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PRESSURE DISTRIBUTION IN THE OIL RESERVOIR IN A TWO-DIMENSIONAL PLANE

Purpose. Establishment of regularities of variability of pressure dynamics in the reservoir and development on this basis of methods of control and regulation of hydrocarbon production.

Methodology. To achieve this goal, experimental studies were conducted, and the results of the experiments were summarized.

Findings. Pressure distribution functions for stationary fluid inflow in two planes have been established, which allow monitoring and management of mining operations, especially at late stages of development.

Originality. Based on the established regularities, a model of pressure distribution in the reservoir in a two-dimensional plane has been created. An experimental study of the pressure distribution in the reservoir was carried out, which allowed us to remove the characteristics of the pressure distribution along the axis of the segment of the oil reservoir under varying boundary conditions.

Practical value. A mathematical model of pressure distribution processes along the angle of inclination is proposed, which allows determining the effectiveness of flooding. The influence of the deformability of the formation, the location of the well relative to its impenetrable roof and sole, the length of the horizontal trunk and the power of the opened formation on the magnitude and intensity of the inflow to the horizontal well is estimated.

Keywords: *field, pressure in the oil field, production management*

Introduction. The mass application of various waterflooding systems is due to the need to increase reservoir pressure, which increases the intensity of oil production. At the same time, such maintenance of reservoir pressure creates a threat of flooding due to the advance of oil by water in the most permeable areas. In this case, part of the oil remains in the pillars as residual oil and makes it difficult to extract. Therefore, the management of the waterflooding process is of paramount importance. In this regard, the paper proposes a new method for calculating the pressure distribution in the reservoir and creates new conditions for managing production.

Formulation of the problem. The development of residual oil reserves at the last stages of development is complicated by the techno-economic inexpediency of drilling new wells, which necessitates the establishment of patterns of pressure distribution during waterflooding for effective management of production processes, while the relevance for the conditions of Kazakhstan is high due to the widespread presence of high-viscosity oil.

Literature review. Several works are devoted to the management of production operations in order to increase oil recovery by maximizing the direction of fluids to the bottom-holes of production wells [1, 2], in other works, an increase in oil recovery is solved by controlling and regulating development [3, 4].

Various development systems are used in accordance with the geological and physical conditions [5, 6], however, in the late stage of development, the oil reservoir turns out to be largely occupied by a displacing agent (for example, water or PGS). As is known, some zones with high oil saturation, close to the initial oil content in the reservoir, the so-called oil pillars, remain in the reservoir. To extract oil from pillars, the development system is changed [7, 8] or spot and selective waterflooding is used [9, 10].

Since the methods for maintaining reservoir pressure made it possible to significantly increase oil recovery compared to oil recovery of facilities developed in a natural regime, dispersed waterflooding systems, focal and by changing filtration flows, have been developed [11]. Other methods for maintaining reservoir pressure are also being created [12, 13].

It was shown in [3, 5] that flooding of high-permeability and water-saturated areas leads to partial or complete exclusion of medium and low-permeability layers from the mining process, while the problem of water inflow into wells is relevant not only for wells in operation, but also for newly created ones [2, 8].

In practice, the absence of new deposits, enhanced by fluctuations in oil prices, determines the search for new methods that will bring industrial enterprises into an economically viable zone. Therefore, various methods for improving the control and development of hydrocarbon deposits are rapidly developing.

At present, with the improvement of computer technology in the oil industry, a new applied direction has gradually emerged – geological and hydrodynamic modeling, which is aimed at obtaining new methods and improving existing ones for enhancing oil recovery. However, the fact stands out that, along with the noticeable improvements that the modeling has brought, “it was not possible to solve the main problem – to increase the accuracy of the results obtained” [1]. After some time, the permanent geological and hydrodynamic model of field development deviates the actual dynamics of the current oil production from the design, which is fraught with serious consequences. Therefore, the creation of a new methodology for managing production processes, excluding the identified shortcomings, remains very relevant.

Research methodology. To calculate the distribution of pressure at given points, the contour lines of the formation in the water-driven or elastic regime use known solutions to problems [8, 9]. Fluid inflow from an unrestricted circular reservoir to a runoff point or to the circular contour of an en-

larged well is calculated using the well-known formulas of Carslow and Van Everdingen [9, 14].

To formulate the problem, let us take as an example an oil reduced circular deposit, which has the shape of a circle of radius R . It is surrounded by an infinitely stretching water-bearing area (Fig. 1).

At $t = 0$, the oil deposit began to be developed with a flow rate q (m^3/day). All other reservoir parameters are assumed to be known.

