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MATHEMATICAL MODELS FOR DETERMINING AND ANALYZING THERMAL REGIMES IN MINING INDUSTRY MECHANISM STRUCTURES

Purpose. To develop linear and nonlinear mathematical models of heat conduction for isotropic heterogeneous media with internal heating. This will allow for an increased accuracy in determining temperature fields, which will subsequently impact the effectiveness of designing mechanisms, devices, and individual components of structures that have a layered structure and are subjected to heat stress.

Methodology. For the development of linear and nonlinear mathematical models of the temperature field and the analysis of temperature regimes in layered media with internal thermal heating, the coefficient of thermal conductivity is described as a whole using asymmetric unit functions. This makes it possible to solve a differential equation with singular coefficients in both linear and nonlinear boundary value problems of heat conduction with appropriate boundary conditions.

Findings. Quadratic equations are obtained to determine the analytical solutions of linear and nonlinear boundary problems of heat conduction for a layered plate with internal heat load.

Originality. The scientific novelty lies in the given method of linearization of the nonlinear mathematical model of heat conduction and obtaining analytical solutions, in a closed form, of the corresponding linear and nonlinear boundary value problems for isotropic layered media subjected to internal heating.

Practical value. The developed linear and nonlinear mathematical models for determining the temperature distribution in layered structures with internal heating make it possible to analyze heat exchange processes and ensure the thermal stability of such structures. This also makes it possible to increase the heat resistance of structures and protect them from overheating, which can lead to damage to individual components and elements of mechanisms, as well as to the entire structure as a whole. The resulting analytical solutions can be used to predict temperature fields in mine shafts, underground environments and mechanisms of mining equipment, in particular, in drilling and underground compressor stations, ventilation systems and other equipment, which improves work efficiency and reduces useful energy consumption.

Keywords: *temperature field, thermal conductivity, thermal stability, linearizing function, layered structure, singular coefficients*

Introduction. In mechanical engineering, in particular, for mechanisms of the mining industry, separate nodes of structures and their elements in the form of layered structures that are exposed to temperature effects are widely used. The design and development of such mechanisms, where individual elements and nodes have a piecewise homogeneous structure and often operate under conditions of constant heating or cooling, involves not only expanding their capabilities and improving their performance, but also ensuring stable operation, high reliability, and thermal stability. Increasing the capacity of such mechanisms and their integration into the system significantly complicates the problem of thermal resistance to thermal loads of their structures, which partially or completely fail due to thermal overloads.

Since, as noted above, the above-mentioned structures operate in a wide range of temperatures, their high operating parameters necessitate consideration and solution of nonlinear boundary value problems due to the dependence of the thermal and physical parameters of structural materials on temperature and heat transfer conditions on the temperature of their surfaces, since calculations of temperature fields based on linear mathematical models of heat conduction processes do not always give satisfactory results. Therefore, in order to develop the most adequate mathematical models for the real process, it is necessary to take into account the dependence of thermophysical parameters on temperature, density of surface flows and intensity of internal heat sources, changes in the shape of the medium, and possible phase and structural transformations.

Literature review. Determination of temperature regimes in both homogeneous and heterogeneous structures attracts the attention of many researchers. Temperature plays an important role in determining the physical and chemical characteristics of materials. This effect becomes especially significant when there are significant temperature fluctuations, as observed in heat conduction processes. Temperature changes lead to certain changes in material properties, which makes it difficult to determine the temperature distribution and thermal stress. As a result, determining the thermoelastic state of structures becomes much more difficult.

In [1], the thermoelastic problem of an elliptical cavity in an infinite medium was investigated using the generalized complex variable method. As a result of the analysis of the thermoelastic state of the medium, the temperature dependence of the coefficient of thermal conductivity, modulus of elasticity and coefficient of thermal expansion is taken into account. Taking into account these dependencies, analytical expressions for temperature, heat flux, and thermoelastic fields were obtained.

Analytical solutions of the distribution of temperature, displacements and stresses in layered rectangular plates with a simple support, which are subjected to thermomechanical loads, are presented. The material properties of the layers depend on the temperature [2].

The thermoelastic parameters of functionally graded porous plates with different material distributions were investigated and it was found that thermal stresses are more sensitive to material distribution than temperature and deformations [3].

In paper [4], research is aimed at determining the effect of the temperature dependence of material properties and indicators of the composite gradient in functionally graded rectangular plates on temperature, deformation, and stress.

The solution for the steady-state reaction of thick cylinders subjected to pressure and external heat flow on the inner surface is presented [5].

