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ACCOUNTING FOR A POSITIVE, NEGATIVE AND ZERO SEQUENCES POWER IN A THREE-PHASE UNBALANCED ELECTRICAL SYSTEM

Purpose. Based on the instantaneous electrical power of a three-phase asymmetric system of sinusoidal periodic current, to determine positive, negative, zero-sequences active and reactive power, as well as invariance power factor.

Methodology. In the unbalance case in three-phase electrical system, the electrical energy quality is evaluated by means on voltage and current positive, negative, zero-sequences. At the same time, similar components of active and reactive power have not received practical distribution. But it is precisely in terms of power that electricity is accounted for. The instantaneous power orthogonal components in the time domain are determined using the symmetrical components of voltage and current. Active, reactive powers of positive, negative and zero-sequences are allocated. The result obtained has the property of representativeness, which most of the known results lack.

Findings. The three-phase system's instantaneous power components are analytically determined, including the amplitudes of the oscillating power components. The need to take into account the oscillating instantaneous power components has been proven by means of a graphical interpretation of a special case of the three-phase system mode. As an integral indicator that takes into account the oscillating components of the three-phase system instantaneous power, its root-mean-square value over the repetition period is used.

Originality. By calculating the transformer efficiency of the studied model according to the active power positive sequence and the same indicator according to the active power as a whole, it was established, that the component sequence separation affects the results of calculating the generalized indicators, including the power transmission system objects. This can lead to erroneous judgments about the efficiency of the specified facilities functioning.

Practical value. The invariance power factor was used to characterize the electrical energy quality level of a three-phase sinusoidal current system in an unbalanced mode.

Keywords: *electrical power, root-mean-square power value, electrical power quality*

Introduction. Electrical energy is a product, the quality of which at the stage of consumption depends both on the "generator" ("distributor") and on the "consumer". The need to electrical energy account, taking into account its "high-quality" and "poor-quality" volumes, exists to this day. The availability of information about the specified volumes is a prerequisite for the formation of tactics for improving the electrical energy quality. When characterizing the electrical energy quality in three-phase systems, special attention is paid to the phenomena of asymmetry and non-sinusoidal for voltage and current. As a rule, as the voltage level increases, the phenomenon of non-sinusoidal is suppressed more significantly than the asymmetry phenomenon. Over the past few decades, significant contributions to the study on the electrical energy quality from the instantaneous power standpoint and its integral indicators have been made by groups of researchers led by Professor Emmanuel [1] and Professor Akagi [2]. In the latter case, the theory is used to implement systems for improving the electrical energy quality using real and imaginary powers. Real and imaginary instantaneous powers are related to the symmetrical components of currents and voltages.

A certain rationing of the indicators of the electric energy quality from the calculation methodology standpoint was carried out based on the materials of studies [1] in the standard [3]. This standard is used to evaluate the electrical power components in complex electrical systems [4], to correct the algorithms for the operation of power active filters [5] and to determine the parameters of power circuit elements a power active filters [6]. Numerically, the power quality indicators are normalized by the standard [7] for voltage and by the standard [8] for voltage and current. It is difficult to normalize for certain power components, but there are studies that offer alternative indicators [9], even for electro-hydraulic systems [10].

Research analysis. The asymmetry influence on the elements of the electric power industry seems to be important in the electric power industry. Most often, negative and zero sequence voltages are used as an indicator of unbalance, as, for example, in reference [11]. In work, the authors propose compensation for voltage asymmetry based on the optimal planning of the three-phase branches operation of step voltage regulators with individual control. Load data, including load imbalance information, in many cases can have a significant impact on the correct assessment of many power quality indicators, as shown in reference [12]. The authors investigate the load unbalance effect on several phase unbalance indicators and voltage quality indicators by comparing the values of these indicators. As such indicators, the voltage unbalance factor and the current unbalance factor are used. Those generalized power parameters are not used. A similar approach was used by the authors [13], who noted that a three-phase unbalanced load has a great impact on the safety and efficiency of the distribution network. On the basis of load transfer index, considering the conversion relationship between load and electricity quantity, the electric quantity transfer index is put forward. In this case, when forming indicators or factors, the load current is used.

Power transformers in the presence of asymmetry receive an uneven electromagnetic load of magnetic circuit. As shown in reference [14], the problems of transformer windings vibration during asymmetry operation are due to changes in the winding current, internal magnetic leakage and vibration characteristics of the windings in a three-phase transformer under various operating modes due to electromagnetic-mechanical coupling. It is noted that vibration acceleration occurs most significantly at a frequency of 100 Hz. This is probably due to the oscillating instantaneous power component, as will be shown below. In reference [15], in terms of the electrical energy quality influence of the network, additional losses of the distribution network are studied under a combined perturbation with an asymmetric load and current harmonics. Depending on the asymmetry nature, there is a different effect on the losses level. However, the analysis was performed on the basis of current loads, and not power components, the difference of which at the points of the network determines losses. The increase in the electromagnetic torque pulsations of the turbogenerator was established in reference [16] even under the standard restrictions of the asymmetric operation mode. The authors found that the voltages and power factors of different phase windings also differ markedly, as well as phase currents, the asymmetry of which is the cause of certain complications in turbogenerator operation.

Despite the indicators calculated by current or voltage, it is instantaneous power that is the parameter, which has a balance in electrical circuits, electrical complexes and even systems with the conversion of electrical energy into other forms [10]. Charles Steinmetz, Paul Bouchereau, emphasized this property and the theoretical generalization was realized by Bernard Tellegen in the corresponding theorem. Instantaneous power is also a top priority in the previously mentioned papers [1, 2]. Work on the search for the components of instantaneous power that characterize the asymmetry of a three-phase network mode with a grounded neutral in the negative and zero sequences continues [17]. However, the result of obtaining the equations for complex and instantaneous conventional powers, characterizing the asymmetry of a three-phase network mode with a grounded neutral in the negative and zero sequences, does not allow them to be used in practice due to "conventionality". The definitions included in the standard [3], despite their validity, are criticized. As noted in [18], the instantaneous power is determined exactly and the average power measured over a selected period is generally accepted. Power factor, a measure of the relative efficiency of power delivery, has only a weak relationship with output loss or voltage drop in practical power systems. It is noted that the apparent power, reactive power and non-active power defined in the standard [3] and other standards do not explicitly lead to representative measurements and expose users to the uncertainty inherent in operational measurements in practical systems.

