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# ANALYSIS OF NUMERIC RESULTS FOR ANALOGUE OF GALIN'S PROBLEM IN CURVILINEAR COORDINATES

**Purpose.** Studying the results obtained when applying the perturbation method to the individual contact problems solution. Considering the influence of non-ideal material properties of interaction bodies. Considering the contact area's complex geometry influence.

**Methodology.** Mathematical models of the interaction of an elastic plate with cylindrical anisotropy in the form of a curvilinear sector problems have been constructed and considered. In the process of interaction between the stamp and the plate, areas of slippage and adhesion occur. The original complex problems of the theory of elasticity in the process of applying the perturbation method are reduced to the sequential solution of the potential theory problems.

**Findings.** An analytical solution to the interaction of an elastic anisotropic plate in the form of a curvilinear sector and a rigid stamp problem was obtained considering the existence of slippage and adhesion areas in the contact area. Laws of stress distribution under the stamp were found, as well as dependence of this distribution on the dimensions of the contact area.

**Originality.** The perturbation method is convenient and expedient to use when solving mining mechanics problems.

The study on the stress-strain state of an elastic anisotropic plate for the analogue of Galin's problem was carried out, and the corresponding analytical solutions were obtained. The dependence of the coupling area size on the dimensions of the stamp and the opening angle of the sector, physical properties of the material is analyzed. Possible boundary transitions have been completed.

**Practical value.** The proposed approach makes it possible to obtain analytical solutions to practically important problems in mining, to evaluate the stress-strain state of thick-walled structures with reinforcing elements, stamps, overlays. The results can be useful when designing mine workings.

**Keywords:** *stress distribution, adhesion, anisotropy, sliding, curvilinear sector, asymptotic method*

**Introduction.** The problem of contact interaction is of great importance for modern construction, machine design, etc., because it is related to such important processes and issues as wear, destruction, strength and reliability of structures and building. Therefore, the modeling and study of contact impact is of particular importance for researchers. During the process of mutual contact between the parts, forces and pressures arise, so the need to conduct a correct preliminary assessment of the stress-deformed state of the interaction parts is quite natural and extremely important.

Asymptotic analysis is widely used in the modern theory of elasticity to construct approximation equations and assess the feasibility of applying various hypotheses. This is an effective mathematical tool that allows you to build an adaptive approximation equation and evaluate the feasibility of applying various preliminary assumptions and hypotheses.

Of course, research can be carried out using numerical methods, but the need to obtain analytical solutions remains relevant. First of all, such results are important for problems in which complex real physical properties of the material, such as the presence of anisotropy or nonlinearity, must be taken into account when creating a model. Preliminary analytical processing of the basic equations allows avoiding the accumulation of mathematical complications during calculations and proving the reliability of the results.

Analytical solutions are obvious when qualitatively comparing the results obtained by other methods; they help to identify special points and lines to justify the feasibility of using one or another approximate approach.

The perturbation method used in this study allows expanding the range of important problems of the linear theory of elasticity that can be solved by analytical methods.

**Literature review.** The attention of researchers is largely focused on problems in which the stress-deformed state of a body (in the specific case of a plate) under the action of a rigid stamp or a system of stamps is considered. To obtain analytical solutions of flat contact problems of the theory of elasticity, a significant number of scientists use a mathematical apparatus based on the application of the complex variable functions theory: J. E. Campbell, R. P. Thompson [1], A. F. Ulitko, V. I. Ostryk [2], Yu. O. Antipov [3], and others.

Galin L. A. considered the problem of a stamp moving at a constant speed on the boundary of an elastic body. He also proposed for the first time the delivery and solutions of the interaction problem, when there are areas of adhesion and slippage in the contact area. V. I. Mosakovsky and V. V. Petrov devoted their work to the study on changes in the actual dimensions of the contact area under the influence of the applied load.

Specific contact problems of the theory of elasticity, as well as various approaches to their solution, are considered in their works by R. M. Martynyak [4], V. I. Kuzmenko [5], G. Ya. Popov, and his students [6, 7]. The works by S. I. Homenyuk, O. H. Spits [8], O. H. Nikolaev are devoted to the study on the VAT (stress-strain state) of multicomponent bodies [9, 10].

Many issues considered when solving contact problems are directly related to fracture mechanics [11, 12]. Load transfer during direct contact is taken into account in mining mechanics during the study on the stress state of fractured rocks [13, 14] and the behavior of large mining masses [15].

A significant contribution to the development of asymptotic methods was made by many authors especially by V. M. Aleksandrova, M. V. Vakulenko, S. G. Lehnytskyi, L. A. Agalovyan, and others.

The ideas proposed in the widely known works by A. S. Kosmodamiansky led to the intensification of the asymptotic methods development.

