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EXPERIMENTAL RESEARCH ON MUFFLE FURNACE DYNAMIC PROPERTIES

Purpose. To develop the method and software for building empirical models of muffle furnaces with two heating elements.

Methodology. The method for processing experimental data according to the improved algorithm, which is based on the areas method, is used for constructing empirical models of the muffle furnace. Experimental research of the muffle furnace dynamic is conducted using the following methodology. The muffle furnace is cooled to the room temperature and then the lower heater is turned on and temperatures at the furnace output are fixed using the experimental two-channel temperature controller MIC-344. The second cycle of the experimental research began by cooling the furnace to the room temperature with turning on the upper heater. Observations based on the results of the experiment are conducted until the temperature is stabilized at the two outputs of the muffle furnace. Archiving of temperature trends is carried out using the RS485/Ethernet interface and the SmartReview software (a product of private limited company "Microl").

Findings. The developed software can be used for iteratively selection of the optimal empirical model based on the criterion of the root mean square deviation of the calculated and experimental data. The calculating method for the acceleration curve based on the transfer functions coefficients with a variable discreteness step is improved in order to compare the experimental data with the empirical modeling results. Twelve empirical models were tested for four signal transmission channels in the process of experimental research. It has been established that only four of them are stable (have left-hand roots of the characteristic equation). Among the selected sustainable models, the model with the numerator polynomial equaled to two and denominator polynomial equaled to three has been established as the best model (according to the established criterion). A structural diagram is created based on the synthesized empirical model of the muffle furnace, which includes four transfer functions and cross connections.

Originality. For the first time, the empirical model of a muffle furnace with two electrical energy sources has been developed, which describes its dynamic properties as the object of automatic control with high accuracy, which enables to reveal the presence of internal cross-connections, that significantly complicate the controlling process of the object.

Practical value. The created empirical models with two electric sources (tens) can be used for the synthesis of the high-precision automatic temperature control system of the muffle furnace.

Keywords: *muffle furnace, temperature, control system, dynamics, model, transfer function*

Introduction. Muffle furnaces are used to heat various small products to a given temperature. A design peculiarity of muffle furnaces is the presence of a muffle [1], which is made of heat-resistant material and delimits the working space of the furnace and the heated sample (Fig. 1).

Modern muffle furnaces are universal heating devices that are used both for laboratory research and for heating small products.

To ensure the tightness of muffle furnaces, their doors are made of steel and insulated with thermal insulation. Thermal insulation materials must withstand high temperatures in the range from 100 to 2000 °C, so the muffle is made of heat-resistant materials such as perlite, corundum, fiber or ceramics.

Muffle furnace is a unit of periodic action. Heating processes in a muffle furnace proceed in three stages: heating the furnace to a certain temperature, operating mode and cooling. The first two stages must be performed in the automatic mode and ensure the necessary accuracy of temperature maintenance during the implementation of the second stage. In order to successfully perform the given task, it is necessary to have a mathematical model of the muffle furnace as an object of automatic control.

Literature review. The problem of pattern recognition is one of the most difficult problems of cybernetics [2]. The quality of the processes taking place in the muffle furnace largely depends on the temperature regime.

Maintenance of the necessary temperature regime is carried out by means of automation, the quality indicators of which depend on adequate mathematical models characterizing the dynamic properties of muffle furnaces.

Traditionally, the dynamic properties of a muffle furnace are described [3] by a first-order aperiodic term.

Increasing requirements for the quality and accuracy of temperature regulation in a muffle furnace, especially in complex and precise technological processes, creates a need for more adequate mathematical models.

The authors of the work [4] describe the dynamics of muffle furnace operation by a system of two non-linear differential equations that include a number of parameters, such as fireclay thickness, furnace surface area, heat transfer coefficient of asbestos wool, which can be determined only with a certain approximation. This reduces the value of such a model for analysis and synthesis of automatic temperature control systems of the muffle furnace.

In work [5], the electric resistance furnace is considered as an object with distributed parameters. The resulting mathematical model is a non-linear one and, in addition, it does not explicitly include the input value as a control action, which makes it unsuitable for the synthesis of the control system.

It should be noted that in works [5, 6], mathematical models are used to calculate the technological parameters of muffle furnaces and develop technological regulations.