Purpose. Determination based on the instantaneous electrical power of a three-phase asymmetric system of sinusoidal periodic current, positive, negative, zero-sequences active and reactive power, as well as invariance power factor.

Main material and research results. As noted during the well-known studies analysis, all of them were carried out in order to assess the electric energy quality, its accounting and improve the quality by compensating certain components of electric power. In reference [19], certain evidence is given of what electrical power and, accordingly, electrical energy it is rational to consider as qualitative. The authors noted that in this case, the total electrical energy has no fluctuations and its quality in a three-phase symmetrical system of alternating harmonic current corresponds to a direct current system.

A great contribution to the development of the power theory of three-phase systems, from the standpoint of improving the electrical energy quality, was made by the authors [2]. A very important definition is highlighted in the text of this work. "For a three-phase system with or without a neutral conductor in the steady state or during transients, the three-phase instantaneous active power p_{3ph} describes the total instantaneous energy flow per second between two subsystems". The basic position is used that the three-phase instantaneous active power can be calculated in terms of components in coordinates $\alpha\beta 0$, taking into account the power invariance property when using the Clarke transformation

$$p_{3pih} = u_A i_A + u_B i_B + u_C i_C \Leftrightarrow p_{3ph} = u_\alpha i_\alpha + u_\beta i_\beta + u_0 i_0,$$

where u_A, u_B, u_C are phase voltages; i_A, i_B, i_C – phase currents; u_α, u_β, u_0 – projections of the generalized voltage vector in coordinates $\alpha\beta 0$; i_α, i_β, i_0 – projections of the generalized current vector in coordinates $\alpha\beta 0$.

The authors of [2] define the instantaneous power based on the concept of the instantaneous complex power **s** as the product of the generalized voltage vector **e** and the generalized current vector **i**, using the Clarke transformation [3] as follows

$$\mathbf{s} = \mathbf{e} \cdot \mathbf{i}^* = (u_\alpha + ju_\beta)(i_\alpha + ji_\beta) = \underbrace{(u_\alpha i_\alpha + u_\beta i_\beta)}_{p} + j\underbrace{(u_\beta i_\alpha - u_\alpha i_\beta)}_{q}.$$

As a result, this definition is reduced to a matrix form

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} u_\alpha & u_\beta \\ -u_\beta & u_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix},$$

and developing for three-phase systems with a neutral wire transfer to the coordinate system $\alpha\beta 0$. The corresponding projections of voltage u_0 and current i_0 are introduced, which determine the power

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} u_0 & 0 & 0 \\ 0 & u_\alpha & u_\beta \\ 0 & -u_\beta & u_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix}.$$

Powers are summarized divided into average and oscillating components as follows:

- instantaneous real power: $p = \overline{p} + \tilde{p}$;
- instantaneous imaginary power: $q = \overline{q} + \tilde{q}$;
- instantaneous zero-sequence power: $p_0 = \overline{p}_0 + \tilde{p}_0$.

Attention is paid to the names of the specified powers and measurement units. The instantaneous imaginary power q is proposed to be measured in "volt-ampere imaginary" (vai), by analogy with the traditional reactive power "volt-ampere reactive" (var). At the same time, the source [2] indicates that the terms "instantaneous imaginary power" and "instantaneous reactive power" will be used as synonyms.

As a result, it is noted that the three-phase system instantaneous power is equal to the sum of the instantaneous real power and the instantaneous zero sequence power

$$p_{3ph} = u_A i_A + u_B i_B + u_C i_C = u_\alpha i_\alpha + u_\beta i_\beta + u_0 i_0 = p + p_0,$$

in this case, the imaginary power q is not a component of the instantaneous power.

For a three-phase asymmetrical system, the above powers are determined through symmetrical components as follows

$$\overline{p}_0 = 3U_1^0 I_1^0 \cos(\psi_{u1^0} - \psi_{i1^0});$$
$$\overline{p} = 3U_1^+ I_1^+ \cos(\psi_{u1^+} - \psi_{i1^+}) + 3U_1^- I_1^- \cos(\psi_{u1^-} - \psi_{i1^-});$$
$$\overline{q} = 3U_1^+ I_1^+ \sin(\psi_{u1^+} - \psi_{i1^+}) - 3U_1^- I_1^- \sin(\psi_{u1^-} - \psi_{i1^-});$$
$$\tilde{p}_0 = -3U_1^0 I_1^0 \cos(2\omega t + \psi_{u1^0} + \psi_{i1^0});$$
$$\tilde{p} = -3U_1^+ I_1^- \cos(2\omega t + \psi_{u1^+} + \psi_{i1^-}) -$$
$$-3U_1^- I_1^+ \cos(2\omega t + \psi_{u1^-} + \psi_{i1^+});$$
$$\tilde{q} = -3U_1^+ I_1^- \sin(2\omega t + \psi_{u1^+} + \psi_{i1^-}) +$$
$$+3U_1^- I_1^+ \sin(2\omega t + \psi_{u1^-} + \psi_{i1^+}),$$

where U_1^0, U_1^+, U_1^-, I_1^0, I_1^+, I_1^- are RMS values of zero, positive and negative sequences voltage and current, respectively; ψ_{u1^0}, ψ_{u1^+}, ψ_{u1^-}, ψ_{i1^0}, ψ_{i1^+}, ψ_{i1^-} – phase shift of zero, positive and negative sequences voltage and current, respectively; ω – angular frequency; t – time.

