^{(k)}(\cdot),\tau^I,\nu^{(k)})\right|\leq\varepsilon \tag{13}$$

is not satisfied, we proceed to $(k+2)^{th}$ step of the algorithm, otherwise go to step 6.

6. The completion of the iterative process. The best values are $\lambda_i^*(x)=\lambda_i^{(m)}(x)$, $\nu_{ij}^*=\nu_{ij}^{(m)}$, $i=\overline{1,N}$, $j=\overline{1,M}$, $\psi_i^*=\psi_i^{(m)}$, $i=\overline{1,N}$, and $\eta_j^*=\eta_j^{(m)}, j=\overline{1,M}$, where m is the iteration number at which condition (13) is performed.

7. Calculate the optimal value of the objective function $I(\lambda^*(\cdot), \tau^I, \nu^*)$. We visualize the partition of the set Ω of the correspondingly found optimal values of the components of the vector-function $\lambda^*(\cdot)$.

The end of the algorithm.

The error of the algorithm for solving approximately the problem (equations 1 to 3) with fixed centers consists of the absolute error of the algorithm (which depends on the accuracy of calculating $I_1(\lambda(\cdot), \tau^I, \nu)$), the error arising as a result of the approximate calculation of the integrals when evaluating the center's capacity, rounding errors, and data inaccuracies.

Algorithm 2 below is designed for solving the problem (equations 1 to 3) when the first-stage centers are not given, and we need to find their optimal location. The algorithm utilizes algorithm 1 in conjunction with the method of non-differentiated optimization.

Let us rewrite problem (7) in the following form

$$\max_{\psi\in R^N,\,\eta\in R^M} G(\psi,\eta)=\min_{\tau^I\in\Omega^N}\min_{\nu\in R_{NM}^+}\max_{\psi\in R^N,\,\eta\in R^M} G_1(\{\tau^I,\nu\},\psi,\eta)=$$
$$=\min_{\tau^I\in\Omega^N} Q(\tau^I),$$

where

$$Q(\tau^I)=\min_{\nu\in R_{NM}^+}\max_{\psi\in R^N,\,\eta\in R^M} G_1(\{\tau^I,\nu\},\psi,\eta).$$

At each fixed vector $\tau^I\in\widehat{\Omega}^N$ we obtain the optimal value of the dual function $Q(\tau^I)$ constructed for the continuous problem of optimal set partitioning with the additional connections with given centers formulated in terms of characteristic functions of subsets. Therefore, Algorithm 1 is the structural part of the algorithm below. And to find optimal coordinates of the first stage centers we use the r-algorithm. The one we consider with the constant coefficient of space expansion and adaptive method to regulate step factor [28].

Let the set $\widehat{\Omega}$, where we locate the centers, be rectangular. If it were not, we could put area $\widehat{\Omega}$ within rectangular parallelepiped Π, whose sides are parallel to the Cartesian axes; assume that $\rho(x)=0$, if $x\in\Pi/\widehat{\Omega}$.

Algorithm 2. *Initialization*. Cover area Ω with the rectangular grid $\bar{\Omega}$ and specify the number of first-stage and second-stage centers N and M, the coordinates of placement τ_j^{II}, the values a_i^I,a_j^{II},b_j^{II} $i=\overline{1,N}$, $j=\overline{1,M}$, the constants of priority β_1, $\beta_2\geq 0$; $\varepsilon>0$ the function $\rho(x)\geq 0$ for $x\in\bar{\Omega}$. Specify the initial approximation of vector of coordinates τ_i^I, $i=\overline{1,N}$.

Put $k=0$.

Calculate the values of vector-function $\lambda^{(0)}(x)$ at the nodes of grid $x\in\bar{\Omega}$, and corresponding values $\nu_{ij}^{(0)}$, $i=\overline{1,N}, j=\overline{1,M}$ using the algorithm 1.

Assume that as the calculation result after k, k = 1, 2,... algorithm steps, the coordinates of $\tau^{I(k)}$ are obtained.

Describe the $(k+1)^{th}$ step.

1. Using algorithm 1 calculate values $\lambda^{(k)}(x), \forall x\in\bar{\Omega}$ in terms of the grid nodes, and current values $\nu_{ij}^{(k)}$, $i=\overline{1,N}$, $j=\overline{1,M}$.