Therefore, the mathematical models of the temperature regimes of muffle furnaces proposed by various authors have limited application, since they are approximate or include parameters that require time-consuming and expensive experiments to determine.

Another way to obtain mathematical models suitable for the synthesis of automatic systems for controlling the temperature regime of muffle furnaces is to conduct experimental studies in order to obtain acceleration curves.

Thus, the goal of the work is the construction of empirical mathematical models of the muffle furnace based on experimental research and the development of algorithms and software for the problem of parametric identification of the muffle furnace as an object of automatic control.

Methods of conducting experimental research. The object of experimental research was a high-temperature muffle furnace. Heating of the furnace was carried out with the help of two electric furnaces – lower and upper ones. Two TXA-type thermocouples were used to measure the temperature. The "upper" thermocouple measured the temperature of the upper edge of the sample, and the lower one measured the temperature of the lower edge of the product.

At the beginning of the experimental study, the muffle furnace was cooled to a temperature of 27.75 °C. After that, the upper heater was disabled. The power of the lower heater has been increased by 40 percent.

Fig. 1 shows a scheme for conducting an experimental study on a muffle furnace with the top furnace turned off.

In Fig. 1 the following designations are accepted: U_1 – control action (input value); T_{Upp} – the temperature recorded by the "upper" sensor; T_{Low} – the temperature measured by the "bottom" sensor.

A similar scheme for conducting experimental studies on the muffle furnace was chosen for the upper furnace (Fig. 2).

After conducting an experimental study with the lower heater was turned off, the muffle furnace was cooled to a temperature of 25 °C. Then the upper heater was turned on, while the power of the upper heater was increased by 40 %. In Fig. 2: U_2 is control action (input value); T_{Upp} and T_{Low} – output values.

The results of temperature measurements T_{Upp} and T_{Low} were recorded using an experimental two-channel software temperature controller MIK-344 (manufactured by Mikrol LLC). The temperature measurement error for the temperature range of 0–1300 °C is not worse than 0.02 °C (ADC-16 digits).

The regulator was switched to the manual mode and from its front panel or from the NMI operator's panel, the heating power of the corresponding tank was changed in leaps and bounds through the BUS-31 triac amplifier with numerical-pulse control. Archiving of temperature trends can be implemented both in the NMI and in a computer via the RS485/Ethernet interface. Since the SmartReview program (a product of Mikrol LLC) installed on the computer provides the best service for archiving and processing arrays, xlsx files were created with its help in the Excel software environment, which were then processed in the Matlab software product.

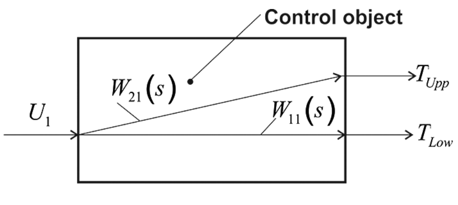

Fig. 1. Scheme of the experimental study on the muffle furnace (lower part)

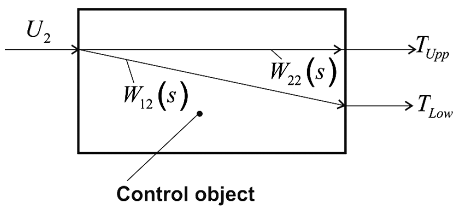

Fig. 2. Scheme of the experimental study on the muffle furnace (upper part)

Synthesis of an empirical model of a muffle furnace. The analysis of experimental data showed that the muffle furnace as an object of automatic control is a multidimensional object and has four signal transmission channels: input $U_1 - T_{Upp}$ and T_{Low} outputs; input $U_2 - T_{Upp}$ and T_{Low} outputs (Fig. 3).

A preliminary analysis of the results of the study on the dynamic properties of the muffle furnace as a control object showed that its transient characteristics are aperiodic in nature and can be described in the general case by the following transfer functions [7]

$$W(p) = \frac{\sum_{j=0}^{m} B_j p^j}{\sum_{i=0}^{n} A_i p^i}, \qquad (1)$$

where $A_i, i = \overline{0,n}$, $B_j, j = \overline{0,m}$ are parameters of the transfer function and are constant values (1).

For real objects, the $m < n$ condition is always met.