Analysis of these results, obtained in [2], shows the following:

1. In the general case, the zero sequence current and voltage have both sine and cosine orthogonal components, while at the same time there is no zero sequence imaginary power q_0.
2. None of the components of both imaginary q and real p powers have an imaginary zero-sequence power q_0, which indicates, in this case, the impossibility of reactive power flowing in the neutral wire.
3. Why is the imaginary power q, which is also a function of time, not included in the instantaneous power of the three-phase system, and, as indicated by the authors, only in some cases coincides with the reactive power Q?

Consider the case of a sinusoidal voltage and current a three-phase system, which we represent as follows

$$u_A = \sqrt{2} U_A \sin(\omega t + \psi_{uA});$$
$$i_A = \sqrt{2} I_A \sin(\omega t + \psi_{iA});$$

$$u_B = \sqrt{2}U_B \sin(\omega t + \psi_{uB} - 2\pi/3);$$
$$i_B = \sqrt{2}I_B \sin(\omega t + \psi_{iB} - 2\pi/3);$$
$$u_C = \sqrt{2}U_C \sin(\omega t + \psi_{uC} - 4\pi/3);$$
$$i_C = \sqrt{2}I_C \sin(\omega t + \psi_{iC} - 4\pi/3),$$

where U_A, U_B, U_C; I_A, I_B, I_C are RMS values of phases voltage and current, respectively; ψ_{iA}, ψ_{iB}, ψ_{iC}; ψ_{uA}, ψ_{uB}, ψ_{uC} – current and voltage phase shift, respectively. In the general case, the voltage currents RMS values and their phase shift in the ABC phases may differ. The power of each phase, in this case, is represented as [19]

$$p_{ph} = P_{ph.a.1-1}\cos(0) + P_{ph.b.1-1}\sin(0) +$$
$$+ P_{ph.a.1+1}\cos(2\omega t) + P_{ph.b.1+1}\sin(2\omega t).$$

Quadrature components' amplitudes of zero frequency for each phases:

- phase active power

$$P_{ph.a.1-1} = U_{ph}I_{ph}\cos(\psi_{u.ph} - \psi_{i.ph}) = P_{ph};$$

- phase reactive power

$$P_{ph.b.1-1} = -U_{ph}I_{ph}\sin(\psi_{u.ph} - \psi_{i.ph}) = Q_{ph}.$$

Quadrature components' amplitudes of double frequency for each phases:

- oscillating power cosine component

$$P_{A.a.1+1} = -U_A I_A \cos(\psi_{uA} + \psi_{iA});$$
$$P_{B.a.1+1} = -U_B I_B \cos(\psi_{uB} + \psi_{iB} - 4\pi/3);$$
$$P_{C.a.1+1} = -U_C I_C \cos(\psi_{uC} + \psi_{iC} - 2\pi/3);$$

- oscillating power sine component

$$P_{A.b.1+1} = U_A I_A \sin(\psi_{uA} + \psi_{iA});$$
$$P_{B.b.1+1} = U_B I_B \sin(\psi_{uB} + \psi_{iB} - 4\pi/3);$$
$$P_{C.b.1+1} = U_C I_C \sin(\psi_{uC} + \psi_{iC} - 2\pi/3).$$

Summing up the power by phases, we determine the three-phase system total instantaneous power

$$p_{3ph} = \sum_{A,B,C} P_{ph.a.1-1}\cos(0) + \sum_{A,B,C} P_{ph.b.1-1}\sin(0) +$$
$$+ \sum_{A,B,C} P_{ph.a.1+1}\cos(2\omega t) + \sum_{A,B,C} P_{ph.b.1+1}\sin(2\omega t) =$$
$$= P_{3ph.a.1-1}\cos(0) + P_{3ph.b.1-1}\sin(0) +$$
$$+ P_{3ph.a.1+1}\cos(2\omega t) + P_{3ph.b.1+1}\sin(2\omega t).$$

In symmetrical mode, consider the power using the example of the data given in Table 1. The vector diagram takes the form shown in Fig. 1. As can be seen from the diagram (Fig. 1, *a*), the total constant power of the three phases consists only of cosine components of zero argument, which in this case are summed up $P_{3\,ph.1-1} = 3P_{ph.a.1-1}$. The sine components of zero argument do not contribute to the instantaneous power and are shown in the diagram by a dotted line. The oscillating powers of a doubled argument component $P_{ph.1+1}$ in this case (Fig. 1, *b*) are the same in absolute value and, summed up, give a zero result. The corresponding total instantaneous power of the three-phase system is constant

$$p_{3ph} = 3P_{a.1-1}\cos(0) + 3P_{b.1-1}\sin(0) =$$
$$= P_{3ph}\cos(0) + Q_{3ph}\sin(0) = P_{3ph},$$