From a technological point of view, it is of interest to determine the change in the pressure dynamics on the contour of the oil reservoir for ΔP compared to the initial reservoir pressure P_0 after some time intervals, considering it to be a well with an enlarged radius [1, 5].

In the well-known calculations by Carslow and Yeger, Van Everdingen and Hurst [1, 2], the following solutions were obtained for the reduced radius of a production well

$$\Delta P = \frac{q\mu}{2\pi kh} f(\tau);$$

$$f(\tau) = 0.5[1 - (1 + \tau)^{-3.81}] + 1.12 \lg(1 + \tau); \quad (1)$$

$$t - \theta t/R^2,$$

where θ is flow rate μ is viscosity of liquid; τ is current time; k is permeability; R is reduced well radius. In calculations, it is assumed that the pressure changes only along the radius vector R .

Since this calculation formula assumes pressure changes only along the radius vector of the influence of the angle of rotation, the problem here is one-dimensional.

Below we consider the flow diagram on the oil-bearing contour (along the circumference), the contour pressure is set, and it is possible to derive a calculation formula when the pressure changes, both along the radius vector and the change in the angle of rotation (Fig. 2).

For example, we take a contour, its model is given, where P_c is circuit pressure. P_w is pressure in the well, all parameters remain inside the circle (since we consider inside the circle). Move along the radius of the vector, enter R . P_c the highest pressure on the circuit. The task is set how the pressure to the well will change $P_w, P_c \rightarrow P_w$.

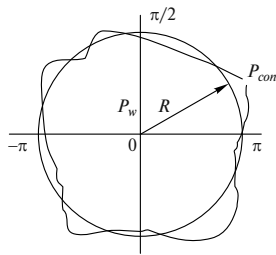


Fig. 1. Scheme of the reduced contour of the oil reservoir

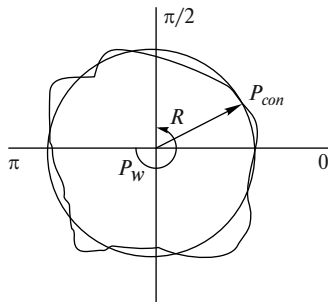


Fig. 2. Schematization of pressure changes along the radius vector and along the angle of rotation for a vertical well

We write all the coefficients a^2 , that is, the same process takes place over a certain period, the stability in time depends on

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = 0 \quad \text{or} \quad \Delta^2 P = 0, \quad (2)$$

it can be solved in cylindrical coordinates

$$\begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \end{cases}$$

Then the equation becomes

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} + \frac{\partial^2 P}{\partial \varphi^2} = 0, \quad (3)$$

or $\Delta^2 P_r = 0$.

To solve this problem, there are two types of mathematical approach. So, in the fundamental book by Zheltov, and in all his other works, the following is done: the pressure is distributed only along the contour, it does not change in the corners, that is $\varphi = \text{const}$ it is required to find a solution to the Laplace equation

$$\Delta P(x, y) = 0 \quad \text{or} \quad \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = 0 \quad (4)$$

inside the circle and satisfying the boundary condition $P = f = P_c$ on the boundary (along the circumference) of the circle, where P_c is the pressure value at the boundary.

This corresponds to the interior Dirichlet problem for a circle, well developed in mathematical physics [10, 12].

It is possible to substantiate the possibility of applying the technique for solving the internal Dirichlet problem for the given problem. The value of the contour pressure at the boundary is constant, that is, the harmonic function reaches its maximum and minimum values only at the boundary at $r = R$ in the near-wellbore zone, that is $r = 0, P \rightarrow \infty$, which corresponds to the technological process-liquid selection.

Passing to the system of polar coordinates, we have the equation [14, 15]

$$\Delta P = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial P}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 P}{\partial \varphi^2} = 0, \quad (5)$$

and boundary conditions

$$P(r, \varphi) = P_c \sin \varphi.$$

The solution to this equation is expressed by the Poisson integral [4]

$$P = (r, \varphi) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\psi) \frac{R^2 - r^2}{r^2 - 2Rr \cos(\varphi - \psi) + R^2} d\psi, \quad (6)$$

and under the integral expression

$$K(r, \varphi, R, \psi) = \frac{R^2 - r^2}{r^2 - 2Rr \cos(\varphi - \psi) + R^2}$$

called the Poisson kernel.

Note that $K(r, \varphi, R, \psi) > 0$ at $r < R$, because of $2Rr < R^2 + r^2$, if $r \neq 0$.