Let us give the ratio (1) in the following form

$$w(p) = \frac{1 + \sum_{j=1}^{m} b_j p^j}{1 + \sum_{i=1}^{n} a_i p^i}, \qquad (2)$$

where $w(p) = \frac{W(p)}{k}$ – the normalized transfer function; $k = \frac{B_0}{A_0}$ – the object transfer coefficient; $a_i = \frac{A_i}{A_0}$, $i = \overline{1,n}$, $b_j = \frac{B_j}{B_0}$, $j = \overline{1,m}$ are parameters of the normalized transfer function.

Let us set the problem: based on the results of experimental studies on the dynamic properties of the muffle furnace, determine the structure and parameters of the normalized transfer function (2).

We will divide the solution of the created problem into two stages. At the first stage, we will choose the structure of the transfer function (2). Such a choice is made by setting the values of the powers of the numerator m and denominator n polynomials.

The criterion for selecting the structure of the normalized transfer function (2) will be the sum of the differences of the squares of the deviations of the calculated values from the corresponding experimental values, i.e.

$$Er = \sum_{k=1}^{N} \left(y(t_k) - y_{ex,k} \right)^2, \qquad (3)$$

where N – the number of observations in an experimental study.

To provide an algorithm for determining the structure of the normalized transfer function (2) of a universal nature, we will calculate the function $y(t)$ using a numerical method.

The following differential equation corresponds to the normalized transfer function

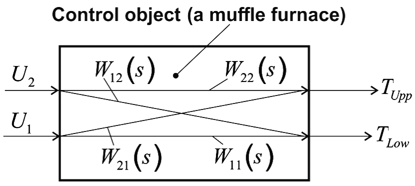

Fig. 3. Structural diagram of the muffle furnace as an object of automatic control

$$\sum_{i=0}^{n} a_i \frac{d^i y(t)}{dt^i} = \sum_{j=0}^{n} b_j \frac{d^j u(t)}{dt^j}. \tag{4}$$

It is assumed that the orders of the derivatives in the right and left parts of the differential equation (4) are equal. If this is not the case, then the coefficients $b_j, j=\overline{m+1,n}$ of equation (4) should be set to zero. Then equation (4) can be represented as a system of differential equations [7, 8], each of which is a first-order differential equation. Therefore,

$$\begin{aligned} &\frac{dx_i}{dt} = x_{i+1} + \beta_i u, \quad i=\overline{1,n-1}; \\ &\frac{dx_n}{dt} = -\frac{1}{a_n}\sum_{j=1}^{n} a_{j-1}x_j + \beta_n u; \\ &y = x_1 + \beta_0 u. \end{aligned} \tag{5}$$

The unknown values β_i, $i=\overline{1,n-1}$ were calculated as the solution of the system of linear algebraic equations

$$\sum_{j=i}^{n} a_j \beta_{j-i} = b_i, \quad i=\overline{0,n}. \tag{6}$$

Let us write the system of equations (6) in matrix-vector form

$$A\bar{\beta} = \bar{b}, \tag{7}$$

where $A = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{n-1} & a_n \\ a_1 & a_2 & a_3 & \cdots & a_n & 0 \\ a_2 & a_3 & a_4 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n-1} & a_n & 0 & \cdots & 0 & 0 \\ a_n & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$; $\bar{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \cdots \\ \beta_n \end{bmatrix}$; $\bar{b} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$.

Since the matrix A is upper triangular, then the solution of the system of equations (5) can be found by the method of backtracking, or as a solution of the vector-matrix equation (7)

$$\bar{\beta} = A^{-1}\bar{b}.$$

The system of equations (5) can also be presented as a matrix-vector equation

$$\begin{aligned} &\frac{d\bar{x}}{dt} = A_r\bar{x} + \bar{\beta}_r u; \\ &y = \bar{n}^T\bar{x} + \beta_0 u, \end{aligned} \tag{8}$$

where $A_r = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & \cdots & -\frac{a_{n-1}}{a_n} \end{bmatrix}$; $\bar{\beta}_r = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \cdots \\ \beta_n \end{bmatrix}$; $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdots \\ x_n \end{bmatrix}$; $\bar{n} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdots \\ 0 \end{bmatrix}$.

Equation (8) will be solved using the Runge-Kutta method [8].