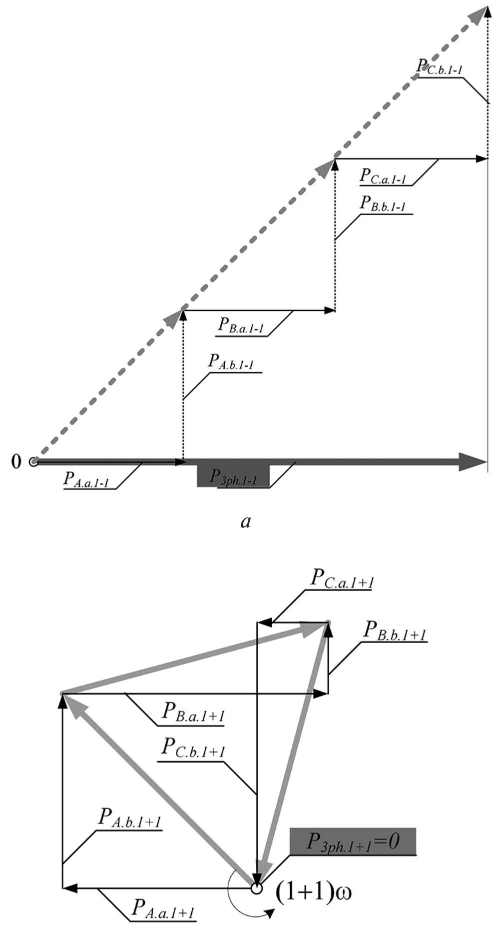
where $P_{3ph} = 3P_{ph.a.1-1} = 3U_{ph}I_{ph}\cos(\psi_{u.ph} - \psi_{i.ph})$ is the three-phase system's active power; $Q_{3ph} = 3P_{ph.b.1-1} = -3U_{ph}I_{ph}\sin(\psi_{u.ph} - \psi_{i.ph})$ – three-phase system reactive power.

The specified power, with asymmetry, Table 2, can be illustrated with a vector diagram shown in Fig. 2. As can be seen from the diagram (Fig. 2, *a*), the total constant power of the three phases consists only of cosine components of zero argument, as in the previous case. The zero argument sine components are shown by the dotted line. Due to the fact that in one of the phases the cosine component of the zero argument power has the opposite sign, the total three-phase power resulting value is not much higher than the power of phase *A* or *C*. The total three-phase power of the doubled argument (oscillated with frequency 2ω) $P_{3ph.1+1}$ (Fig. 2, *b*), is conditioned by the components' vector sum and exceeds the total three-phase power absolute value $P_{3ph.1-1}$. That is, in this case, the fluctuations in the total three-phase system power exceed the average value, respectively, the instantaneous power will be sign-alternating.

Table 1

Initial data for symmetrical mode

Ph	$\dot{U}$, V	$\dot{I}$, A	P, W	Q, Ar	$P_{a.1+1}$, VA	$P_{b.1+1}$, VA
A	50 ∠ 0°	1 ∠ 45°	35.35	35.35	–35.35	35.35
B	50 ∠ 120°	1 ∠ 165°	35.35	35.35	48.29	12.94
C	50 ∠ 240°	1 ∠ 285°	35.35	35.35	–12.94	–48.29



b

Fig. 1. Three-phase system's power vector diagrams with symmetry (Table 1)

Consider an asymmetric system in which a sinusoidal current flows and a sinusoidal voltage acts, represented by zero (0), positive (+) and negative (–) sequences

$$u_A = \sqrt{2}U_1^0 \sin(\omega t + \psi_{u1^0}) + \sqrt{2}U_1^+ \sin(\omega t + \psi_{u1^+}) +$$
$$+ \sqrt{2}U_1^- \sin(\omega t + \psi_{u1^-});$$
$$u_B = \sqrt{2}U_1^0 \sin(\omega t + \psi_{u1^0}) + \sqrt{2}U_1^+ \sin(\omega t + \psi_{u1^+} - 2\pi/3) +$$
$$+ \sqrt{2}U_1^- \sin(\omega t + \psi_{u1^-} - 4\pi/3);$$
$$u_C = \sqrt{2}U_1^0 \sin(\omega t + \psi_{u1^0}) + \sqrt{2}U_1^+ \sin(\omega t + \psi_{u1^+} - 4\pi/3) +$$
$$+ \sqrt{2}U_1^- \sin(\omega t + \psi_{u1^-} - 2\pi/3);$$

Table 2

Initial data for asymmetrical mode

Ph	$\dot{U}$, V	I, A	P, W	Q, VAr	$P_{a.1+1}$, VA	$P_{b.1+1}$, VA
A	$50 \angle 0°$	$1 \angle 45°$	35.35	35.35	–35.35	35.35
B	$50 \angle 200°$	$0.8 \angle -20°$	–30.64	25.71	–20	34.64
C	$40 \angle 270°$	$1 \angle 300°$	34.64	20	–34.64	–20

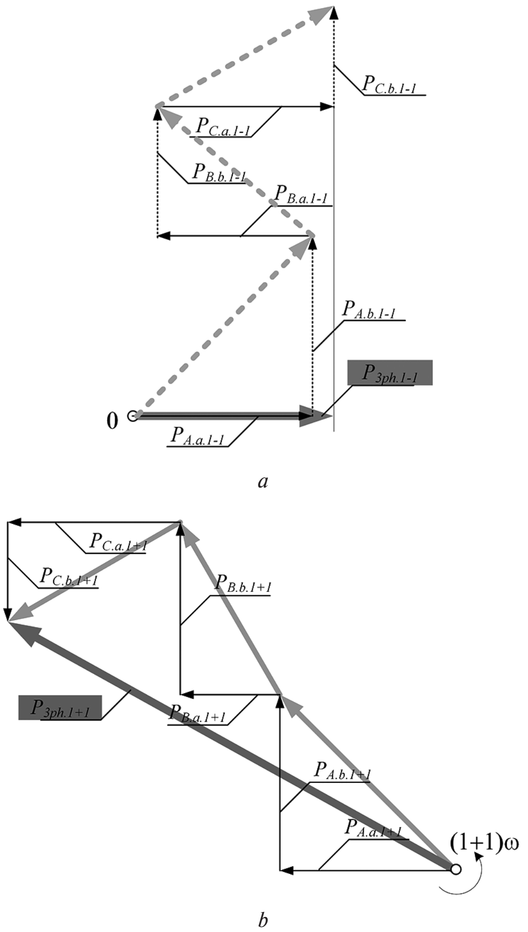


Fig. 2. Three-phase system's power vector diagrams with asymmetry (*Table 2*)

$$i_A = \sqrt{2} I_1^0 \sin(\omega t + \psi_{i1^0}) + \sqrt{2} I_1^+ \sin(\omega t + \psi_{i1^+}) + \sqrt{2} I_1^- \sin(\omega t + \psi_{i1^-});$$

$$i_B = \sqrt{2} I_1^0 \sin(\omega t + \psi_{i1^0}) + \sqrt{2} I_1^+ \sin(\omega t + \psi_{i1^+} - 2\pi/3) + \sqrt{2} I_1^- \sin(\omega t + \psi_{i1^-} - 4\pi/3);$$

$$i_C = \sqrt{2} I_1^0 \sin(\omega t + \psi_{i1^0}) + \sqrt{2} I_1^+ \sin(\omega t + \psi_{i1^+} - 4\pi/3) + \sqrt{2} I_1^- \sin(\omega t + \psi_{i1^-} - 2\pi/3).$$