The thermal analysis of cylinders of different thicknesses made of functionally graded materials under the influence of heterogeneous heat flows concentrated on the inner and outer layers was performed [6, 7].

The work [8] is dedicated to the determination of the solution of the non-stationary problem of thermal conductivity and thermoelasticity for functional gradient thick spheres. Thermophysical and thermoelastic parameters of materials, with the exception of Poisson's coefficient, are arbitrary functions of the radial coordinate. The axisymmetric stationary problem of thermal conductivity and thermoelasticity for hollow functional gradient areas relative to the heat source is considered.

Thermal modeling of electronic devices is one of the most important tools for assessing their reliability in various operating modes. In [9], a thermal model of electronic devices is presented, which is based on experimental temperature measurement data obtained by an infrared camera. These data are input for the developed mathematical model, which is based on the method of finite differences and some known physical dependencies. The developed model was verified by comparing the simulation data with the experimentally obtained ones. It can be used to study the thermal behavior of the device under various operating conditions.

In most portable electronic devices, in addition to the temperature of multiple heat sources, i. e., the connection temperature, the body temperature, i. e., the skin temperature, should also be monitored to protect the user' s work. Thus, the creation of a compact device-level thermal model for predicting skin temperature will not only improve the efficiency of thermal design at an early stage, but also help develop a model-based temperature control strategy. In the paper [10], dynamic compact thermal models of two portable electronic devices, including a smartphone and a laptop, were first created based on the convolution method. Under the assumption of linear time-invariant systems, the skin temperature of the two test devices can be quickly calculated after obtaining the step response of each heat source.

The increase in specific power of electronic devices, due to higher performance and miniaturization requirements, has prompted researchers to search for new and alternative methods of temperature control. Since most electronic devices are often subjected to high-frequency power cycles, cooling systems must also be able to manage transient thermal profiles to delay the temperature response and reduce temperature gradients within the device that can lead to thermal stress and, in the long run, electronic device failure. The integration of phase change materials (PCM) into heatsinks for electronic devices represents an interesting technical system to increase the thermal inertia of the cooling system while providing a more stable operating temperature in the electronic components. Article [11] discusses the latest research trends in this area, with a special focus on electric batteries, power electronics, and the use of portable devices.

Much of the effort in electronics temperature management has been focused on developing cooling solutions that provide steady-state operation. However, electronic devices are increasingly used in applications involving time-varying workloads. These include microprocessors (including those used in portable devices), power electronic devices, and arrays of powerful semiconductor laser diodes. Transient solutions for temperature management are becoming essential to ensure the performance and reliability of such devices. New requirements for temperature control in transient processes are defined in [12], and cooling solutions described in the literature for such applications are presented, focused on the time scales of the thermal response.

Existing methods have been improved and new approaches have been developed to create mathematical models that allow analyzing heat exchange in piecewise homogeneous media. Flat and spatial models of heat transfer are presented, in which the differential equations contain coefficients that depend on the thermophysical properties of the phases and the geometric structure. Approaches for determining analytical and analytical-numerical solutions of boundary value problems of heat conduction are presented. Heat exchange processes occurring in homogeneous and layered structures with inclusions of canonical form were analyzed [13].

Problem formulations. 1. Uneven heating is one of the factors that cause deformations and stresses in elastic media. If nothing prevents its expansion as the temperature rises, then this medium is deformed and no stresses arise. However, if the temperature rises unevenly there and the environment is not homogeneous, temperature stresses are formed as a result of expansion. Let us remind you that the stress-strain state is a set of internal stresses and deformations of the structure or its elements that occur as a result of external loads, temperature fields, or other factors acting on it. The so-called thermostressed deformation state of structurally heterogeneous media is determined by calculation and experimental methods in the form of the distribution of stresses, deformations and movements in the structure, which is important for assessing the static strength and resource of structures at all stages of their life cycle. If we do not take into account the temperature distribution of the influence of stresses and deformations caused by force factors inherent in most practical problems, then the first and independent step for the study on temperature stresses is the determination of temperature fields, which is the main task of the analytical theory of heat conduction. In some cases, the determination of temperature fields is an independent technical problem, the solution of which makes it possible to determine temperature stresses. In this regard, we will present a method for determining the temperature distribution by spatial coordinate for a thermally active layered medium, which is in conditions of thermal stress due to concentrated, uniformly distributed heat sources with a certain density on the entire surface of the structure. This structure consists of n heterogeneous layers, on the contact surfaces $y_i\ (i=\overline{1,n-1})$ of which the conditions of equality of temperatures $t_i = t_{i+1}$ and heat flows $\lambda_i \dfrac{\partial t_i}{\partial y} = \lambda_{i+1} \dfrac{\partial t_{i+1}}{\partial y}$ (conditions of ideal thermal contact) are set, and the boundary surfaces of the medium are maintained by a certain constant temperature t_k (Fig. 1).