After the structure of the transfer function (2) is selected, at the second stage of solving the given problem, based on the results of an experimental study on the dynamics of the muffle furnace, it is necessary to determine the parameters of the transfer function (2). For this, we will use the area method [9, 10].

The following normalized transfer function corresponds to the differential equation (4)

$$w(p) = \frac{1+\sum_{j=1}^{n} b_j p^j}{1+\sum_{i=1}^{n} a_i p^i}. \tag{9}$$

The essence of the area method is that the function, which is the inverse of the transfer function (9), is expanded into a Taylor series. As a result, the following expression is obtained

$$\frac{1}{w(p)} = 1 + S_1 p + S_2 p^2 + S_3 p^3 + \cdots. \tag{10}$$

The coefficients S_i, i = 1, 2, 3,... of the infinite series (10) have the content of the area of the k^{th} order [9]. If formula (9) is considered, expression (10) makes it possible to determine [10] the relationship between the values S_i, i = 1, 2, 3,... and the parameters of the transfer function (9)

$$a_k = \sum_{i=1}^{k-1} b_i S_{k-i} + b_k + S_k, \quad k = 1, 2, \ldots, N. \tag{11}$$

The unknowns $a_i, i=\overline{1,n}$ and $b_j, j=\overline{1,m}$ in the system of equations (11) are the coefficients and the normalized transfer function (2). To determine them, we present the system of equations in a matrix-vector form [10]. Therefore,

$$A_s\bar{\alpha} = \bar{S}, \tag{12}$$

where $N = m + n$; $A_s = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & \cdots & 0 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & \cdots & 0 & -S_1 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & \cdots & 0 & -S_2 & -S_1 & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & \cdots & 0 & -S_{k-1} & -S_{k-2} & -S_{k-3} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & -S_{N-1} & -S_{N-2} & -S_{N-3} & \cdots & -1 \end{bmatrix}$; $\bar{\alpha} = \begin{bmatrix} a_1 \\ a_2 \\ \cdots \\ a_n \\ b_1 \\ b_2 \\ \cdots \\ b_m \end{bmatrix}$; $\bar{S} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_k \\ \cdots \\ S_N \end{bmatrix}$.

The matrix A_s, which has the $(m + n) \times (m + n)$ size, is formed as follows [10]: in the upper left part, write the unit matrix of the $n \times n$ size; a zero matrix of the $m \times n$ size is placed under it. The remaining part of the matrix A_s is recommended to be filled with columns, starting from the $(n + 1)^{th}$ column. The first element of the $(n + 1)^{th}$ column will be –1. Other elements of the $(n+1)^{th}$ column will be the values $-S_i$, $i=\overline{1,N-1}$. The next columns of the matrix A_s are formed from the previous ones with a zero shift by one element down. The elements $m + n$ column of the matrix A_s are zeros, except for the $m + n$ – element. Its value is –1.

The values of the areas S_i, $i=\overline{1,N}$ are calculated [10] as a solution of the matrix-vector equation

$$\Lambda\bar{S} = \bar{\mu}, \tag{13}$$

$$\text{where } \Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -\mu_1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ -\mu_2 & -\mu_1 & 1 & 0 & \cdots & 0 & 0 \\ -\mu_3 & -\mu_2 & -\mu_1 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -\mu_{N-1} & -\mu_{N-2} & -\mu_{N-3} & -\mu_{N-4} & \cdots & -\mu_1 & 1 \end{bmatrix}; \quad \bar{S} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ \cdots \\ S_N \end{bmatrix}; \quad \bar{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \cdots \\ \mu_N \end{bmatrix}.$$

In the matrix-vector equation, the following notations are adopted

$$\mu_k = \frac{1}{(k-1)!}\int_0^\infty (-t)^{k-1}\varphi(t)dt, \quad k = 1,2,\ldots; \tag{14}$$

$$\varphi(t) = 1 - h(t), \tag{15}$$

where $h(t)$ – the transient characteristic corresponding to the transfer function (2).

The matrix Λ is a lower diagonal matrix of the $N \times N$ size.

The function $\varphi(t)$ is built based on the results of an experimental study of object dynamics. As an example, Fig. 4 shows the graph of the dependence built on the results of an experimental study on the dynamics of a muffle furnace using the signal transmission channel $U_1 - T_{Upp}$.