Nowadays various numerical methods are widely known and the search for approximate results for contact problems of the theory of elasticity is carried out by many authors and in different directions. But at the same time, obtaining analytical solutions remains relevant. They make it possible to carry out a correct preliminary assessment of the interaction details stress-strain state, justify the choice of one or another numerical method for obtaining final results.

Analysis of the current state of the contact interaction problem confirms the relevance of the proposed approach and the obtained solutions.

**Unsolved aspects of the problem.** The presence of a large number of contacting parts in real practical problems, whose material has complex properties and geometry, arouses interest in any solutions of the corresponding problems: analytical, numerical, numerical-analytical. When applying numerical methods, as a rule, special points and solutions cannot be taken into account, which in practice can lead to undesirable behavior or the destruction of the structure. The need to solve many important issues in the mentioned direction determined the importance of developing analytical methods for calculating the stress-strain state during the contact interaction of bodies. Taking into account anisotropy, both linear and curvilinear, becomes another reason for mandatory preliminary research on relevant problems, additional modeling before applying the known numerical method. The complex geometry of the contact area often leads to the uniqueness of the problem statement, while the reliability of the results requires additional proof.

The analytical approach developed by the authors has been well tested on various types of problems, the practical accuracy of the results has been proven, and possible comparisons with known solutions by other authors have been made. Therefore, this method is convenient to use for quite complex productions taking into account the real properties of materials.

**The purpose.** The purpose of this study is to analyze the most significant results for contact problems of the linear theory of elasticity about the action of a rigid stamp on an elastic orthotropic curvilinear sector with cylindrical anisotropy.

It is necessary to obtain the laws of stress distribution under a stamp acting on an elastic orthotropic semi-finite or finite curvilinear sector with cylindrical anisotropy. In addition, you need to find the dimensions of the clutch area, provided that it is located in the contact zone symmetrically between the two sliding areas.

The behavior of the main functions must be investigated for different materials, the influence of certain characteristics should be taken into account, interesting regularities should be found and their justification are also should be found.

**Research methodology.** The perturbation method proposed and developed in the works by A. V. Pavlenko and T. S. Kagadiy and their students [16, 17] was used to solve the contact problems of the theory of elasticity. This method has been sufficiently tested and gives good and reliable results when solving a large class of contact problems of the theory of elasticity in a flat [18] and spatial formulation.

According to this method, the asymptotic analysis of the equations of the theory of elasticity for orthotropic media is carried out using a parameter that characterizes the anisotropic properties of the plate material (a small parameter). In the solution process the initial complex problems of the linear theory of elasticity are reduced to the solution of simpler consecutive boundary value problems of the potential theory, that is, the stress-strain state of an orthotropic body is decomposed into two components, whose properties differ significantly from each other.

In order to take into account all possible relationships that may arise between the components of the displacement vector and their rates of change, affine transformations (1, 2) of coordinates and unknown functions are introduced [19]

$$\xi_1 = a\varepsilon^{\frac{1}{2}}x; \quad \eta_1 = y; \quad u = \tilde{u}^{(1)}; \quad \upsilon = \varepsilon^{\frac{3}{2}}\tilde{\upsilon}^{(1)}; \tag{1}$$

$$\xi_2 = x; \quad \eta_2 = b\varepsilon^{\frac{1}{2}}y; \quad u = \varepsilon^{\frac{3}{2}}\tilde{u}^{(2)}; \quad \upsilon = \tilde{\upsilon}^{(2)}. \tag{2}$$

The transformations mentioned above depend on a small parameter $\varepsilon = \dfrac{G}{A_1}$; this relation is of a physical nature and contains characteristics of the body's material stiffness under investigation.

After applying transformations, we move on to two stress-strain states with different properties. These states are connected to each other through boundary conditions on tangential stresses.

Finally, the solution of the initial problem is obtained in the form of a two-separate-component superposition and the unknown functions $\tilde{u}^{(n)}$, $\tilde{\upsilon}^{(n)}$, where $n$ = 1, 2 are found by decomposing the physical parameter ε (3) into series by fractional powers. In this case, the appearance of the asymptotic sequence is determined by the structure of the equations and the order of ε errors in the boundary conditions. It arises after solving the problem in the previous approximation.

To take into account all possible cases, the unknown functions $\tilde{u}^{(n)}$ and $\tilde{\upsilon}^{(n)}$, where $n$ = 1, 2, are defined in the form of series by parameter $\varepsilon^{\frac{1}{2}}$, because due to the transformations (1, 2), lower powers of the parameter ε cannot occur

$$\tilde{u}^{(n)} = \sum_{j=0}^{\infty} \varepsilon^{\frac{j}{2}} \tilde{u}^{n,j}; \quad \tilde{\upsilon}^{(n)} = \sum_{j=0}^{\infty} \varepsilon^{\frac{j}{2}} \tilde{\upsilon}^{n,j}. \tag{3}$$

The expansions coefficients $a$ and $b$ are also given in the form of the rows by parameter $\varepsilon^{\frac{1}{2}}$

$$a = \sum_{j=0}^{\infty} \varepsilon^{\frac{j}{2}} a_j; \quad b = \sum_{j=0}^{\infty} \varepsilon^{\frac{j}{2}} b_j, \tag{4}$$

here $a_0 = b_0 = 1$, and coefficients $a_j$; $b_j$, where $j$ = 1, 2,..., are calculated during the solution and are used to simplify the equations in the following approximations.