The instantaneous power of such a three-phase system has a general form in accordance with [20]

$$p_{3ph} = P_{a.1-1} \cos(0) + P_{b.1-1} \sin(0) + P_{a.1+1} \cos(2\omega t) + P_{b.1+1} \sin(2\omega t).$$

The power orthogonal components' amplitudes of the zero frequency of a three-phase system are

$$P_{a.1-1} = 3U_1^0 I_1^0 \cos(\psi_{u1^0} - \psi_{i1^0}) + 3U_1^+ I_1^+ \cos(\psi_{u1^+} - \psi_{i1^+}) + 3U_1^- I_1^- \cos(\psi_{u1^-} - \psi_{i1^-}) = P_{a.1-1}^{00} + P_{a.1-1}^{++} + P_{a.1-1}^{--} = P_{3ph}$$

– three-phase system's active power, consisting of the corresponding sequences active powers;

$$P_{b.1-1} = -3U_1^0 I_1^0 \sin(\psi_{u1^0} - \psi_{i1^0}) - 3U_1^+ I_1^+ \sin(\psi_{u1^+} - \psi_{i1^+}) - 3U_1^- I_1^- \sin(\psi_{u1^-} - \psi_{i1^-}) = P_{b.1-1}^{00} + P_{b.1-1}^{++} + P_{b.1-1}^{--} = Q_{3ph}$$

– three-phase system's reactive power, consisting of the corresponding sequences reactive powers.

The oscillating power orthogonal components' amplitudes of the doubled frequency of a three-phase system are

$$P_{a.1+1} = -3U_1^0 I_1^0 \cos(\psi_{u1^0} + \psi_{i1^0}) - 3U_1^+ I_1^- \cos(\psi_{u1^+} + \psi_{i1^-}) - 3U_1^- I_1^+ \cos(\psi_{u1^-} + \psi_{i1^+}) = P_{a.1+1}^{00} + P_{a.1+1}^{+-} + P_{a.1+1}^{-+}$$

– amplitude of the cosine component of the three-phase system's oscillating power;

$$P_{b.1+1} = 3U_1^0 I_1^0 \sin(\psi_{u1^0} + \psi_{i1^0}) + 3U_1^+ I_1^- \sin(\psi_{u1^+} + \psi_{i1^-}) + 3U_1^- I_1^+ \sin(\psi_{u1^-} + \psi_{i1^+}) = P_{b.1+1}^{00} + P_{b.1+1}^{+-} + P_{b.1+1}^{-+}$$

– amplitude of the sine component of the three-phase system's oscillating power.

That is, in the general case, the power of a three-phase system is pulsating. By analogy with [20], we implement an approach related to determining the invariance power factor to generalize the electrical energy quality in a three-phase system

$$q_{inv} = \frac{P_{3ph}}{P_{rms.3ph}}.$$

To do this, we determine the root-mean-square value of the power of a three-phase system

$$P_{rms.3ph} = \sqrt{\frac{1}{T}\int_0^T p_{3ph}^2 dt} = \left[P_{a.1-1}^2 + \frac{P_{a.1+1}^2}{2} + \frac{P_{b.1+1}^2}{2}\right]^{0.5} =$$
$$= 3\Big[\left(U_1^0 I_1^0\right)^2 \cos^2(\psi_{u1^0} - \psi_{i1^0}) + \left(U_1^+ I_1^+\right)^2 \cos^2(\psi_{u1^+} - \psi_{i1^+}) + \left(U_1^- I_1^-\right)^2 \cos^2(\psi_{u1^-} - \psi_{i1^-}) +$$
$$+ 0.5\left(U_1^0 I_1^0\right)^2 + 0.5\left(U_1^+ I_1^-\right)^2 + 0.5\left(U_1^- I_1^+\right)^2 +$$
$$+ 2U_1^0 I_1^0 U_1^+ I_1^+ \cos(\psi_{u1^0} - \psi_{i1^0})\cos(\psi_{u1^+} - \psi_{i1^+}) +$$
$$+ 2U_1^0 I_1^0 U_1^- I_1^- \cos(\psi_{u1^0} - \psi_{i1^0})\cos(\psi_{u1^-} - \psi_{i1^-}) +$$
$$+ 2U_1^+ I_1^+ U_1^- I_1^- \cos(\psi_{u1^+} - \psi_{i1^+})\cos(\psi_{u1^-} - \psi_{i1^-}) +$$
$$+ U_1^0 I_1^0 U_1^- I_1^+ \cos(\psi_{u1^0} + \psi_{i1^0} - \psi_{u1^-} - \psi_{i1^+}) +$$
$$+ U_1^0 I_1^0 U_1^+ I_1^- \cos(\psi_{u1^0} + \psi_{i1^0} - \psi_{u1^+} - \psi_{i1^-}) +$$
$$+ U_1^+ I_1^+ U_1^- I_1^- \cos(\psi_{u1^+} - \psi_{i1^+} - \psi_{u1^-} + \psi_{i1^-})\Big]^{0.5}.$$