Since the value of the function $\varphi(t)$ is known only at discrete moments of time t_i, the integration of the expression in formula (14) is possible only by a numerical method. The smallest integration error is provided by Simpson's method [10], which is based on three-point Lagrange interpolation.

We assume that the interpolation step along the abscissa axis is uneven. Then the Lagrange function will be as follows [8, 10]

$$L(t) = \sum_{k=1}^{N_1} \varphi_k \frac{\prod_{i=1, i\neq k}^{N_1}(t - t_i)}{\prod_{i=1, i\neq k}^{N_1}(t_k - t_i)}, \tag{16}$$

where $\varphi_k = \varphi(t_k)$; N_1 is the number of interpolation points.

The interpolation polynomial (16) for three points (Fig. 5) will be as follows

$$L(\tau) = \frac{y_1}{h_1(h_1+h_2)}(\tau^2 - (\tau_2+\tau_3)\tau + \tau_2\tau_3) -$$
$$-\frac{y_2}{h_1 h_2}(\tau^2 - (\tau_1+\tau_3)\tau + \tau_1\tau_3) + \frac{y_3}{h_2(h_1+h_2)}(\tau^2 - (\tau_1+\tau_2)\tau + \tau_1\tau_2),$$

where $\tau_1 = t_i$; $\tau_2 = t_{i+1}$; $\tau_3 = t_{i+2}$; $y_1 = (-t_i)^{k-1}\varphi(t_i)$; $y_2 = (-t_{i+1})^{k-1}\varphi(t_{i+1})$; $y_3 = (t_{i+2})^{k-1}\varphi(t_{i+2})$; $k = 1, 2, \ldots$; $h_1 = \tau_2 - \tau_1$; $h_2 = \tau_3 - \tau_2$; $h_1 + h_2 = \tau_3 - \tau_1$.

Let us write the integral separately, which is included in formula (14)

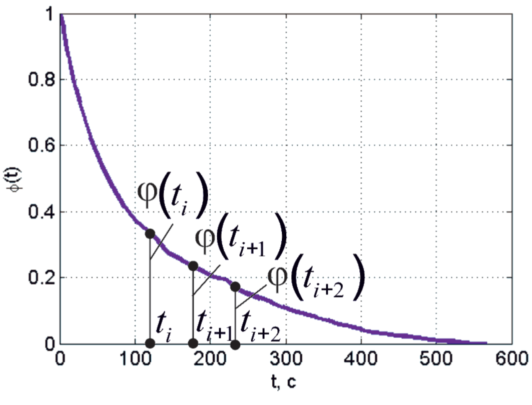

Fig. 4. Function $\varphi(t)$ with interpolation node

$$I_k = \int_0^\infty (-t)^{k-1}\varphi(t)dt, \quad k = 1,2,\ldots. \tag{17}$$

It is obvious that the value of the integral (17) will be equal to the sum of the areas, each of which is calculated as the area bounded by the curve $L(\tau)$ on the segment $\tau \in [\tau_1, \tau_3]$. Therefore,

$$s = \int_{\tau_1}^{\tau_3} L(\tau)d\tau.$$

Considering the value of $L(\tau)$, we find

$$s = \frac{y_1}{h_1(h_1+h_2)}\left(\frac{z_3}{3} - (\tau_2+\tau_3)\frac{z_2}{2} + \tau_2\tau_3 z_1\right) -$$
$$-\frac{y_2}{h_1 h_2}\left(\frac{z_3}{3} - (\tau_1+\tau_3)\frac{z_2}{2} + \tau_1\tau_3 z_1\right) + \tag{18}$$
$$+\frac{y_3}{h_2(h_1+h_2)}\left(\frac{z_3}{3} - (\tau_1+\tau_2)\frac{z_2}{3} + \tau_1\tau_2 z_1\right),$$

where $z_1 = \tau_3 - \tau_1$; $z_2 = \tau_3^2 - \tau_1^2$; $z_3 = \tau_3^3 - \tau_1^3$.

Since for the calculation of each area s according to formula (18) it is necessary to have three points, the total number of such areas will be $N/2$, where N – the number of observations in the experimental study. Then

$$I_k \simeq \sum_{j=1}^{N/2} s_j^{(k)}. \tag{19}$$

The values of areas $s_j^{(k)}$ are calculated by formula (18) for index values $j = \overline{0, N}$, where N must be an even number. If N – is an odd number, then the last y_N ordinate is discarded.