The possibility of formulating boundary conditions for finding the main unknown functions corresponding to stress-strain states of the first and second type is proved. The initial boundary value problem of the theory of elasticity is reduced to the solution of Laplace's equations under appropriate boundary conditions.

**Research results.** This paper presents the results of solving two contact problems about pressing a rigid stamp into the free face of a plate with curvilinear anisotropy. The nature of the contact is similar to the classic formulation of the problem with adhesion and sliding, but taking into account the complex properties of the material and geometry makes the formulation of the problem new and relevant, and the solution much more difficult.

Two cases were studied – for a semi-infinite sector and a sector of finite dimensions (Fig. 1).

***Both problems consider the action of a rigid stamp on the free face of a semi-infinite (or finite) elastic orthotropic plate under the conditions of a generalized plane stress state.*** It is assumed that the elastic plate is fixed at the edges: $\varphi = \omega$, $\varphi = -\omega$, $r_0 \le r < \infty$, $-\omega \le \varphi \le \omega$ (for the second case $r_0 \le r \le r_1$, $-\omega \le \varphi \le \omega$). The material of the sector is orthotropic, the main directions of anisotropy correspond to polar coordinates.

A hard stamp $r = r_0$ with normal force $Q$ is pressed into the border $-\chi \le \varphi \le \chi$ of the sector on the site and its base coincides with the border $r = r_0$.

As in L. Galin's problem, it is assumed that there are adhesion and slip zones under the stamp. The first of them is located between two areas of sliding adjacent to the end points of the stamp (no peeling occurs). One of the main tasks that must be solved with such a formulation of the problem is finding the position of the points of intersection of the adhesion and sliding zones $\varphi = \pm\beta$ (Fig. 1) depending on the characteristics of the material of the sector. It is necessary to take into account the mandatory limitation at these points and continuity of stresses. In addition, in the problem, it is necessary to determine the laws of stress distribution under the stamp.

By introducing dimensionless coordinates $\xi$, $\eta$ instead of polar coordinates $r$, $\varphi$ with the use of transformations $r = r_0 e^{\xi}$, $\varphi = \eta$ the original problem can be reduced to the integration of the equilibrium equations of the plane sector in displacements

$$A_1 \cdot u_{\xi\xi} + G \cdot u_{\eta\eta} - A_2 \cdot (\upsilon_\eta + u) + G \cdot m \cdot \upsilon_{\xi\eta} - G \cdot \upsilon_\eta = 0;$$
$$G \cdot v_{\xi\xi} + A_2 \cdot \upsilon_{\eta\eta} + A_2 \cdot u_\eta + G \cdot m \cdot u_{\xi\eta} + G \cdot (u_\eta - \upsilon) = 0.$$

The boundary conditions are defined as follows. At the outside of the stamp one has

$$\sigma_1 = \frac{A_1\left(u_\xi + \vartheta_2(\upsilon_\eta + u)\right)}{r_0 e^{\xi}} = 0;$$
$$\tau = \frac{G(u_\eta + \upsilon_\xi - \upsilon)}{r_0 e^{\xi}} = 0 \quad \text{for} \quad \xi = 0, \quad \chi < |\eta| < \omega;$$
$$u = \upsilon = 0 \quad \text{for} \quad \eta = \pm\omega.$$

Under the stamp one has

$$u = c_0 = \text{const} \qquad \text{for} \quad \xi = 0, |\eta| \le \chi;$$
$$\upsilon = 0 \qquad \text{for} \quad \xi = 0, |\eta| \le \beta;$$
$$\tau = \text{sign}(\eta)\rho\sigma_1 \qquad \text{for} \quad \xi = 0, \beta < |\eta| < \chi.$$

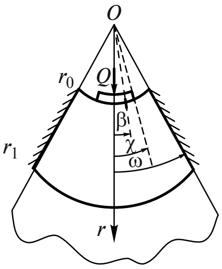


*Fig. 1. Semi-infinite (dotted) or finite (solid arc) plate under the action of a hard stamp, if sliding and adhesion areas are available*

Displacement and stress at infinity are assumed to be zero.

In addition, it is necessary to comply with the equilibrium conditions of the stamp.

The perturbation method used in the work for solving the given problem makes it possible to separate two types of stress-strain state of a flat elastic sector into two components that are interconnected through boundary conditions [18].