We describe the thermophysical parameters for an isotropic piecewise homogeneous medium in the form of a function on the spatial coordinate

$$p(y) = p_1 + \sum_{i=1}^{n-1} (p_{i+1} - p_i) S_+(y - y_i), \tag{1}$$

where $p_i (i = \overline{1,n})$ is the thermophysical parameter of the i^{th} layer of the medium.

The relation (1) and the conditions of ideal thermal contact make it possible to write down one differential equation of thermal conductivity to determine the temperature field $t(y)$ in a piecewise homogeneous structure

$$\operatorname{div}[\lambda(y) \operatorname{grad} t(y)] = -q_0 \tag{2}$$

with a boundary condition on the boundary surfaces

$$t(0) = t(l) = t_k, \tag{3}$$

where t_k is the temperature value given on the boundary surface; $\lambda(y)$ is the coefficient of thermal conductivity of a piece-homogeneous structure, described by the expression (1); l is a thickness of a piece-homogeneous structure; q_0 = const is the power of uniformly concentrated internal heat sources.

Let us enter the function

$$T(y) = \lambda(y) t(y), \tag{4}$$

and we differentiate it by the variable y, taking into account the expression (3) for the thermal conductivity coefficient $\lambda(y)$. As a result, we get the ratio

$$\lambda(y) \frac{dt}{dy} = \frac{dT}{dy} - \sum_{i=1}^{n-1} (\lambda_{i+1} - \lambda_i) t(y_i) \delta_+(y - y_i),$$

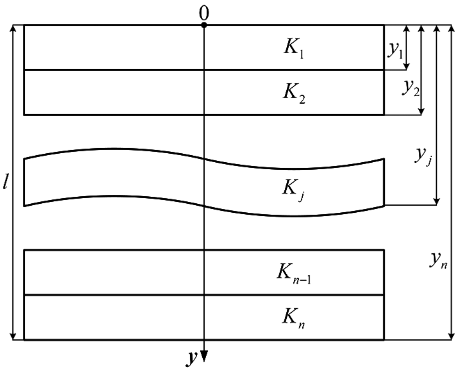

Fig. 1. Isotropic piecewise homogeneous structure

considering which we will rewrite the original equation (2) in the form

$$\frac{d^2T}{dy^2}-\sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)t(y_i)\delta'_+(y-y_i)=-q_0.$$

Here $S_+(\zeta)=\begin{cases}1, & \zeta\ge 0\\ 0, & \zeta<0\end{cases}$, $\delta_+(\zeta)=\frac{dS_+(\zeta)}{d\zeta}$ is the asymmetric Dirac delta function; $S_+(\zeta)$ is the asymmetric unit function [13].

The general solution of this equation is

$$T(y)=\sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)t(y_i)S_+(y-y_i)-\frac{q_0}{2}y^2+c_1y+c_2. \quad (5)$$

Let us determine the values of $t(y_i)(i=\overline{1,n-1})$, using the relations (4, 5)

$$t(y_1)=\frac{1}{\lambda_1}\left(c_1y_1+c_2-\frac{q_0}{2}y_1^2\right);$$

$$t(y_i)=c_1\left[\sum_{j=1}^{i-1}\left(\frac{1}{\lambda_j}-\frac{1}{\lambda_{j+1}}\right)y_j+\frac{y_i}{\lambda_i}\right]+\frac{c_2}{\lambda_1}-\frac{q_0}{2}\left[\sum_{j=1}^{i-1}\left(\frac{1}{\lambda_j}-\frac{1}{\lambda_{j+1}}\right)y_j^2+\frac{y_i^2}{\lambda_i}\right].$$

Taking into account the boundary condition (3) and the relation (5), we find the integration constants c_1 and c_2

$$c_1=\frac{q_0}{2}\frac{\lambda_n\sum_{i=1}^{n-1}\left(\frac{1}{\lambda_i}-\frac{1}{\lambda_{i+1}}\right)y_i^2+y_n^2}{\lambda_n\sum_{i=1}^{n-1}\left(\frac{1}{\lambda_i}-\frac{1}{\lambda_{i+1}}\right)y_i+y_n};\quad c_2=\lambda_1t_k.$$

As a result, the temperature field in a layered plate with uniformly distributed internal heat sources is determined by expression (5).