Solution is

$$\begin{aligned} \varphi - \psi &= t; \\ \varphi &= 0; \quad t = \varphi; \\ \varphi &= \pi; \quad t = \varphi - \pi; \\ a &= R^2 + r^2; \\ b &= 2Rr; \end{aligned}$$

$$\begin{aligned}
P(r, \varphi) &= \frac{P_c}{\pi} \int_{-\pi}^{\pi} \frac{(R^2 - r^2) \sin(\varphi - \psi)}{R^2 - 2Rr \cos(\varphi - \psi) + r^2} d\varphi = \\
&= -\frac{P_c(R^2 - r^2)}{\pi} \int_0^{\pi} \frac{\sin t}{R^2 - 2Rr \cos t + r^2} dt = \\
&= \frac{P_c(R^2 - r^2)}{\pi} \int_0^{\pi} \frac{\sin t dt}{a - b \cos t} = -\frac{1}{b} \frac{P_c(R^2 - r^2)}{\pi} \int_0^{\pi} \frac{\sin t dt}{\frac{a}{b} - \cos t} = \\
&= -\frac{P_c(R^2 - r^2)}{\pi b} \int_0^{\pi} \frac{d\left(\frac{a}{b} - \cos t\right)}{\frac{a}{b} - \cos t} = \frac{P_c(R^2 - r^2)}{\pi Rr} \ln \left| \frac{a}{b} - \cos t \right| = \\
&= \frac{P_c(R^2 - r^2)}{\pi Rr} \left[\ln \left| \frac{a}{b} - \cos(\varphi - \pi) \right| \right] = \ln \left| \frac{a}{b} - \cos \varphi \right| = \\
&= \frac{P_c(R^2 - r^2)}{\pi Rr} \ln \left| \frac{R^2 + r^2 - \cos(\varphi - \pi)}{2Rr} \right| = \\
&= \frac{P_c(R^2 - r^2)}{\pi Rr} \ln \left| \frac{R^2 + r^2 - \cos \varphi}{2Rr} \right| = \\
&= \frac{P_c(R^2 - r^2)}{\pi Rr} \ln \left| \frac{R^2 - 2Rr \cos(\varphi - \pi) + r^2}{R^2 - 2Rr \cos \varphi + r^2} \right|.
\end{aligned}$$

The Poisson integral is derived under the assumption $r < R$, $r = R$ representation (5) loses its meaning. However,

$$\lim_{r \rightarrow R} P(r, \varphi) = f(\varphi_0) = P_c.$$

Substituting boundary conditions into equation (5), we obtain solutions to the first boundary value problem inside the circle.

The solution is determined by the expression

$$\begin{aligned}
P(r, \varphi) &= \\
&= \left\{ \frac{P_c(R^2 - r^2)}{\pi R^2} \ln \left| \frac{R^2 - 2Rr \cos(\varphi - \pi) + r^2}{R^2 - 2Rr \cos \varphi + r^2} \right| \text{ at } r > R, \right. \\
&\quad \left. P_c \text{ at } r = R. \right. \quad (7)
\end{aligned}$$

This solution shows for any value $r < R$ and $-\pi \leq \varphi \leq \pi$ it is possible to calculate the dynamics of the pressure distribution inside the circle and takes a given value at the boundary of the circle.

Considering the pressure distribution for an injection well, we make the following physical assumptions. The environment of the bottomhole zone is assumed to be homogeneous in terms of physical parameters and close enough to the injection well, and the pressure P_w is assumed to be given and constant. In this case, the problem reduces to the external Dirichlet problem for the circle of the Laplace equation.

Solution is,

$$\begin{aligned}
&\left\{ \begin{aligned} a &= R^2 + r^2; \\ b &= 2Rr \end{aligned} \right\}; \\
&t = \varphi - \psi; \\
&\psi = \varphi - t; \\
&d\psi = -dt; \\
&\psi = -\pi; \quad t = \varphi + \pi; \\
&\psi = \pi; \quad t = \varphi - \pi; \\
&x = \operatorname{tg} \frac{t}{2}; \\
&t = \operatorname{tg} \frac{x}{2};
\end{aligned}$$