In the software implementation of the model parameter calculation algorithm (2), the number of values μ_k rarely exceeds five.

Knowing the value of I_k and using the (14), it is possible to calculate the values of μ_k, $k = \overline{1, N_f}$, where $N_f \leq 5$.

After determining the value of μ_k, $k = \overline{1, N_f}$ from equation (13) we get the following value

$$\bar{S} = \Lambda^{-1}\bar{\mu}. \tag{20}$$

Having found the areas $\bar{S}$, you can calculate the parameters of the mathematical model (2) using (12). Therefore,

$$\bar{\alpha} = A_s^{-1}\bar{S}. \tag{21}$$

Thus, in order to determine the parameters of the transfer function based on the results of the experimental study, it is necessary to determine the moments μ_k, $k = \overline{1, N_f}$ of the auxiliary function $\varphi(t)$, and then the components of the vector $\bar{S}$ are calculated according to the formula (20), which makes it possible to calculate the vector of the parameters of the transfer function (2) using the formula (21).

Empirical models of the muffle furnace. To calculate the parameters of the normalized transfer function (2), software was developed in the MatLab environment in the form of a main program (Script file) and subprograms (M-files). Subroutines solve the following problems: calculation of areas $s_j^{(k)}$,

$j=\overline{0,N}$, $k=\overline{1,N_f}$ according to (18); calculation of μ_k, $k=\overline{1,N_f}$ moments according to (14 and 19); calculation of the transient characteristic based on the known parameters of the transfer function of the object; matrix A_s formation.

The program works in dialog mode: the researcher enters the orders of the polynomials of the numerator *m* and denominator *n*. At the same time, the ratio $m \le n$ must be fulfilled.

By setting different values of *m* and *n*, one can get a set of empirical models that approximate experimental data with varying accuracy. The selection of the best model from the obtained set is carried out according to criterion (3).

The experimental data, which form a time series for each signal transmission channel, were pre-processed so that the data values at the initial time point were subtracted from them. Therefore, the first ordinate of the time series for each of the four channels was zero.

The values *m* and *n* were chosen so that the condition $m < n$ was fulfilled. This is explained by the fact that the initial ordinate of the transient characteristic is different from zero, which contradicts the experimental data.

Calculations of the ordinates of the numerical series of experimental data were made with a variable step of discreteness. Therefore, in the numerical method for solving the empirical mathematical model using the Runge-Kutta method, the integration step is a selected variable and was calculated according to the formula

$$h_k = t_{k+1} - t_k, \quad k=\overline{0,N-1}.$$

The choice of a variable integration step is explained by the need to synchronize in time the calculated ordinates of the transient characteristic with the ordinates of the time series of experimental data.

Table 1 shows the results of the synthesis of empirical models based on the results of experimental data, which reflect the dynamics of the muffle furnace as an object of automatic control.

In order to compare empirical models for different signal transmission channels with experimental data, the approximation error was calculated as the sum of squares of the differences between normalized and overclocking characteristics. The latter were reduced to a dimensionless form with a unit transmission coefficient, i.e., in formula (3) $y(t_k)$ – the transient characteristic is normalized, and $y_{ex.k}$ – the acceleration characteristic, the ordinates of which are calculated according to the following formula

$$y_{ex,k} = \frac{Y_{ex,k}}{Y_{ex,N}},$$

where $Y_{ex,k}$ – are the values of the dimensional ordinates of the acceleration characteristic at discrete moments of time $k=\overline{0,N}$; $Y_{ex,N}$ – is the value of the ordinate of the dimensional acceleration characteristic at the final moment of time t_N.

From Table 1 it follows that for different values of *m* and *n* we have different accuracy of approximation. At the same time, empirical models are also stable only for certain values of *m* and *n*, that is, only for some values $n \le 3$ the coefficients of the characteristic equation of the normalized transfer function are positive numbers.

As can be seen from Table 1, the highest approximation accuracy is obtained for the following values: $m = 2$ and $n = 3$. Such values *m* and *n* ensure, in most cases, that the accuracy of approximation is an order of magnitude higher than other values *m* and *n*.