For mining industry mechanisms, tasks that consider the process of heating or cooling of various systems with internal heat sources concentrated in the area of a particular node or its element are important. The reliability of the operation of parts, assemblies, component structures, and in some cases the entire structure cannot be guaranteed without observing the proper thermal condition. Thermal and temperature conditions limit the operational characteristics of the equipment, affect the choice of structural materials, impair the dynamic capabilities of the device as an object of regulation and control, determine technical and economic indicators, dimensional and weight parameters, etc. For this, there are clearly defined requirements for the operating modes of such structures, primarily optimal, transitional and basic thermal conditions for their operation. In this regard, we will consider the case when the internal heat sources are concentrated on the surface of one of the layers of a piece-homogeneous medium. Then the thermal conductivity equation (2) will take the following form

$$\mathrm{div}[l(y)\ \mathrm{grad}\,t(y)]=-q_0N(y,y_j), \quad (6)$$

where

$$N(\zeta,\zeta_j)=S_+(\zeta-\zeta_{j-1})-S_+(\zeta-\zeta_j).$$

Let us determine the general solution of equation (6) in the form

$$T(y)=\sum_{i=1}^{n-1}(\lambda_{i+1}-\lambda_i)t(y_i)S_+(y-y_i)-q_0F(y)+c_1y+c_2. \quad (7)$$

Here

$$F(y)=0.5y^2N(y,y_j)+F_1(y)-F_2(y);$$

$$F_1(y)=y_{j-1}(0.5y_{j-1}-y)S_+(y-y_{j-1});$$

$$F_2(y)=y_j(0.5y_j-y)S_+(y-y_j).$$

Using ratios (4, 7), we determine the values $t(y_i)\,(i=\overline{1,n-1})$ (temperature values at the interface surfaces $y=y_i$ of dissimilar layers of the medium)

$$t(y_1)=\frac{1}{\lambda_1}\left(c_1y_1+c_2-\frac{q_0F(y)}{2}y_1^2\right);$$

$$t(y_i)=c_1\left[\sum_{j=1}^{i-1}\left(\frac{1}{\lambda_j}-\frac{1}{\lambda_{j+1}}\right)y_j+\frac{y_i}{\lambda_i}\right]+\frac{c_2}{\lambda_1}-\frac{q_0F(y)}{2}\left[\sum_{j=1}^{i-1}\left(\frac{1}{\lambda_j}-\frac{1}{\lambda_{j+1}}\right)y_j^2+\frac{y_i^2}{\lambda_i}\right].$$

We obtain the constants of integration c_1 and c_2 using boundary condition (3) and relation (7)

$$c_1=0.5q_0F(y)\frac{\lambda_n\sum_{i=1}^{n-1}\left(\frac{1}{\lambda_i}-\frac{1}{\lambda_{i+1}}\right)y_i^2+y_n^2}{\lambda_n\sum_{i=1}^{n-1}\left(\frac{1}{\lambda_i}-\frac{1}{\lambda_{i+1}}\right)y_i+y_n};\quad c_2=\lambda_1t_k.$$

2. Consider an isotropic thermosensitive layered structure. Due to the thermal sensitivity of the materials of the medium, the conditions of ideal thermal contact at the junction of the layers $y_i\,(i=\overline{1,n-1})$ will be rewritten as $t_i=t_{i+1}$, $\lambda_i(t)\frac{\partial t_i}{\partial y}=\lambda_{i+1}(t)\frac{\partial t_{i+1}}{\partial y}$ (Fig. 1). In the given structure, it is necessary to determine the temperature distribution $t(y)$ along the spatial coordinate y, which will be obtained by solving the nonlinear heat conduction equation

$$\frac{d}{dy}\left[\lambda(y,t)\frac{dt}{dy}\right]=-q_0 \quad (8)$$

with the boundary condition (3), where

$$\lambda(y,t)=\lambda_1(t)+\sum_{i=1}^{n-1}(\lambda_{i+1}(t)-\lambda_i(t))S_+(y-y_i) \quad (9)$$

is the thermal conductivity coefficient of the thermosensitive layered system; $\lambda_i(i=\overline{1,n})$ is the thermal conductivity of the i^{th} layer of the structure.