Then the invariance power factor of the three-phase system will be presented as follows

$$q_{inv} = \frac{P_{3ph}}{P_{rms.3ph}} = \left[\frac{P_{a.1-1}^2}{P_{a.1-1}^2 + \frac{P_{a.1+1}^2}{2} + \frac{P_{b.1+1}^2}{2}}\right]^{0.5} = \left[1 + \frac{P_{a.1+1}^2 + P_{b.1+1}^2}{2P_{a.1-1}^2}\right]^{-0.5}.$$

To characterize the asymmetry level of a three-phase system, negative sequence voltage coefficients are usually used $K_U^- = U_1^- / U_1^+$ and for current $K_I^- = I_1^- / I_1^+$; voltage zero sequence coefficients $K_U^0 = U_1^0 / U_1^+$ and for current $K_I^0 = I_1^0 / I_1^+$. Then, in the general case, the invariance power factors are

$$q_{inv} = (1 + (\cos(\psi_{u1^+} - \psi_{i1^+}) + K_U^- K_I^- \cos(\psi_{u1^-} - \psi_{i1^-}) + K_U^0 K_I^0 \cos(\psi_{u1^0} - \psi_{i1^0}))^{-2} \times$$
$$\times [K_U^0 K_I^0 K_U^- \cos((\psi_{u1^0} + \psi_{i1^0}) - (\psi_{u1^-} + \psi_{i1^+})) +$$
$$+ K_U^0 K_I^0 K_I^- \cos((\psi_{u1^0} + \psi_{i1^0}) - (\psi_{u1^+} + \psi_{i1^-})) +$$
$$+ K_U^- K_I^- \cos((\psi_{u1^+} + \psi_{i1^+}) - (\psi_{u1^-} + \psi_{i1^-}))])^{-0.5}.$$

The invariance power factor has a complex dependence not only on the asymmetry coefficient indicators of voltage

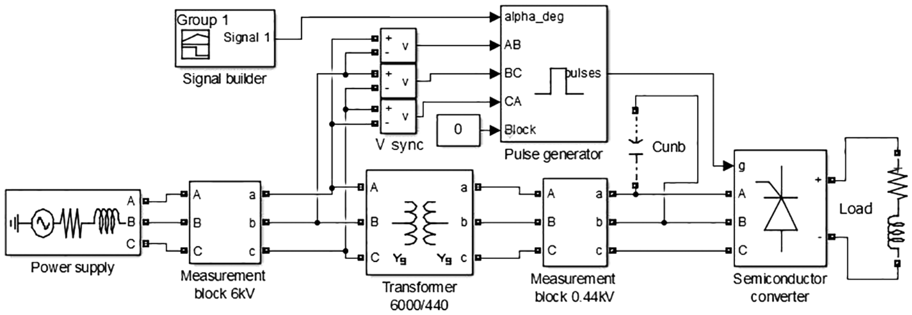


Fig. 3. Power part diagram of the studied model

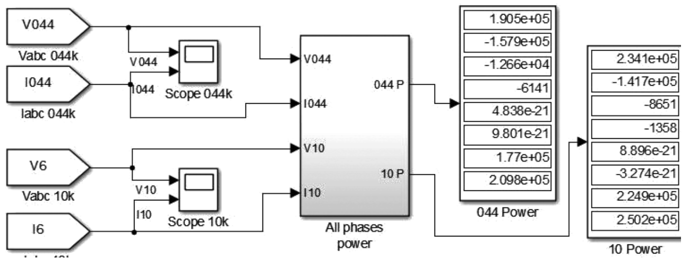


Fig. 4. Power component calculation system

and current, but also on the phase shift between the voltage (current) of the negative sequence and the current (voltage) of the positive sequence.

Consider the practical use of the proposed electric power components for asymmetric modes in a simple power system shown in Fig. 3. The scheme consists of a power supply (Power supply: $U_{rms} = 6$ kV; $R_s = 10$ Ohm; $L_s = 5$ mH), a step-down transformer (Transformer 6000/430: Yg/Yg; $S_r = 250$ kVA; $U_1 = 6$ kV; $R_1 = 1.066$ Ohm; $L_1 = 9.74$ mH; $U_2 = 430$ V; $R_2 = 16.5$ mOhm; $L_2 = 39$ μH; $R_\mu = 925$ Ohm; $L_\mu = 23.94$ H), controlled converter (Semiconductor converter) and active-inductive load (Load; $R_{ld} = 1.63$ Ohm; $L_{ld} = 5$ mH) in the DC circuit. Control of phase voltages and currents takes place on the high voltage side by the measuring block (Measurement block 6 kV) and on the low voltage side by the measuring block (Measurement block 0.44 kV). The converter elements are controlled by a pulse-phase control unit (Pulse generator), which is synchronized with the network by voltage meters unit (V sync). The control angle is formed by the corresponding signal setting unit (Signal builder).

The system, shown in Fig. 4, is built to determine the power components. The system's input elements are units that transmit voltage and current signals from measuring units from the low voltage side (Vabc 044k, Iabc 044k) and from the high voltage side (Vabc 6k, Iabc 6k). The signals of these units enter the corresponding inputs of the block for power components determining unit (All phases power). The power components calculated values on the low voltage side are formed at the output 044P, high voltage – 6P. Additionally, phase voltage and current signals are output to the oscilloscope inputs (Scope 044k, Scope 6k). The power components values are indicated on the displays (044k Power components, 6k Power components).