The study on the stress-strain state of the first type (there is a slow change in parameters in the direction of the coordinate $\xi$) in the first approximation consists in the integration of the equilibrium equations in displacements [19] under the given boundary conditions

$$\begin{aligned}
&\sigma_1^{1,0} = \frac{A_1 u_\xi^{1,0}}{r_0} = 0 && \text{for} \quad \xi = 0, \quad \chi < |\eta| < \omega;\\
&\sigma_1^{1,0} = \frac{A_1 u_\xi^{1,0}}{r_0} = 0 && \text{for} \quad \xi = 0, \quad \chi < |\eta| < \omega;\\
&u^{1,0} = \text{const} && \text{for} \quad \xi = 0, \quad |\eta| \le \chi;\\
&u^{1,0} = 0 && \text{for} \quad \eta = \pm\omega.
\end{aligned} \tag{5}$$

The sought function and its derivatives at infinity are equal to zero.

The following formulas are used to find the normal stress $\sigma_1^0$ and the tangential stress component $\tau^{1,0}$, corresponding to the function $u^{1,0}$

$$\sigma_1^0 = \frac{A_1 u_\xi^{1,0}}{r_0 e^{\xi}} = \frac{\sqrt{GA_1}\, u_{x_1}^{1,0}}{r_0 e^{\xi}}; \quad \tau^{1,0} = \frac{G u_\eta^{1,0}}{r_0 e^{\xi}} = \frac{G u_{y_1}^{1,0}}{r_0 e^{\xi}}.$$

The normal stresses in the first approximation are determined from the expression directly below the stamp

$$\sigma_1^0 = -\frac{Q_0 \pi}{\sqrt{\beta_1^2 - \eta_1^2} \cdot 4\omega \cdot K(\beta_1)}. \tag{6}$$

The formula for finding the tangential stress $\tau^{1,0}$ for $\xi_1 = 0$, $|\eta_1| > \beta_1$ has the following form

$$\tau^{1,0}(\eta_1) = -\frac{Q}{4\omega K(\beta_1)} \sqrt{\frac{G}{A_1(\eta_1^2 - \beta_1^2)}}. \tag{7}$$

Determination of the stress state of the second type (functions slowly change in the direction of the coordinate $\eta$) is carried out in a similar way

Here, the tangential stresses in the contact area ($\xi = 0$, $|\eta| < \chi$) are determined using the following formulas in the first approximation

$$\tau = \text{sign}(\eta)\rho\sigma_1^0 \qquad \text{for} \quad \beta \le |\eta| < \chi;$$
$$\tau = \frac{G\upsilon_\xi^{2,0}}{r_0} = \frac{\sqrt{GA_2}}{r_0} \cdot \upsilon_{x_2}^{2,0} \qquad \text{for} \quad |\eta| \le \beta.$$

In the process of successively solving the problems of the theory of a complex variable function to which the initial problem is reduced due to the application of the perturbation method, among other important results, the value of the parameter, $\beta_*$ was found which characterizes the size of the adhesion area between the sector face and the rigid stamp. The equation below relates the parameter $\beta_*$ and material stiffness characteristics, taking into account the effect of the friction coefficient and the size of the contact area

$$F(\gamma, \beta_*) = \rho \cdot \sqrt{\frac{A_1}{G}} \cdot K'(\beta_*).$$

Here $\gamma = \arcsin\sqrt{1 - \beta_1^2 / 1 - \tilde{\beta}_1^2}$, $\tilde{\beta}_1 = \sin(\pi\beta/2\omega)$, $F(\gamma, \beta_*)$ is incomplete elliptic integral of the first kind and $K'(\beta_*)$ is complete elliptic integral of the first kind.

In order to carry out a correct and reliable analysis of the numerical results of the original complex problem, several cases of sector materials and the dependence on different values of the friction coefficient ρ were considered.

During the study, the fact that the friction coefficient is primarily an empirical value that depends on many factors is also taken into account.

To obtain the numerical results presented in the work and their further analysis, the values of the friction coefficient given in the literature for specific materials depending on the method of processing their surfaces were used.

For the convenience of calculations and recording, a value $\sqrt{G/A_1}$ is introduced that is important from the point of view of describing the elastic properties of the material. This value takes into account the ratio of stiffness characteristics and takes the following values for the materials selected for consideration:

$\sqrt{G/A_1}=0.154$ – a type of composite carbon fiber;

$\sqrt{G/A_1}=0.407$ – a type of composite fiberglass;

$\sqrt{G/A_1}=0.524$ – plumbum.