Let us introduce a linearizing function

$$\vartheta(y)=\int_0^{t(y)}\lambda_1(\zeta)d\zeta+\sum_{i=1}^{n-1}S_+(y-y_i)\int_{t(y_i)}^{t(y)}[\lambda_{i+1}(\zeta)-\lambda_i(\zeta)]d\zeta. \quad (10)$$

We differentiate the expression (10) by the variable y taking into account the relation (9), as a result of which we obtain

$$\lambda(y,t)\frac{\partial t}{\partial y}=\frac{\partial\vartheta}{\partial y}.$$

Based on this, the original nonlinear thermal conductivity equation (8) is reduced to an ordinary differential equation of the second order with constant coefficients relative to the function $\vartheta(y)$

$$\frac{d^2\vartheta}{dy^2} = -q_0, \tag{11}$$

the general solution of which will be obtained in the form

$$\vartheta(y) = -\frac{q_0}{2}y^2 + c_1 y + c_2.$$

Let us determine the constants c_1, c_2 using the boundary conditions (3). Taking into account the relation (10), we transform this boundary condition and

$$\vartheta\Big|_{y=0} = \int_0^{t_k}\lambda_1(t)dt, \vartheta\Big|_{y=y_n} = \int_0^{t_k}\lambda_1(t)dt + \sum_{i=1}^{n-1}\int_{t(y_i)}^{t_k}[\lambda_{i+1}(t) - \lambda_i(t)]dt. \tag{12}$$

Based on the obtained integration constants c_1, c_2 using boundary condition (12), we write the partial solution of problems (11, 12) in the form

$$\vartheta(y) = y\left[\frac{q_0}{2}(y_n - y) + \frac{1}{y_n}\sum_{i=1}^{n-1}\int_{t(y_i)}^{t_k}\left[\lambda_{i+1}(\zeta) - \lambda_i(\zeta)d\zeta\right]d\zeta\right] + \int_0^{t_k}\lambda_1(\zeta)d\zeta. \tag{13}$$

If one of the layers of a thermosensitive piecewise homogeneous medium is thermally active, then the nonlinear thermal conductivity equation (8) is rewritten as

$$\frac{d}{dy}\left[\lambda(y,t)\frac{dt}{dy}\right] = -q_0 N(y, y_j). \tag{14}$$

Taking into account the introduced linearizing function (10), after performing certain mathematical transformations, we obtain an ordinary differential equation of the second order with constant coefficients

$$\frac{d^2\vartheta}{dy^2} = -q_0 N(y, y_j), \tag{15}$$

the general solution of which is

$$\vartheta(y) = -q_0 F(y) + c_1 y + c_2.$$

Boundary conditions (12) make it possible to determine the constants of integration and, as a consequence, the final partial solution of the boundary value problem (13, 15)

$$\vartheta(y) = -q_0 F(y) + \frac{y}{y_n}\sum_{i=1}^{n-1}\int_{t(y_i)}^{t_k}\left[\lambda_{i+1}(\zeta) - \lambda_i(\zeta)d\zeta\right]d\zeta + \int_0^{t_k}\lambda_1(\zeta)d\zeta.$$

As an example, consider an isotropic two-layer thermosensitive plate. For many structural materials in certain temperature ranges, a linear dependence on temperature of thermophysical parameters is observed in the form

$$p(t) = p_m^0 - \kappa_m t, \tag{16}$$

where p_m^0 is a reference thermophysical parameter of materials for the first ($m = 1$) and second ($m = 2$) layers of the structure; κ_m is a parameter that is in some way related to the temperature coefficient of thermal conductivity. Let us choose silicon as the material of the first layer of a piece-homogeneous structure, and germanium as the second layer. Having performed interpolation of the coefficient of thermal conductivity $\lambda(t)$ as a discrete function of temperature for selected construction materials in the range [0; 1127 °C], we obtain relations that are a partial case of expression (16):

- $\lambda(t) = (67.9 - 0.03395t)$ for silicon;
- $\lambda(t) = (60.3 - 0.00088t)$ for germanium. (17)

Taking into account these dependencies and expressions (10, 13) to determine the temperature distribution $t(y)$ in the given structure, we obtain quadratic equations for:

- *the first* $0 \le y \le y_1$

$$\lambda_1^0 k_1 t^2 - 2\lambda_1^0 t + \lambda_1^0 t_k(2 - k_1 t_k) + \vartheta(y) = 0; \tag{18}$$