The content of a power component determination subsystem is shown in Fig. 5. In the scheme shown in Fig. 3, during the operation of the controlled converter, the current and voltage are generally non-sinusoidal, so the power component determination subsystem is configured only for the fundamental harmonic (50 Hz). The voltage and current signals from the input ports (V044, I044) are fed to the units for calculating the positive sequence active P^+ and reactive Q^+ powers (044 Power (+), negative sequence active P^- and reactive Q^- powers (044 Power (–)), zero sequence active P^0 and reactive Q^0 power (044 Power (0)). Additionally, the three phases' instantaneous power is calculated using the multiplication unit and the summation unit. Using the average value determining unit (044P Mean), the three-phase system's active power P_{3ph} is calculated. The root-mean-square value calculation of the three-phase system's power $P_{rms.3ph}$ is provided by the unit (044P RMS). The calculated power components are multiplexed and transmitted to the output port (044 P). In a similar way, the power components' determining subsystem for the 6 kV part was built.

We will investigate modes of the model shown in Fig. 3. We will research the influence of load asymmetry on power components. First, let us make the transformer "weak". To do this, we will increase the secondary three-phase windings' active resistance of the transformer by four times. We will follow modes simulate:

a) symmetrical mode;

b) asymmetrical mode with additional capacitor connection between phases A and B at the controlled converter input;

c) asymmetrical mode with additional capacitor connection between phase A and ground.

Fig. 6 shows the voltages' and currents' time diagrams, respectively, for the modes of symmetry (*a*), interphase asymmetry (*b*) and phase asymmetry (*c*). Intentional "weakening" of the transformer led to the fact that, in addition to current distortion (Fig. 6, *a*), under the current asymmetry condition, voltage asymmetry arose (Figs. 6, *b* and *c*). Disregarding cur-

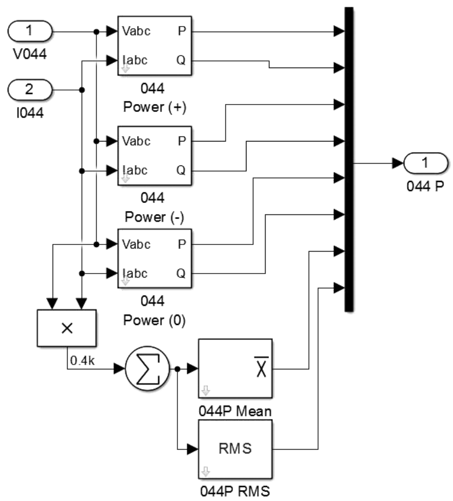
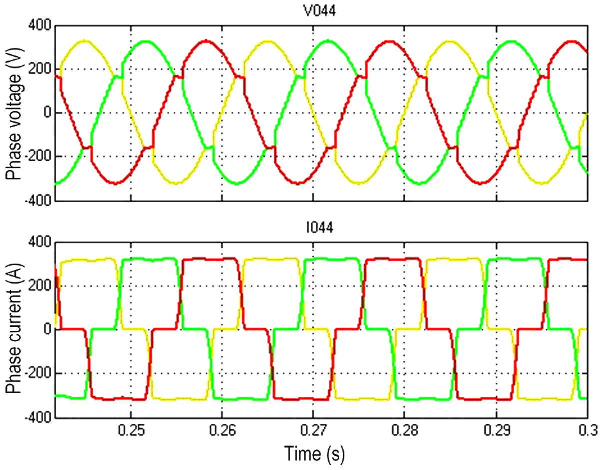
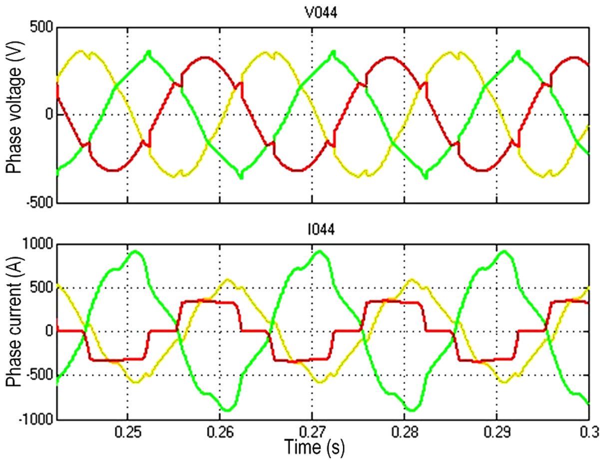


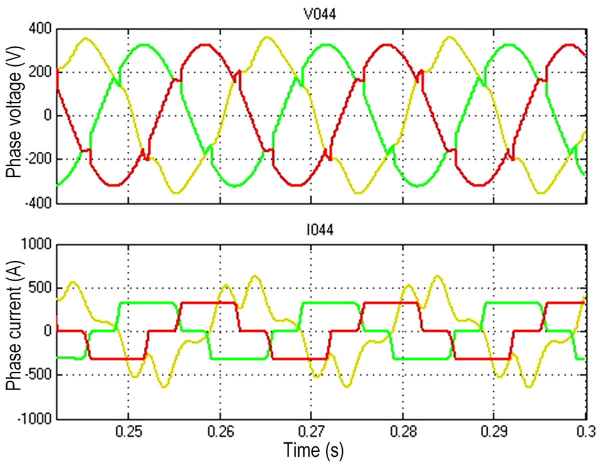
Fig. 5. Power component determination subsystem (part 044k)



a



b



c

Fig. 6. Transformer voltages' and currents' time diagrams on the voltage side of 0.44 kV:

a – symmetrical load; b – interphase asymmetry; c – asymmetry of one phase

rent and voltage asymmetry coefficients, let us immediately turn to the power components, which are summarized for the corresponding modes in Table 3.

Analysis of the data presented in Table 3 allows us to make the following judgments. In the symmetrical mode, the active power of the direct sequence P^+ slightly differs from the active power P: $\Delta P = P - P^+ = -9 \cdot 10^2$ W. In this mode, the invariance power factor is $q_{inv} = 0.998$. Probable influence of higher harmonics. The implementation of mode (*b*) leads to a significant increase in the positive sequence power P^+, its difference from the active power P increases by $\Delta P = P - P^+ = -13.5 \cdot 10^3$ W. This value, with a slight difference, corresponds to the negative sequence's active power P^-. Thus, the negative sequence's generated power leads to a decrease in the active power, which is counted by the accounting means.