So, for all four signal transmission channels, the transfer functions will be as follows

$$W(p) = k\frac{b_0p^2 + b_1p + b_2}{a_0p^3 + a_1p^2 + a_2p + a_3}. \quad (22)$$

Table 1

Results of construction of empirical models

The order of the numerator polynomial, *m*	The order of the denominator polynomial, *n*	Properties of empirical models		The error value, *Er*	
		Lower heater	Upper heater		
		$U_1 - T_{Low}$	$U_2 - T_{Low}$	$U_1 - T_{Low}$	$U_2 - T_{Low}$
0	1	stable	stable	0.0794	0.01308
0	2	unstable	stable	–	0.04521
0	3	unstable	unstable	–	–
1	2	unstable	unstable	–	0.0109
1	3	unstable	unstable	–	–
2	3	stable	stable	0.00623	0.007435
–	–	$U_1 - T_{Upp}$	$U_2 - T_{Upp}$	–	–
0	1	stable	stable	0.0533	0.01527
0	2	unstable	stable	–	0.03374
0	3	unstable	unstable	–	–
1	2	unstable	stable	–	0.01506
1	3	unstable	unstable	–	–
2	3	stable	stable	0.00299	0.00333

Table 2 contains the values of the parameters of the normalized functions, described by formula (22), and in Fig. 5, as an example, transient and overclocking characteristics for the channel $U_1 - T_{Low}$ are shown.

The obtained results make it possible to write down the mathematical model of the muffle furnace in matrix-vector form

$$\bar{T}(p) = W(p)\bar{U}(p), \quad (23)$$

where $W(p) = \begin{bmatrix} W_{11}(p) & W_{12}(p) \\ W_{21}(p) & W_{22}(p) \end{bmatrix}$ – the matrix transfer function of the object; $W_{ij}(p)$; $i = j = 1, 2$ – the transfer function of the object of the i^{th} output relative to the j^{th} input; $\bar{T}(p) = \begin{bmatrix} T_{Low}(p) \\ T_{Upp}(p) \end{bmatrix}$ – the vector of object outputs; $\bar{U}(p) = \begin{bmatrix} U_1(p) \\ U_2(p) \end{bmatrix}$ – the vector of input values of the object.

If we consider the notations made, then expression (23) in expanded form will be as follows

$$\begin{bmatrix} T_{Low}(p) \\ T_{Upp}(p) \end{bmatrix} = \begin{bmatrix} W_{11}(p) & W_{12}(p) \\ W_{21}(p) & W_{22}(p) \end{bmatrix} \cdot \begin{bmatrix} U_1(p) \\ U_2(p) \end{bmatrix}.$$

Having performed the appropriate actions on the matrices, we arrive at the following result

$$T_{Low}(p) = W_{11}(p)U_1(p) + W_{12}(p)U_2(p); \quad (24)$$

$$T_{Upp}(p) = W_{21}(p)U_1(p) + W_{22}(p)U_2(p). \quad (25)$$

The system of equations (24, 25) forms a mathematical model of the muffle furnace as an object of automatic control, based on which the structural diagram of the object is created (Fig. 6).

Thus, the muffle furnace as an object of automatic control is a multidimensional object with internal cross-connections, which are represented by the transfer functions $W_{12}(p)$ and $W_{21}(p)$. The presence of such internal connections significantly complicates the automatic control of the object to stabilize the temperature regime of the muffle furnace.

Table 2

Parameters of empirical models

Signal transmission channels	k	Numerator parameters of FP			Denominator parameters of FP			
		b_0	b_1	b_2	a_0	a_1	a_2	a_3
$U_1 - T_{Low}$	468.25	7828.23	97.77	1	559,116.88	16,854.16	206.53	1
$U_1 - T_{Upp}$	459.25	7724.32	97.20	1	612,374.61	17,821.93	211.50	1
$U_2 - T_{Low}$	150,00	10,401.34	136.80	1	1,596,016.46	34,051.10	290.59	1
$U_2 - T_{Upp}$	172.88	9560.48	111.38	1	1,133,264.06	26,531.02	254.91	1