- t*he second* $0 \le y \le y_2$

$$\lambda_2^0 k_2 t^2 - 2\lambda_2^0 t + \lambda_1^0 t_k(2 - k_1 t_k) + t(y_1)\left[\lambda_2^0(2 - k_2 t(y_1)) - \lambda_1^0(2 - k_1 t(y_1))\right] + \vartheta(y) = 0 \tag{19}$$

layers of the structure and on the surface of their junction $y = y_1$

$$\left[\lambda_1^0 k_1 + \frac{y_1}{y_2}(\lambda_2^0 k_2 - \lambda_1^0 k_1)\right]t^2(y_1) - 2\left[\lambda_1^0 + \frac{y_1}{y_2}(\lambda_2^0 - \lambda_1^0)\right]t(y_1) + \vartheta(y_1) + \lambda_1^0 t_k(2 - k_1 t_k) = 0, \tag{20}$$

where

$$\vartheta(y) = y\left\{q_0(y_2 - y) + \frac{1}{y_2}\left[t_k\Lambda_1 - t(y_1)\Lambda_2\right]\right\};$$

$$\vartheta(y_1) = y_1\left[q_0(y_2 - y_1) + \frac{t_k}{y_2}\Lambda_1\right];$$

$$\Lambda_1 = \lambda_2^0(2 - k_2 t_k) - \lambda_1^0(2 - k_1 t_k);$$

$$\Lambda_2 = \lambda_2^0(2 - k_2 t(y_1)) - \lambda_1^0(2 - k_1 t(y_1));$$

$k_m = \frac{\kappa_m}{\lambda_m^0}$, λ_m^0 are the temperature and resistance coefficients of thermal conductivity, respectively.

As a result, the temperature field is completely determined by relations (16–18).

Analysis of numerical results. Fig. 2 shows the behavior of the temperature field determined by formula (5) in the construction of a five-layer assembly of a lithium-ion battery, in which the material of the first, third and fifth layers is aluminum ($\lambda_1 = \lambda_3 = \lambda_5 = 282$ W/(degree · m) at a temperature of 627 °C), and the second and fourth layers are lithium ($\lambda_2 = \lambda_4 =$ $= 52.9$ W/(degree·m) at a temperature of 627 °C) for the following values of the layer thickness: $y_1 = 0.05$; $y_2 = 0.25$; $y_3 =$ $= 0.3$; $y_4 = 0.5$; $y_5 = 0.55$ m.

As can be seen from the figure, the temperature reaches its highest value in the middle aluminum layer and monotonically decreases as a function of the spatial coordinate y to the value $t_k = 627$ °C, set in the boundary conditions (2).

The presence of corner points, which are observed on the curves in the area of the inner surfaces of the first and fifth aluminum layers of the lithium-ion battery assembly, indicates the continuity of the temperature as a function of the spatial coordinate y and that at these points a phase transition from

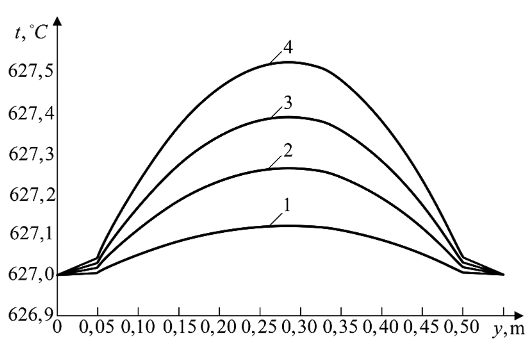

Fig. 2. Temperature distribution $t(y)$ in the design of the lithium-ion battery unit for different power values q_0 of the heating sources:

1 – $q_0 = 250$; 2 – $q_0 = 500$; 3 – $q_0 = 750$; 4 – $q_0 = 1000$ w/m^3

the aluminum medium (solid state) into the lithium environment (liquid state) takes place.

The table shows the value of the temperature field $t(y)$ in a piecewise homogeneous structure, which contains two layers for a linear model (constant values of the coefficient of thermal conductivity of the structural materials of the medium for the first layer $\lambda_1 = 67.9$ W/(degree·m) and the second layer $\lambda_2 = 60.3$ (W/(degree·m) at a temperature of 27 °C) and a nonlinear model (the coefficient of thermal conductivity of structural materials of a heat-sensitive medium varies depending on the temperature according to relations (17)). The values of power of internally concentrated heat sources q_0 and temperature t_k on the boundary surfaces of the plate are equal to 200 W/m^3 and 100 °C, respectively. The values of the thickness of the plate layers are $y_1 = 0.2$ and $y_2 = 0.4$ m. The results of numerical calculations of the temperature field values indicate the continuity of the temperature as a function of $t(y)$ of the spatial coordinate y without corner points on the surfaces of the junction of heterogeneous layers (temperature as the spatial coordinate function is a smooth function). This confirms the correctness of both linear and non-linear mathematical models for determining the temperature field, since the conditions of ideal thermal contact (equality of temperatures and thermal fluxes) are set on the surfaces of the conjugation of heterogeneous layers of the medium. The results obtained for the selected materials based on the linear dependence of the coefficient of thermal conductivity on temperature differ from the results obtained for a constant coefficient of thermal conductivity (Table) by 5 %. Their insignificant difference is explained by the fact that the values of the temperature coefficient of thermal conductivity for the considered materials, as shown by the ratio (17), are small, and taking into account thermal sensitivity leads to a decrease in the temperature values $t(y)$ for the given materials of the structure layers.