2. Realize the current iteration of the $r(\alpha)$-algorithm and define the vector t_q$^{I(k+1)}$.

3. Project t_q$^{I(l+1)}$ on Π^N and obtain the next approximation of $\tau^{I(k+1)}$.

4. If none of the conditions

$$|Q^{(k+1)} - Q^{(k)}| < \varepsilon \quad \text{or} \quad \tau^{I(k+1)} - \tau^{I(k)} < \varepsilon, \qquad (14)$$

is met, then move on to $(k+2)^{th}$ step of the algorithm; in other case, move on to point 5.

5. Assume that $\tau^{I*} = \tau^{I(m)}$, $\lambda_i^*(\cdot) = \lambda_i^{(m)}(\cdot)$, $\nu_{ij}^* = \nu_{ij}^{(m)}$, $i = \overline{1, N}$, $j = \overline{1, M}$, where m is the number of the iteration at which condition (14) is met.

6. Calculate values $b_i^{I*} = \int_\Omega \rho(x)\lambda_i^*(x)dx$, $i = \overline{1, N}$ by any cubature formula. Calculate the values of objective functional $I^* = I(\lambda^*(\cdot), \tau^{I*}, \nu^*)$. Visualize the optimal partition of the set $\bar{\Omega}$ according to the found optimal vector-function $\lambda_i^*(\cdot)$. End of the algorithm.

Algorithm 2 is described.

Analysis of the model problems' optimal solutions. The algorithms, denoted as Algorithms 1 and 2, have been implemented within the Visual Studio 2022 development environment using the C# programming language. As demonstrated in [26], the simultaneous redistribution of resources across all logistical stages plays a pivotal role in optimizing complex logistical processes. This is achieved by formulating the problems in the form of equations (1) to (3), thereby embracing a systemic approach to the analysis of such processes. Consequently, these formulations facilitate a reduction in transportation and associated costs.

In the following sections, we will elucidate the operation of these algorithms through the use of model problems.

The preparatory stage of processing electronic maps is to remove places that do not belong to the territory of the region from the map drawing using a graphic editor. Next, we introduce a rectangular coordinate system by choosing the origin and scale unit so that the region under consideration is completely contained in the rectangle $\Pi = \{(x_1, x_2)$: $0 \le x_1 \le 12$; $0 \le x_2 \le 12\}$. In all figures we present a colorful partition of area Ω, where each color corresponds to certain zone. In the numerical implementation of the algorithms, the given area is discretized with the steps $h_1 = h_2 = 0.02$. To calculate multiple integrals, we use the cubature trapezoid formula, the problem (16–19) is solved using the potential method. We set the following parameters and error values of the $r(\alpha)$-algorithm: $\alpha = 3$, $\beta = 0.9$, $\varepsilon = 0.0001$. In all the tasks below, it is assumed that the total capacity of the centers of the second stage is 1 conventional unit. And, therefore, in accordance with condition (9), the total number of the population covered in the territory of Ω by all the centers of the first stage is also equal to 1.

To compute the distance between two points $c_i^{I}(x, \tau_i)$ and $c_{ij}^{II}(\tau_i, \tau_j)$ we use Minkovsky metrics $c(x, y) = ((x_1 - y_1)^p + (x_2 - y_2)^p)^{1/p}$ with certain value of parameter p. The function $\rho(x) = 1$ for all points in the area Ω; $\beta_1 = \beta_2 = 0.5$.

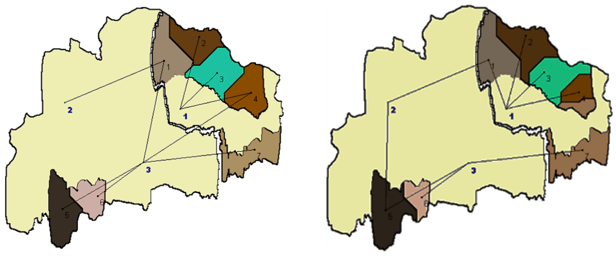
Problem 1. *The continuous problem OSPAC with fixed centers of first stage.*

The initial data: $N = 7$, $M = 3$; $\tau_1^{I} = (6.006; 3.454)$, $\tau_2^{I} = (7.392; 2.552)$, $\tau_3^{I} = (8.074; 3.916)$, $\tau_4^{I} = (9.372; 4.664)$, $\tau_5^{I} = (2.178; 9.064)$, $\tau_6^{I} = (3.52; 8.58)$, $\tau_7^{I} = (9.504; 6.82)$; $\tau_1^{II} = (6.67; 5.28)$, $\tau_2^{II} = (2.266; 5.038)$, $\tau_3^{II} = (5.258; 7.304)$; $b^{II} = (0.416; 0.156; 0.428)$.