From the graphs presented in Fig. 2, it is possible to draw conclusions about the direct influence of the friction coefficient ρ on the size of the clutch area. As the value of this coefficient increases, the adhesion area between the flat sector and the stamp increases. The elastic characteristics of the material from which the plate is made are also significantly affected. Thus, it can be seen from Fig. 2 that for carbon fiber $\sqrt{G/A_1}=0.154$ and fiberglass $\sqrt{G/A_1}=0.407$ as materials with more pronounced anisotropic properties, the increase in the size of the adhesion area occurs faster than for lead $\sqrt{G/A_1}=0.524$.

An interesting and important fact is that in the general case for materials with stronger anisotropy, the size of the adhesion area that occurs when the stamp is pressed into the face of the sector is larger. The described fact is well illustrated by the curve shown below in Fig. 3 for a constant value of the friction coefficient ρ = 0.5 and different values $\sqrt{G/A_1}$. It should be noted that the parameter $0<\sqrt{G/A_1}<1$ clearly characterizes

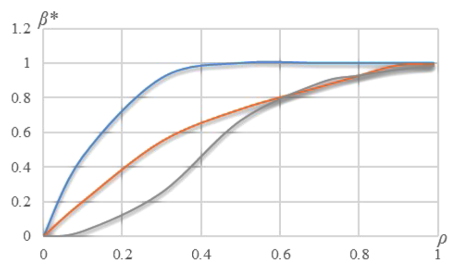


*Fig. 2. Dependence of the coupling area size for the orthotropic sector made of carbon fiber, fiberglass and lead on the value of the friction coefficient* ρ

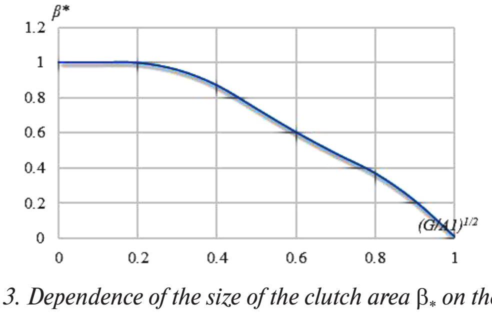


*Fig. 3. Dependence of the size of the clutch area* $\beta_*$ *on the value of the parameter* $\sqrt{G/A_1}$ *for a fixed value of the friction coefficient* ρ = 0.5

the anisotropic properties: the closer its value is to zero, the stronger the anisotropy of the material under consideration is.

The results obtained above allow us to draw a conclusion about the relationship between the values $\sqrt{G/A_1}$, $\beta_*$, ρ, ω. It is possible to trace the influence of the increase in the value of the friction coefficient on the increase in the size of the clutch area.

In the case when the friction coefficient ρ is equal to zero, the parameters are $q=0$, $\beta_*=0$, that is, there is no clutch section. With the growth of ρ the size of the coupling area increases. It should also be noted that the size of the coupling area significantly depends on the stiffness characteristics of the material of the plate $\sqrt{G/A_1}$. For example, with a decrease in $\sqrt{G/A_1}$ (ρ ≠ 0 and constant) the size of the clutch area increases.

The obtained relationship between the friction coefficient and the size of the clutch area were compared with those given in the work by V. I. Ostryk [2] and qualitatively coincide. In the work by Y. O. Antipov [3], it is also noted that the friction coefficient and Poisson's ratio directly affect the size of the clutch area.

Fig. 4 shows the results for the case $\sqrt{G/A_1}=0.407$. A real material with such stiffness characteristics is one of the types of composite fiberglass, for such type of material ε ≈ 0.09 – it is an anisotropic material. The value $h_1/\omega=0.29$ was used during the calculations, this is a ratio that characterizes the relationship between the length of the plate along the edges and the opening angle of the sector. The graph shown in Fig. 4 demonstrates the effect of an increase in the parameter dependent on the friction coefficient $B=\frac{\rho}{\sqrt{G/A_1}}$ on the increase in the area of adhesion between the stamp and the plate for $\sqrt{G/A_1}=0.407$ (fiberglass), $\sqrt{G/A_1}=0.154$ (carbon fiber).

We will study the influence of the dimensions of the interaction surfaces on the stress-strain state of the sector in the contact area using the parameter χ/ω. This parameter is introduced for convenience and it well describes the contact area size ratio to the size of the plate's free border.

Let the stress under the stamp have the form $\sigma^*=-\frac{4\omega}{Q\pi}\sigma_1^0$, then with account of (6), one gets

$$\sigma^*=\frac{1}{K(\beta_1)}\frac{1}{\sqrt{\beta_1^2-\eta_1^2}}=\frac{1}{K(\beta_1)\cdot\beta_1}\frac{1}{\sqrt{1-\left(\frac{\eta_1}{\beta_1}\right)^2}}.$$

If $l=\frac{\eta_1}{\beta_1}=\frac{\sin(\pi y_1/2\omega)}{\sin(\pi\chi/2\omega)}$ for $x_1=0$, $|y_1|<\chi$ and taking into account the fact that the friction effect on the stresses distribu-