Discussion of results. A method of linearization of the nonlinear mathematical model of thermal conductivity is proposed and analytical solutions of the corresponding linear and nonlinear boundary value problems for isotropic layered media subjected to internal thermal heating are obtained in a closed form.

Based on the obtained analytical solutions for linear and nonlinear boundary value problems of heat conduction in isotropic layered media with internal heating, it is possible to develop computational algorithms and software for numerical implementation. This will make it possible to analyze temperature regimes in individual structural elements and nodes of mechanisms and devices exposed to various thermal effects, in particular, to identify unknown parameters, to increase thermal resistance, which helps to increase their service life.

Conclusions. In the process of developing and studying linear and nonlinear mathematical models for determining temperature fields and analyzing temperature regimes caused by internal heat sources for structures geometrically described by isotropic layered structures, a minor influence of taking into account the thermal sensitivity of structural materials on the temperature distribution in these environments was revealed. This is explained by the fact that the value of the temperature coefficient of thermal conductivity for the selected materials, which characterizes their thermal sensitivity at a given temperature interval, as shown by the ratio (17), is small. In the future, research will be conducted for a number of materials used in the process of designing mechanisms and digital devices for their control, regarding the effect of thermal sensitivity on temperature distribution using the above developed linear and nonlinear mathematical models for determining temperature fields, and based on this, the analysis of temperature regimes in thermosensitive layered environments. Taking into account the thermal sensitivity of structural materials significantly complicates the process of solving the corresponding nonlinear boundary value problems of thermal conductivity, but the sought solutions of these problems describe the temperature behavior as a function of spatial coordinates more adequately to the real physical process.

Table

Values of the temperature field $t_1(y)$ and $t_2(y)$ for a constant coefficient of thermal conductivity of the material of the medium layers (linear model) and linearly variable with temperature, respectively

y	0.0125	0.0250	0.0750	0.1250	0.1750
$t_1(y)$	100	100.007	100.034	100.053	100.063
$t_2(y)$	100	100.006	100.032	100.049	100.059
y	0.2250	0.2750	0.3250	0.3750	0.4125
$t_1(y)$	100.066	100.061	100.047	100.023	100
$t_2(y)$	100.062	100.057	100.043	100.021	100

$t_1(y)$ denotes the temperature value for the linear model, and $t_2(y)$ denotes the temperature value for the nonlinear model.

References.

1. Haopeng, S., Kunkun, X., & Cunfa, G. (2021). Temperature, thermal flux and thermal stress distribution around an elliptic cavity with temperature-dependent material properties. *International Journal of Solids and Structures, 216*, 136-144. https://doi.org/10.1016/j.ijsolstr.2021.01.010.

2. Zhang, Z., Zhou, D., Fang, H., Zhang, J., & Li, X. (2021). Analysis of layered rectangular plates under thermo-mechanical loads considering temperature-dependent material properties. *Applied Mathematical Modelling, 92*, 244-260. https://doi.org/10.1016/j.apm.2020.10.036.

3. Gong, J., Xuan, L., Ying, B., & Wang, H. (2019). Thermoelastic analysis of functionally graded porous materials with temperature-dependent properties by a staggered finite volume method. *Composite Structures, 224*, 111071. https://doi.org/10.1016/j.compstruct.2019.111071.

4. Demirbas, M. D. (2017). Thermal stress analysis of functionally graded plates with temperature-dependent material properties using theory of elasticity. *Composites Part B: Engineering, 131*, 100-124.

5. Ghannad, M., & Yaghoobi, M. P. (2015). A thermoelasticity solution for thick cylinders subjected to thermo-mechanical loads under various boundary conditions. *International Journal of Advanced Design & Manufacturing Technology, 8*(4), 1-12.