In addition, let us pay attention to the display indication in Fig. 4. The difference in the active powers of the positive sequence P^+ (first line) on the low and high voltage side is the power loss in the transformer from this sequence, W

$$\Delta P_{TV}^{+} = P_{6k}^{+} - P_{044k}^{+} = 2.341 \cdot 10^5 - 1.905 \cdot 10^5 = 43.6 \cdot 10^3.$$

In addition, the difference in the negative sequence's active powers P^- (third line) on the low and high voltage side also adds to the power losses in the transformer, W

$$\Delta P_{TV}^{-} = P_{6k}^{-} - P_{044k}^{-} = -8651 + 1.266 \cdot 10^4 = 4009.$$

That is, in this case, this is an additional 9.1 % of power losses. If we calculate the transformer efficiency by active power as a whole, we will get

$$\eta_{TV} = P_{044k}/P_{6k} = 1.77 \cdot 10^5/2.249 \cdot 10^5 = 0.787,$$

and the same indicator according to the positive sequence power

$$\eta_{TV} = P_{044k}^{+} \big/ P_{6k}^{+} = 1.905 \cdot 10^5 \big/ 2.341 \cdot 10^5 = 0.814.$$

Therefore, the isolation of the component sequences affects the calculations of generalized indicators, including the objects of the power transmission system, in this case the efficiency of the transformer. This can lead to erroneous judgments about the efficiency of the functioning of the specified facilities. Let us pay attention to the fact that in this case the coefficient of power constancy also changed significantly from $q_{inv} = 0.998$ for symmetrical mode to $q_{inv} = 0.843$, signaling the deterioration of the quality of electric energy.

When implementing an asymmetric mode with the introduction of a capacitor between phase A and ground (mode c), the difference between the direct sequence power P^+ and the active power P was $\Delta P = P - P^+ = -8.4 \cdot 10^3$ W. At the same time, components of zero sequence power, both active and reactive, naturally appeared. Inverse sequence power components also arose. Considering the relatively insignificant level of these capacities, the coefficient of constancy takes the value $q_{inv} = 0.968$. This indicates a deterioration in the quality of electrical power (electrical energy) of mode *c* relative to mode *a*. Of course, higher harmonics should be taken into account in regimes with non-sinusoidal currents and voltages [19].

Conclusions.

1. According to the analysis results of the instantaneous power components according to the *pq* theory, it was found that none of the components, both real and imaginary instantaneous power components of the three-phase system, take into account the zero-sequence reactive power.

2. The three-phase system's instantaneous power components are analytically determined, including the amplitudes of the oscillating power components. The need to take into account the oscillating instantaneous power components has been proven by means of a graphical interpretation of the three-phase system mode special case. As an integral indicator that takes into account the oscillating components of the three-phase system's instantaneous power, its root-mean-square value over the repetition period is used.

3. The invariance power factor was used to characterize the electrical energy quality level of a three-phase sinusoidal current system in an asymmetrical mode. This indicator is determined analytically using the negative and zero sequence coefficients for voltage and current, respectively, taking into account the difference in the phase shift of voltages and currents of the same and different sequences.

4. A model of an elementary electric power system with a "weak" transformer and a controlled three-phase rectifier was

developed for the practical testing of the obtained theoretical results. By adding a capacitor, asymmetrical modes have been realized and an increase in the power losses level in the transformer due to the negative and zero sequences power components has been noted. This leads to a change of invariance power factor.

5. By calculating the transformer efficiency of the studied model according to the positive sequence's active power components and the same indicator according to the active power as a whole, it was established that the separation of the component sequences affects the calculation results for generalized indicators, including the power transmission system objects. This can lead to erroneous judgments about the efficiency of the specified objects functioning.

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Облік потужностей прямої, оберненої та нульової послідовностей у несиметричній трифазній електричній системі

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Мета. Визначення, на підставі миттєвої електричної потужності трифазної несиметричної системи синусоїдального періодичного струму, активної та реактивної потужності прямої, оберненої та нульової послідовностей і фактору незмінності потужності.

Методика. У разі несиметрії у трифазній електричній системі якість електричної енергії оцінюється за напругою та струмом прямої, оберненої, нульової послідовності. При цьому аналогічні складові активної й реактивної потужності не отримали практичного поширення. Але саме по потужності ведеться облік електроенергії. Ортогональні компоненти миттєвої потужності у часовій області визначені за допомогою симетричних компонентів напруги та струму. Виділені активна, реактивна потужності прямої, оберненої й нульової послідовностей. Отриманий результат має властивість репрезентативності, якої не має більшість відомих результатів.

Результати. Аналітично визначені миттєві складові потужності трифазної системи, у тому числі амплітуди коливальних складових потужності. За допомогою графічної інтерпретації окремого випадку режиму трифазної системи доведена необхідність урахування коливальних складових миттєвої потужності. Як інтегральний показник, що враховує коливальні складові миттєвої потужності трифазної системи, використовується її середньоквадратичне значення за період повторення.

Наукова новизна. Шляхом розрахунку коефіцієнту корисної дії трансформатора досліджуваної моделі за активної потужності прямої послідовності й такого ж показника за активної потужності у цілому встановлено, що на результати розрахунку узагальнених показників, у тому числі об'єктів електроенергетичної системи, впливає розмежування послідовностей компонент потужності. Це може призвести до помилкових суджень щодо ефективності функціонування зазначених об'єктів електроенергетичної системи.

Практична значимість. Для характеристики рівня якості електричної енергії трифазної системи синусоїдального струму в несиметричному режимі використано коефіцієнт незмінності потужності.

Ключові слова: *електрична потужність, середньоквадратичне значення потужності, якість електроенергії*

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