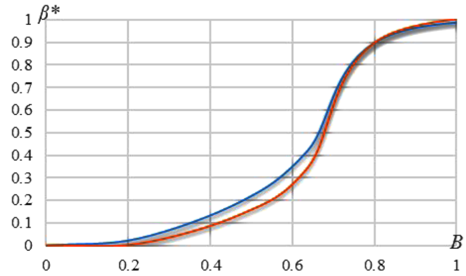


*Fig. 4. The size of the clutch area for the semi-infinite sector depending on the value of* $B=\frac{\rho}{\sqrt{G/A_1}}$ *for fiberglass* $\sqrt{G/A_1}=0.407$ *and carbon fiber* $\sqrt{G/A_1}=0.154$ *with variable* ρ: 0; 0.2; 0.4; 0.6; 0.8

*Table 1*

Dependence $\beta_1 = \sin(\pi\chi/2\omega)$ on the expression $\chi/\omega$

| Parameters | Values | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\chi/\omega$ | 0.007 | 0.059 | 0.1301 | 0.3321 | 0.409 | 0.5901 | 0.869 | 0.974 |
| $\beta_1$ | 0.018 | 0.1895 | 0.3661 | 0.7642 | 0.5992 | 0.8462 | 0.9509 | 0.9968 |

*Table 2*

Distribution of normal stresses under the stamp taking into account the influence of the plate's free face size and the size of the stamp

| | l | 0 | 0.2 | 0.3 | 0.5 | 0.6 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|
| $\sigma^*$ | $\beta_1 = 0.0094$ | 66.39 | 68.58 | 71.34 | 76.72 | 82.31 | 87.51 | 152.1 |
| | $\beta_1 = 0.1004$ | 6.195 | 6.312 | 6.521 | 7.105 | 7.801 | 8.122 | 13.79 |
| | $\beta_1 = 0.1998$ | 3.019 | 3.093 | 3.174 | 3.523 | 3.792 | 3.981 | 6.941 |
| | $\beta_1 = 0.4971$ | 1.094 | 1.101 | 1.128 | 1.251 | 1.362 | 1.401 | 2.453 |
| | $\beta_1 = 0.5991$ | 0.841 | 0.869 | 0.898 | 0.987 | 1.094 | 1.131 | 1.971 |
| | $\beta_1 = 0.7989$ | 0.549 | 0.564 | 0.596 | 0.622 | 0.683 | 0.714 | 1.321 |
| | $\beta_1 = 0.9889$ | 0.306 | 0.312 | 0.321 | 0.351 | 0.379 | 0.404 | 0.687 |
| | $\beta_1 = 0.9992$ | 0.209 | 0.215 | 0.225 | 0.244 | 0.261 | 0.275 | 0.482 |

tion under the stamp is noted only from the third approximation, we have

$$\sigma^* = \frac{1}{K(\beta_1)\cdot\beta_1}\frac{1}{\sqrt{1-l^2}} \quad \text{for} \quad -1<\beta_1<1; \quad -1<l<1.$$

The value $\sigma^*$ depends on $\beta_1 = \sin(\pi\chi/2\omega)$, which characterizes the area size under the stamp due to its dependence on the parameter $\chi/\omega$. Let us establish the dependence $\beta_1 = \sin(\pi\chi/2\omega)$ on the relation $\chi/\omega$ (Table 1).

In a further study (using the calculated results from Table 1), the dependence of the normal stress under the stamp on the values of the contact area size ratio to the size of the plate's free face was established (Table 2).

As shown in Fig. 5, the size of the stamp also has a significant effect on the value of normal stresses occurring in the contact area. At the same time, the following pattern is clearly observed: with an increase in the size of the contact area between the stamp and the plate (we are talking about the value $\beta_1$) the values of the normal stresses under the stamp decrease.

The problem where the finite dimensions of the sector are taken into account is solved in a similar way. The following are the most interesting results for this case.

The value of the quantity describing the settlement of the stamp is important from the point of view of contact analysis

$$\delta_0 = -\frac{Q\cdot r_0}{\pi}\frac{K(k_1)\cdot\ln(\beta_1)}{\sqrt{GA_1}\cdot A\cos(\pi\chi/2\omega)}.$$

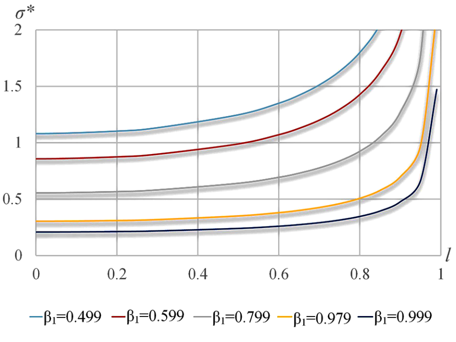


*Fig. 5. Distribution of normal stresses under the stamp depending on the size of the plate's free face and the contact area (size of the stamp)*