6. Yaghoobi, M. P., & Ghannad, M. (2020). An analytical solution for heat conduction of FGM cylinders with varying thickness subjected to non-uniform heat flux using a first-order temperature theory and perturbation technique. *International Communications in Heat and Mass Transfer, 116*, 104684. https://doi.org/10.1016/j.icheatmasstransfer.2020.104684.

7. Eker, M., Yarımpabuç, D., & Celebi, K. (2020). Thermal stress analysis of functionally graded solid and hollow thick-walled structures with heat generation. *Engineering Computations, 38*(1), 371-391. https://doi.org/10.1108/EC-02-2020-0120.

8. Bayat, A., Moosavi, H., & Bayat, Y. (2015). Thermo-mechanical analysis of functionally graded thick spheres with linearly time-dependent temperature. *Scientia Iranica, 22*(5), 1801-1812.

9. Evstatieva, N., & Evstatiev, B. (2023). Modelling the Temperature Field of Electronic Devices with the Use of Infrared Thermography. *13th International Symposium on Advanced Topics in Electrical Engineering (ATEE)*, (pp. 1-5). Bucharest, Romania. https://doi.org/10.1109/ATEE58038.2023.10108375.

10. Haoran, L., Jiaqi, Y., & Ruzhu, W. (2023). Dynamic compact thermal models for skin temperature prediction of porta-ble electronic devices based on convolution and fitting methods. *International Journal of Heat and Mass Trans-fer, 210*, 124170. ISSN 0017-9310. https://doi.org/10.1016/j.ijheatmasstransfer.2023.124170.

11. Bianco, V., De Rosa, M., & Vafai, K. (2022). Phase-change materials for thermal manage-ment of electronic devices. *Applied Thermal Engineering, 214*, 118839. ISSN 1359–4311. https://doi.org/10.1016/j.applthermaleng.2022.118839.

12. Mathew, J., & Krishnan, S. (2021). A Review On Transient Thermal Management of Electronic Devices. *Journal of Electronic Packaging*. https://doi.org/10.1115/1.4050002.

13. Havrysh, V., Pelekh, Ya., Kolyasa, L., Ovchar, I., Ivasyk, H., & Bilas, O. (2017). Examining the temperature fields in flat piecewiseuniform structures. *Eastern European Journal of Enterprise Technologies, 2*(5-86), 23-32.

Математичні моделі визначення та аналізу теплових режимів у конструкціях механізмів гірничої промисловості

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Мета. Розроблення лінійних і нелінійних математичних моделей теплопровідності для ізотропних неоднорідних середовищ із внутрішнім нагріванням, унаслідок чого є можливість підвищити точність визначення температурних полів, що в подальшому вплине на ефективність методів проєктування механізмів і пристроїв, окремі елементи й вузли конструкцій яких є шаруватої структури та піддаються тепловому навантаженню.

Методика. Для розроблення лінійних і нелінійних математичних моделей температурного поля та аналізу температурних режимів у шаруватих середовищах із внутрішнім тепловим нагріванням, коефіцієнт теплопровідності описано як єдине ціле за допомогою асиметричних одиничних функцій. Це приводить до розв'язування одного диференціального рівняння з сингулярними коефіцієнтами як у лінійній, так і в нелінійній крайових задачах теплопровідності з відповідними крайовими умовами.

Результати. Отримані квадратні рівняння, якими визначаються аналітичні розв'язки лінійної й нелінійної крайових задач теплопровідності для шаруватої пластини із внутрішнім тепловим навантаженням.

Наукова новизна. Полягає в наведеному способі лінеаризації нелінійної математичної моделі теплопровідності та отриманні в замкнутому вигляді аналітичних розв'язків відповідних лінійної й нелінійної крайових задач для ізотропних шаруватих середовищ, що піддаються внутрішньому тепловому нагріванню.

Практична значимість. Розроблені лінійна й нелінійна математичні моделі визначення температурного розподілу у шаруватих конструкціях при внутрішньому нагріванні дають змогу аналізувати процеси теплообміну та забезпечити термостійкість таких конструкцій, а також підвищити її та захистити ці конструкції від перегрівання, що може призвести до пошкоджень як окремих вузлів і елементів механізмів, так і всієї конструкції в цілому. Отримані аналітичні розв'язки можуть бути використані для прогнозування температурних полів у шахтах, підземних середовищах і механізмах гірничого обладнання, зокрема, у бурових і підземних компресорних станціях, вентиляційних системах та іншому обладнанні, що покращує ефективність роботи та зменшує витрати корисної енергії.

Ключові слова: *температурне поле, теплопровідність, термостійкість, лінеаризуюча функція, шарувата структура, сингулярні коефіцієнти*

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