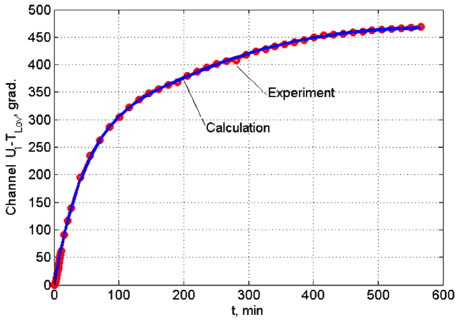

Fig. 5. Transient and acceleration characteristics of the muffle furnace along the channel $U_1 - T_{Low}$

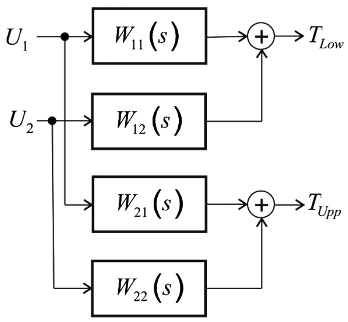

Fig. 6. Structural diagram of muffle furnace dynamics as an object of automatic control

Conclusions. A method for conducting experimental research was developed, based on which the acceleration characteristics of the muffle furnace for four signal transmission channels were obtained.

The area method for calculating the parameters of the controlled object based on its acceleration characteristic has been improved in terms of the algorithmic support of the method, which made it possible to create effective software and, based on it, to iteratively select an empirical model of the object for each signal transmission channel from a set of synthesized models.

The structural scheme of the object was developed, based on which it was concluded that the muffle furnace as an object of automatic control is a multidimensional object with internal cross connections, the presence of which significantly complicates the stabilization of the temperature regime of the furnace.

The obtained mathematical models of the muffle furnace will be used in the future for the development of highly efficient automatic control systems to produce parts when high accuracy of maintaining the temperature regime is required.

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Експериментальні дослідження динамічних властивостей муфельних печей

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Мета. Розроблення методу та програмного забезпечення побудови емпіричних моделей муфельних печей із двома елементами нагріву.

Методика. Для побудови емпіричних моделей муфельної печі використана методика з обробленням експериментальних даних за вдосконаленим алгоритмом, в основі якого лежить метод площ. Експериментальні дослідження динаміки муфельної печі проводились за такою методикою. Муфельна піч охолоджувалась до кімнатної температури, потім включався нижній тен і фіксувалась температура на виходах печі за допомогою експериментального двоканального програмного регулятора

температури МИК-344. Другий цикл експериментального дослідження починався з охолодження печі до кімнатної температури з наступним включенням верхнього тену. Спостереження за результатами експерименту велись до стабілізації температури на двох виходах муфельної печі. Архівування трендів температур здійснювалось за допомогою інтерфейсу RS485/Ethernet і програмного забезпечення SmartReview (продукт ТОВ «Мікрол»).

Результати. Розроблене авторами програмне забезпечення дає змогу в ітеративному режимі вибрати оптимальну емпіричну модель за критерієм середньоквадратичного відхилення розрахункових від експериментальних даних. З метою порівняння експериментальних даних із результатами емпіричного моделювання удосконалено метод побудови кривої розгону за коефіцієнтами передавальних функцій зі змінним кроком дискретності. У процесі експериментального дослідження були апробовані дванадцять емпіричних моделей за чотирма каналами передачі сигналів. Було встановлено, що лише чотири з них є стійкими (мають ліві корені характеристичного рівняння). Серед відібраних стійких моделей найкращою (за сформованим критерієм) виявилася модель, поліном чисельника якої дорівнює двом, а знаменника – трьом. На основі синтезованої емпіричної моделі муфельної печі створена структурна схема, що включає в себе чотири передавальні функції та перехресні зв'язки.

Наукова новизна. Уперше розроблена емпірична модель муфельної печі з двома джерелами електричної енергії, яка з високою точністю описує її динамічні властивості як об'єкта автоматичного керування, що дало змогу виявити наявність внутрішніх перехресних зв'язків, які значно ускладнюють процес керування таким об'єктом.

Практична значимість. Практична цінність роботи полягає в тому, що створені емпіричні моделі з двома електричними джерелами (тенами) будуть використані для синтезу високоточної автоматичної системи керування температурним режимом муфельної печі.

Ключові слова: *муфельна піч, температура, система керування, динаміка, модель, передавальна функція*

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