Fig. 3 shows the optimal partition of the area Ω into seven zones and the connections between centers of the first and second stages in two cases:

a) when in $c_i^{I}(x, \tau_i)$ and $c_{ij}^{II}(\tau_i, \tau_j)$ $p = 1$ and $p = 2$ correspondingly, $a_i^{I} = 0, i = \overline{1, 7}$; $a_j^{II} = 0, j = \overline{1, 3}$;

b) $p = 1$ for $c_i^{I}(x, \tau_i)$ and $c_{ij}^{II}(\tau_i, \tau_j)$; $a^{I} = (1, 1, 2, 3, 1, 2, 3)$, $a_j^{II} = 0, j = \overline{1, 3}$.



a *b*

Fig. 3. Optimal set partition of the area Ω and the scheme of resource transportation between centers of the first and second stages

The capacities of the first-stage centers calculated with the accuracy 0.001 are the following:

a) $b^{I} = (0.158; 0.129; 0.155; 0.146; 0.16; 0.096; 0.153)$;

b) $b^{I} = (0.172; 0.168; 0.15; 0.063; 0.19; 0.067; 0.19)$.

The optimal costs are 14.973 and 36.004, respectively.

Clearly, considering the additional costs for constructing or organizing centers in the first stage not only increases the value of the target function but also affects the capacity of these centers. This capacity relates to the number of resources they can accommodate, which, in the context of an emergency evacuation problem, translates to the number of people gathered in these centers for organizing further movement. As shown in Fig. 3, not only do the zones of responsibility of the first-stage centers change (fixed territories can even become disconnected), but the connections between all centers also vary. The quantity of resources dispatched from the first-stage centers to the second-stage centers is provided in Table.

Problem 2. *The continuous problem OSPAC with locating the centers of first stage.*

We solved it in two variants.

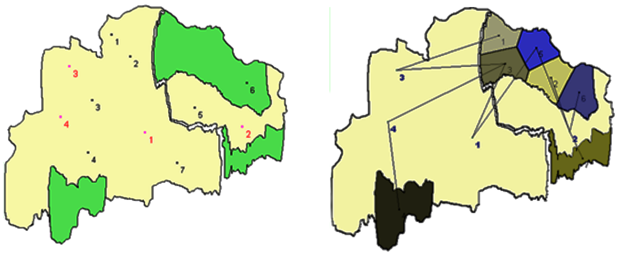
Problem 2.1. The initial data: $N = 7$, $M = 4$; $\tau_1^{II} = (5.302; 6.314)$, $\tau_2^{II} = (8.954; 6.094)$, $\tau_3^{II} = (2.464; 3.784)$, $\tau_4^{II} = (2.134; 5.72)$; $b^{II} = (0.123; 0.413; 0.192; 0.272)$; $a_i = 0, i = \overline{1, 4}$, $p^{I} = p^{II} = 2$. The initial placement of the centers of the first stage is presented in Fig. 4, *a*.

The optimal solution: $\tau_1^{I} = (6.297; 2.482)$, $\tau_2^{I} = (8.24; 4.064)$, $\tau_3^{I} = (6.527; 3.538)$, $\tau_4^{I} = (2.541; 9.016)$, $\tau_5^{I} = (7.689; 2.944)$, $\tau_6^{I} = (9.314; 4.156)$, $\tau_7^{I} = (9.433; 7.096)$; $\nu_{31} = 0.0164$, $\nu_{51} = 0.107$, $\nu_{22} = 0.122$, $\nu_{52} = 0.004$, $\nu_{62} = 0.135$, $\nu_{72} = 0.153$,