$$\begin{aligned}
P(r, \varphi) &= \int_{-\pi}^{\pi} P_w \frac{R^2 - r^2}{r^2 - 2Rr \cos(\varphi - \psi) + R^2} d\psi = \\
&= \frac{P_w}{2\pi} (r^2 - R^2) \int_{-\pi}^{\pi} \frac{d\psi}{r^2 - 2Rr \cos(\varphi - \psi) + R^2} = \\
&= \frac{P_w}{2\pi} (r^2 - R^2) \int_{-\pi}^{\pi} \frac{d\psi}{a - b \cos(\varphi - \psi)} = \\
&= \frac{P_w}{2\pi} (r^2 - R^2) \int_{\varphi - \pi}^{\varphi + \pi} \frac{dt}{a - b \cos t} = \int_{\varphi - \pi}^{\varphi + \pi} \frac{dt}{a - b \cos t} = \\
&= \int_{\varphi - \pi}^{\varphi + \pi} \frac{2}{1 + x^2} dt = 2 \int_{\varphi - \pi}^{\varphi + \pi} \frac{dx}{a + ax^2 - b + bx^2} = \\
&= 2 \int_{\varphi - \pi}^{\varphi + \pi} \frac{dx}{(a - b) + (a + b)x^2} = \frac{2}{a + b} \int_{\varphi - \pi}^{\varphi + \pi} \frac{dx}{\frac{a - b}{a + b} + t^2} = \\
&= \frac{2}{a + b} \sqrt{\frac{a + b}{a - b}} \operatorname{arctg} \sqrt{\frac{a + b}{a - b}} x = \frac{2}{\sqrt{a^2 - b^2}} \operatorname{arctg} \sqrt{\frac{a + b}{a - b}} x = \\
&= \frac{2}{R^2 - r^2} \operatorname{arctg} \frac{R + r}{R - r} x = \frac{2}{R^2 - r^2} \operatorname{arctg} \left[2 \frac{R + r}{R - r} \operatorname{tg} t \right] = \\
&= -\frac{P_w}{\pi} (r^2 - R^2) \frac{1}{R^2 - r^2} \operatorname{arctg} \left[\frac{R + r}{R - r} (\operatorname{tg}(\varphi + \pi) - \operatorname{tg}(\varphi - \pi)) \right] = \\
&= \frac{P_w}{\pi} \operatorname{arctg} \left[2 \frac{R + r}{R - r} \operatorname{tg} \varphi \right].
\end{aligned}$$

From the Poisson integral we obtain the following solution

$$\begin{aligned}
P(r, \varphi) &= \left\{ \frac{P_w}{\pi} \operatorname{arctg} \left(2 \frac{R + r}{R - r} \operatorname{tg} \varphi \right) \text{ at } r > R, \right. \\
&\quad \left. P_w \text{ at } r = R. \right. \quad (8)
\end{aligned}$$

This solution continuously adjoins the boundary of the circle with the given values and remains valid not only for a constant value, but also for continuously or piecewise continuous variable values on the supply circuit.

The next solution to the problem will consider the definitions of the pressure distribution function in the case of a stationary fluid inflow to a horizontal well in a finite circular formation with a deformable reservoir.

The problem to be solved is formulated as follows. Assuming the filtration regime to be stationary, it is necessary to determine the fluid inflow to a horizontal well located parallel to the relatively impermeable top and bottom of the final isotropic reservoir with a deformable reservoir at a distance Z_0 above the bottom of the reservoir (Fig. 3). A constant pressure is maintained on the reservoir contour. Liquid – slightly compressible, with constant viscosity. Liquid filtration obeys Darcy's law.

We replace the horizontal section of the wellbore with a length of $l_{1,2}$ with a horizontal linear flow of the same length,

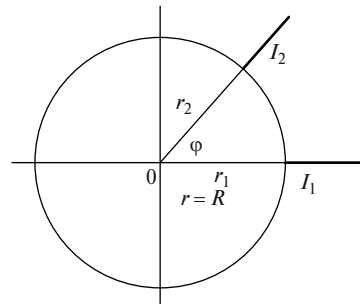


Fig. 3. Projection of two horizontal wells on the formation soil

consisting of point flows with a constant stable – intensity (flow density)

$$Q = \frac{q}{l},$$

where Q is the flow rate of a horizontal well.

The following method can be used to determine the pressure drop in the reservoir from the action of all point flows that make up linear flows, which (Fig. 3) replace horizontal wells. To solve the problem, the well flow rate is stably uniformly distributed along the length of a given horizontal well, which leads to calculations under this condition, the pressure distribution values on the surface of a horizontal well will change from point to point, respectively. On the surface of a horizontal well, you can find a point at which the value of the bottom-hole pressure will be equal to its average value over the entire surface of the well.