After entering the notation $\delta_0^* = -\frac{\pi\delta_0}{Qr_0}$, we will have an easier-to-understand formula for calculating the above-mentioned sought-after constant

$$\delta_0^* = \frac{K(k_1)\cdot\ln(\beta_1)}{\sqrt{GA_1}\cdot A\cos(\pi\chi/2\omega)}.$$

The value of the quantity $\delta_0^*$ was found for different characteristics of the stiffness of the material $\sqrt{G/A_1}$. Cases for lead, fiberglass and carbon fiber are considered. At the same time, a constant value of the contact area size ratio to the opening angle of the plate $\chi/\omega = 0.4$ was chosen. The numerical results of the research are given in Table 3.

It can be seen from Table 3 that the settling $\delta_0$ takes the greatest importance for lead $\sqrt{G/A_1} = 0.524$, and the least for carbon fiber $\sqrt{G/A_1} = 0.154$. That is, the anisotropic properties of the material are also of great importance here. For materials with a smaller value of the parameter $\sqrt{G/A_1}$ (stronger anisotropy) the settlement of the stamp is significantly reduced.

The curve shown in Fig. 6 allows us to track the effect of an increase in the value of the friction coefficient $\rho$ on the increase in the size of the clutch area. The values of $\rho$ in the calculations were chosen taking into account the method for processing real materials with the stiffness characteristics specified above.

The behavior of the curve in Fig. 6 is similar to the case in which the pressing of a rigid stamp into the free face of a semi-infinite sector is considered.

Table 4 shows the dependence $\beta_1 = \sin(\pi\chi/2\omega)$ on the relation $\chi/\omega$ for a finite sector (similar to Table 1 for the first problem)

*Table 3*

The value $\delta_0^*$ (stamp settlement) for lead $\sqrt{G/A_1} = 0.524$, fiberglass $\sqrt{G/A_1} = 0.407$ and carbon fiber $\sqrt{G/A_1} = 0.154$ at constant $\chi/\omega = 0.4$

| $\sqrt{G/A_1}$ | $A$ | $\chi/\omega$ | $\beta_1$ | $K(k_1)$ | $\delta_0$ |
|---|---|---|---|---|---|
| 0.524 | 2.989 | 0.4 | 0.882338 | 3.362 | –0.02108 |
| 0.407 | 4.024 | 0.4 | 0.963352 | 5.163 | –0.00602 |
| 0.154 | 12.339 | 0.4 | 0.999995 | 17.011 | $-7.9\cdot 10^{-7}$ |

*Table 4*

Dependence of the auxiliary parameter $\beta_1 = \sin(\pi\chi/2\omega)$ on the relation $\chi/\omega$

| Para-meters | Values | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\chi/\omega$ | 0.007 | 0.063 | 0.1281 | 0.3333 | 0.4092 | 0.591 | 0.8722 | 0.9712 |
| $\beta_1$ | 0.0093 | 0.1003 | 0.1998 | 0.4995 | 0.5992 | 0.7996 | 0.9799 | 0.9993 |

*Table 5*

Distribution of normal stresses under the stamp, taking into account the influence of the dimensions of the free face of the finite plate and the stamp

| | 1 | 0 | 0.2 | 0.3 | 0.4 | 0.6 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|
| $\sigma^*$ | $\beta_1 = 0.0089$ | 66.12 | 67.08 | 72.14 | 75.21 | 82.49 | 87.51 | 151.62 |
| | $\beta_1 = 0.0901$ | 6.154 | 6.311 | 6.462 | 7.111 | 7.802 | 7.992 | 13.78 |
| | $\beta_1 = 0.1979$ | 3.106 | 3.092 | 3.159 | 3.375 | 3.769 | 3.984 | 6.945 |
| | $\beta_1 = 0.4899$ | 1.063 | 1.114 | 1.129 | 1.239 | 1.363 | 1.419 | 2.458 |
| | $\beta_1 = 0.6002$ | 0.838 | 0.869 | 0.702 | 0.991 | 1.101 | 1.131 | 1.956 |
| | $\beta_1 = 0.8041$ | 0.562 | 0.571 | 0.594 | 0.628 | 0.689 | 0.736 | 1.268 |
| | $\beta_1 = 0.9806$ | 0.302 | 0.313 | 0.321 | 0.358 | 0.391 | 0.411 | 0.705 |
| | $\beta_1 = 0.9987$ | 0.204 | 0.213 | 0.219 | 0.253 | 0.261 | 0.277 | 0.483 |