Table

The structure of connections between the centers of two stages in the problem 1

Under conditions a			Under conditions b		
The ID number of I stage's center i	The ID number of II stage's center j	The ID number of I stage's center i	The ID number of II stage's center j	The ID number of I stage's center i	The ID number of II stage's center j
2	1	0.1291	1	1	0.0347
3	1	0.1561	2	1	0.1682
4	1	0.1305	3	1	0.1498
1	2	0.1562	4	1	0.0627
1	3	0.0023	1	2	0.1375
4	3	0.0156	5	2	0.0182
5	3	0.1605	5	3	0.1714
6	3	0.0961	6	3	0.0669
7	3	0.1533	7	3	0.1902



a *b*

Fig. 4. Illustrations for the problem 2.1:

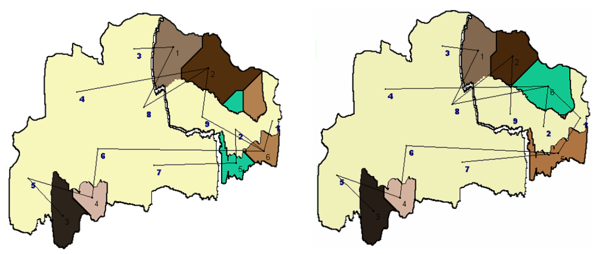
a – initial placement of the centers of the first (black) and second (red) stages and zoned area; b – optimal partition of the set and the scheme of resource transportation between the centers of the first and second stages

$\nu_{13} = 0.101$, $\nu_{33} = 0.091$, $\nu_{34} = 0.014$, $\nu_{44} = 0.2565$. The optimal partition and additional connections are shown in Fig. 4, *b*. The optimal value of the functional is 12.182. As seen in Fig. 4, even if you initially try to locate the centers of the first stage at arbitrary points of the region under consideration, placing them on the territory of the area being divided will still be the best.

Problem 2.2. Initial data: the region and the divided area Ω are the same; $N = 6$, $M = 9$; $\tau_1^{II} = (10.23; 5.61)$, $\tau_2^{II} = (8.8; 5.94)$, $\tau_3^{II} = (4.86; 2.82)$, $\tau_4^{II} = (2.64; 4.49)$, $\tau_5^{II} = (0.75; 7.83)$, $\tau_6^{II} = (3.45; 6.69)$, $\tau_7^{II} = (5.63; 7.33)$, $\tau_8^{II} = (5.24; 5.08)$, $\tau_9^{II} = (7.5; 5.46)$; $b^{II} = (0.056; 0.047; 0.163; 0.046; 0.191; 0.134; 0.073; 0.171; 0.114)$; $a^I = (0.9; 0.7; 0.3; 0.9; 0.5; 0.2)$, $a_i^{II} = 0, i = \overline{1,9}$, $p^{I,II} = 1$.

Fig. 5, *a* shows the initial location of the first-stage centers, the corresponding partition of Ω and the additional connections between centers of both stages obtained by solving the continuous OSPAC problem with fixed centers. The value of the functional (1) at these data is 26.786, and capacities of the centers of first stage are the following: $b^I = (0.212; 0.268; 0.162; 0.093; 0.107; 0.154)$.

When we solved the OSPAC problem with locating centers we obtained such components of the optimal solution: $\tau_1^I = (6.25; 2.98)$, $\tau_2^I = (7.58; 3.20)$, $\tau_3^I = (2.17; 9.21)$, $\tau_4^I = (3.21; 8.7)$, $\tau_5^I = (9.38; 6.97)$, $\tau_6^I = (8.97; 4.37)$; $\nu_{51} = 0.012$, $\nu_{61} = 0.044$, $\nu_{62} = 0.048$, $\nu_{13} = 0.163$, $\nu_{64} = 0.047$, $\nu_{35} = 0.165$, $\nu_{45} = 0.027$, $\nu_{46} = 0.065$, $\nu_{56} = 0.07$, $\nu_{57} = 0.074$, $\nu_{18} = 0.017$, $\nu_{28} = 0.07$, $\nu_{68} = 0.085$, $\nu_{29} = 0.114$. The amount of gathered resource by corresponding centers of the located centers we calculated with precision 0.001. It was the following: $b^I = (0.18; 0.184; 0.165; 0.092; 0.152; 0.224)$. Fig. 5, *b* demonstrates the optimal zoning of the territory and additional connections between centers. The functional equals to 23.639. As we can see, through the redistribution of some of the centers of the first stage and their connections with the centers of the second stage, it was possible to reduce the total costs by 12 %.



a *b*

Fig. 5. Illustrations for the problem 2.2:

a – solution of the OSPAC problem with fixed centers; b – the optimal placement of centers, the corresponding partition of the set and the transportation scheme

In the following examples, the area subject to zoning will be the one that is not shaded in Fig. 4, a. On light green we will place the centers of the second stage.

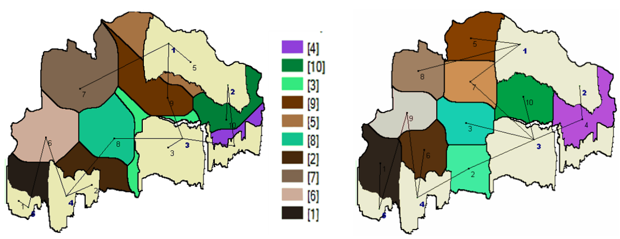
Problem 2.3. The initial data: $N = 10$, $M = 5$; $\tau_1^{II} = (6.69; 2.77)$, $\tau_2^{II} = (9.13; 4.48)$, $\tau_3^{II} = (7.26; 6.69)$, $\tau_4^{II} = (2.46; 9.06)$, $\tau_5^{II} = (0.9; 9.53)$; $b^{II} = (0.292; 0.093; 0.311; 0.104; 0.197)$; $a^I = \Theta$, $a^{II} = \Theta$, $p^I = 8$; $p^{II} = 2$. The initial location of the centers of the first stage, the zones corresponding to them and the scheme of additional connections with the centers of the second stage are shown in Fig. 6, *a*. On the right in the figure, the used color palette is shown. They indicate the numbers of the centers to which they are assigned. The value of the objective functional (1) with such data is 41.978.

When we solved problem (1–3) with the locating the centers of the first stage with the same initial data, we got the following optimal solution: $\tau_1^I = (0.95; 7.66)$, $\tau_2^I = (4.58; 7.87)$, $\tau_3^I = (4.46; 6.0)$, $\tau_4^I = (9.23; 5.85)$, $\tau_5^I = (4.67; 2.53)$, $\tau_6^I = (2.75; 7.09)$, $\tau_7^I = (4.65; 4.31)$, $\tau_8^I = (2.51; 3.87)$, $\tau_9^I = (2.05; 5.58)$, $\tau_{10}^I = (6.83; 4.92)$. The amount of resource collected by the corresponding centers of the first stage (with accuracy 0.001) amounted to: b^I = (0.107; 0.088; 0.099; 0.103; 0.1; 0.092; 0.099; 0.101; 0.099; 0.107). Then the resources are redistributed at the centers of second stage in such a way: $\nu_{51} = 0.101$, $\nu_{71} = 0.091$, $\nu_{81} = 0.101$, $\nu_{42} = 0.094$, $\nu_{23} = 0.086$, $\nu_{33} = 0.099$, $\nu_{43} = 0.01$, $\nu_{73} = 0.009$, $\nu_{10.3} = 0.107$, $\nu_{24} = 0.002$, $\nu_{64} = 0.092$, $\nu_{94} = 0.01$, $\nu_{15} = 0.108$, $\nu_{95} = 0.09$. The optimal partition and additional connections are shown in Fig. 6, *b*. The value of the functional is 28.058. In this problem, through the optimal locating centers of the first stage and the almost uniform distribution of the resource between these centers (as indicated by the composition of the resulting vector b^I) it was possible to reduce the objective functional value by 33 %.

Discussion. In practice, to enhance the road network and search for the new shortest path between the two centers of the first and second stages, it is advisable to utilize the Google Maps Distance Matrix API library. To reduce the number of API calls and, consequently, the computing resources required, we can address problems (1) to (3) in two stages. Initially, we locate the centers of the first stage using established theoretical metrics for distance functions. Subsequently, by employing GIS, we calculate the actual distance between the identified centers and points within the region, thereby determining the optimal partition of the given region and the cost of transporting resources between the first and second stage centers.

Integrating the developed approach to enhance the efficiency of the humanitarian logistics system with the methodology for assessing the country's sustainable development in the context of global threats, as developed in [29], is advisable. Furthermore, there is interest in generalizing models of two-stage humanitarian logistics problems when using multimodal transport terminals [30], as well as in allocating specific territories to two or three centers.