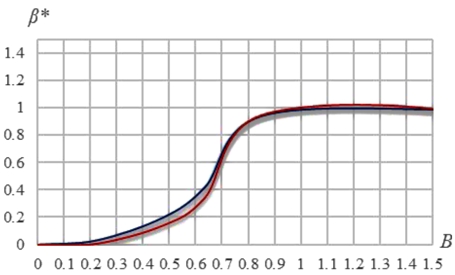


*Fig. 6. Dependence of the size of the clutch area $\beta_*$ for the finite sector on the parameter $B = \frac{\rho}{\sqrt{G/A_1}}$ for fiberglass $\sqrt{G/A_1} = 0.407$ and carbon fiber $\sqrt{G/A_1} = 0.154$ with variable $\rho$*

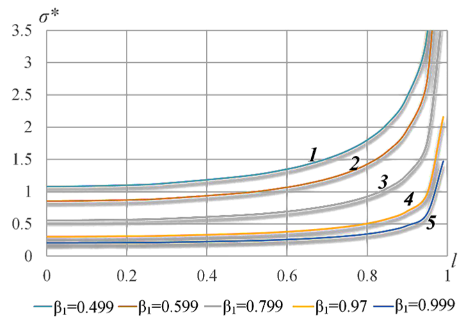


*Fig. 7. Dependence of the normal stress values under the stamp on the dimensions of the free face of the finite sector and the contact area*

In further calculations (the results of Table 4 are used), we will establish the dependence of the normal stress under the stamp on the value of the contact area size ratio to the size of the plate's free face (Table 5).

Based on the results of Table 5, the graphical dependences between the values of normal stresses $\sigma^*$ arising under the stamp as a result of interaction and the dimensions of the free face of the plate are shown below. The influence of the dimensions of the rigid stamp was also taken into account during the calculations.

It can be seen from Fig. 7 that the size of the stamp has a significant effect on the value of normal stresses occurring in the contact area.

The graphical results shown in Fig. 7 allow us to state that the values of normal stresses $\sigma^*$, which correspond to $\beta_1 = 0.489$ (curve *1*) are significantly greater than those obtained for the value $\beta_1 = 0.999$ (curve *5*).

That is, with an increase in the size of the stamp relative to the size of the free face of the sector, the values of normal stresses occurring in the contact area decrease.

**Conclusions.** Using the applied approach, it is possible to carry out a preliminary assessment of the stress-strain state of structures, mechanisms or parts under different contact conditions, analytical solutions of various practically important and relevant contact problems can be obtained.

A study on the influence of the material's stiffness characteristics of the finite and semi-infinite plate on the stress distribution under the stamp at different values of the geometric parameter (the ratio of the size of the contact area to the opening angle of the plate) was carried out. Corresponding dependencies are plotted graphically and the influence of the following parameters on the stress-strain state of the plate is analyzed: the size of the interaction area, the opening angle of the sector, the coefficient of friction.

The small parameter method proposed by the authors can be successfully used for solving problems taking into account viscoelastic, physically or geometrically nonlinear properties of materials; it can be useful in studying electroelasticity problems taking into account SMART materials, as well as for problems related to the fracture mechanics.

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## Аналіз числових результатів для аналога задачі Галіна у криволінійних координатах

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**Мета.** Дослідження результатів, отриманих при застосуванні методу збурень до розв'язання окремих контактних задач. Урахування впливу неідеальних властивостей матеріалу тіл взаємодії. Урахування впливу складної геометрії області контакту.

**Методика.** Побудовані й розглянуті математичні моделі задач про взаємодію пружної пластини з циліндричною анізотропією у вигляді криволінійного сектора. У процесі взаємодії штампа та пластини виникають ділянки проковзування та зчеплення. Вихідні складні задачі теорії пружності у процесі застосування методу збурень зводяться до послідовного розв'язання задач теорії потенціалу.

**Результати.** Аналітичний розв'язок задачі про взаємодію пружної анізотропної пластини у вигляді криволінійного сектора й жорсткого штампа було отримано з урахуванням існування ділянок проковзування та зчеплення в області контакту. Знайдені закони розподілу напружень під штампом, а також залежність цього розподілу від розмірів області контакту.

**Наукова новизна.** Метод збурень зручно й доцільно застосовувати під час розв'язання задач гірничої механіки.

Проведено дослідження напружено-деформованого стану пружної анізотропної пластини для аналогу задачі Галіна, отримані відповідні аналітичні розв'язки. Проаналізована залежність розміру ділянки зчеплення від розмірів штампа й кута розкриття сектора, фізичних властивостей матеріалу. Виконані можливі граничні переходи.

**Практична значимість.** Запропонований підхід дозволяє отримати аналітичні розв'язки практично важливих задач гірничої справи, провести оцінки напружено-деформованого стану товстостінних конструкцій з підкріплюючими елементами, штампами, накладками. Результати можуть бути корисними при проектуванні гірничих виробок.

**Ключові слова:** *розподіл напружень, зчеплення, анізотропія, ковзання, криволінійний сектор, асимптотичний метод*

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