Conclusion. So, the paper is related to mathematical modeling and methods for solving problems in emergency and human-



a *b*

Fig. 6. Optimal partitioning of the area Ω and the scheme of resource transportation between the centers of the first and second stages in the problem 2.3

itarian logistics. We have defined the problem of optimally distributing human flows within the transport and logistics system. The structural elements of such a system are primary aid centers that gather residents from affected areas (first-stage centers) and specialized emergency assistance units that provide further services to the evacuated population (second-stage centers). The optimality criterion is to maximize the coverage of affected territories and minimize the evacuation time for victims.

The proposed mathematical model for the described problem is a continuous problem of optimal set partitioning with the placement of subset centers and additional connections. We have detailed the method and the iterative algorithms for solving this problem. The results of computational experiments confirm the validity of formulating a continuous problem of optimal set partitioning with additional connections, especially when determining the locations of new objects within a territory while considering multi-stage evacuation processes. Additionally, the proposed model can be applied to study issues related to the optimal placement of warehouses for material and technical resources, food, and medical supplies (locations for the concentration of material resources and resources within the emergency zone, establishment of supply and distribution points for essential items, personal protective equipment, etc.). Through modeled scenarios, we have demonstrated the potential for reducing costs associated with the management of material, human, and other flows across the entire logistics chain, from the moment the flow originates until it reaches its destination.

The presented material can be valuable in substantiating and making informed decisions regarding the zoning of territories, the placement of emergency shelters or protective structures, and it will aid in proactively shaping emergency operations to mitigate the impact of hazards.

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Двоетапні задачі оптимального розміщення–розподілення структурних підрозділів системи гуманітарної логістики

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Мета. Забезпечення раціональної організації евакуації людей із регіону, постраждалого від надзвичайної ситуації, за рахунок розробки математичного та алгоритмічного забезпечення, що дозволить завчасно розподіляти транспортні й матеріальні ресурси, максимально охоплюючи постраждалі райони та мінімізуючи евакуаційний час.

Методика. Системний аналіз евакуаційних процесів; математичне моделювання, теорія неперервних задач оптимального розбиття множин, недиференційована оптимізація.

Результати. Об'єктом дослідження є двоетапні евакуаційні логістичні процеси, що виникають при наданні допомоги населенню територій, які постраждали від надзвичайної ситуації природного чи техногенного характеру. У дослідженні розглянута можливість оптимального розподілу людських потоків у транспортній системі з підрозділами двох рівнів – центри першої черги (медичні пункти, що здійснюють прийом громадян із постраждалих районів) і другої черги (спеціалізовані підрозділи системи екстреної допомоги, що здійснюють подальше обслуговування евакуйованого населення). Запропоновані математичні моделі є задачами оптимального розбиття континуальних множин з розміщенням центрів підмножин і додатковими зв'язками. Описані методи їх розв'язання. Продемонстрована універсальність указаних моделей за рахунок використання їх для опису як евакуаційних процесів, ураховуючи необхідність організації збірних, проміжних і приймальних пунктів евакуації, так і процесів, пов'язаних із наданням первинної допомоги постраждалому населенню, розраховуючи й доставляючи відповідну кількість продуктів першої необхідності з наявних складів через розподільчі центри в райони лиха.

Наукова новизна. Як превентивні заходи з підвищення рівня безпеки населення при надзвичайних ситуаціях розглядаються оптимальне розміщення рятувальних засобів і зонування території для розподілу евакуаційного руху. Також вирішується задача оптимального розподілу людських потоків у транспортно-логістичній системі.

Практична значимість. Представлені моделі, методи та алгоритми дозволяють вирішити низку практичних завдань, пов'язаних із розробкою профілактичних заходів і плануванням рятувальних робіт із забезпечення безпеки населення при виникненні надзвичайних ситуацій, у тому числі техногенного характеру. Отримані теоретичні результати дають можливість розробляти конкретні рекомендації щодо виконання логістичних завдань, пов'язаних з організацією первинної евакуації населення з постраждалих районів і його транспортуванням до безпечного місця для подальшого надання допомоги.

Ключові слова: *гуманітарна логістика, двоетапна евакуація, територіальний розподіл, математичне моделювання*

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