Then the pressure distribution at any point is determined by the following formula

$$F(x, y, z) = \frac{2\mu Q}{2\pi k_0 h \rho} \left[\frac{R_c}{r} + 2 \sum_{\sigma=1}^{\infty} K_0 \left(\frac{\sigma \pi r}{h} \right) \cos \frac{\sigma \rho z}{h} \cos \frac{\sigma \pi z_1}{h} \right], \quad (9)$$

where K_0 is Bessel function of an imaginary argument of the second kind of zero order; μ is fluid viscosity; ρ is density

$$F(x, y, z) = \int_{P_c}^P k(p) dp; \\ \bar{k}(p) = \frac{k(p)}{k_0}, \quad (10)$$

where k_0 – the value of the reservoir rock permeability at pressure P_0 is equal to

$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

in $x_0 = 0, y_0 = 0, z = z_0, y = R_w, r = \sqrt{S + R_w^2}$.

From formula (10) we have

$$F_c - F_w = \frac{\mu Q}{2\pi k_0 h \rho} \left[\ln \frac{R_c}{\sqrt{S + R_w^2}} + 2 \sum_{\sigma=1}^{\infty} \left[K_0 \left(\frac{\sigma \pi \sqrt{S + R_c^2}}{h} \right) \cos^2 \frac{\sigma \pi z_0}{h} \right] \right]. \quad (11)$$

To determine the pressure drop from the action of all points of wastewater, considering that the flow rate is equal to $Q = \frac{q}{l}$, integrating from $-f$ to $l-f$ with respect to the variable s we obtain

$$F_c - F_w = \frac{\mu Q}{2\pi k_0 h \rho} \left[1 + \ln \frac{R_c}{l-f} + \frac{f}{l} \ln \frac{l-f}{f} + \frac{h}{l} \cdot \ln \left(\frac{h}{2\pi R_w} \cdot \frac{1}{\sin \left(\frac{\pi z_0}{h} \right)} \right) \right]. \quad (12)$$

To eliminate the parameter f in the last expression, the right side of (12) is needed, multiplying it by $\frac{df}{l}$, obtaining the integral over f from 0 to 1, we obtain (13) the equation

$$F_c - F_w = \frac{\mu Q}{2\pi k_0 h \rho} \left[1.5 + \ln \frac{R_c}{l} + \frac{h}{l} \ln \left(\frac{h}{2\pi R_w} \cdot \frac{1}{\sin \left(\frac{\pi z_0}{h} \right)} \right) \right]. \quad (13)$$

The resulting (13) integral means the averaging of the function $F_c - F_w$ by length l given from two horizontal wells.

For the case when a battery of two horizontal wells with the lengths of their trunks l_1 is operated in the formation, l_1, l_2 and angle φ between their projections on the plane of the bottom of the reservoir, and which are located at heights h_1

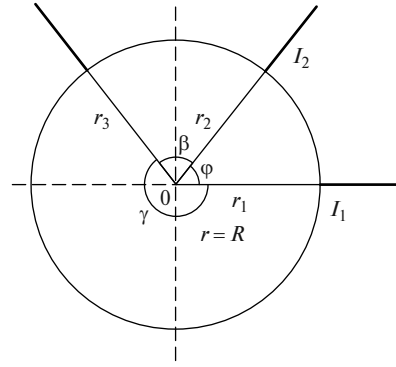


Fig. 4. Projection of three horizontal wells on the formation soil

and h_2 from the bottom of the formation (Fig. 4), the corresponding hydrodynamic problem is solved in a similar way.

$$F_c - F_w = \frac{\mu Q}{2\pi k} \left\{ \frac{3}{2} + \frac{1}{2(l_1 + l_2)} \left(l_1 \ln \frac{r_c}{l_1} + l_2 \ln \frac{r_c}{l_2} \right) + \frac{h}{(l_1 + l_2)} \left[\ln \frac{2h}{2\pi r_c} - \frac{1}{2} \ln \left(\sin \frac{\pi h_1}{h} \sin \frac{\pi h_2}{h} \right) \right] + \frac{1}{4(l_1 + l_2)} \left(\frac{1}{l_1} + \frac{1}{l_2} \right) F(x, y, \varphi) \oint_{x_1, y_1}^{x_2, y_2} 0 \right\}; \quad (14)$$

$$F(x, y, \varphi) = \left[xy - \frac{1}{2}(x^2 + y^2) \cos \varphi \right] \ln \frac{r_c^2}{x^2 + y^2 - 2xy \cos \varphi} - \sin \varphi \left[x^2 \operatorname{arctg} \frac{y - x \cos \varphi}{x \sin \varphi} + y^2 \operatorname{arctg} \frac{x - y \cos \varphi}{y \sin \varphi} \right], \quad (15)$$

where

$$F(x, y, \varphi) \oint_{x_1, y_1}^{x_2, y_2} 0 = F(x_2, y_2, \varphi) + F(x_1, y_1, \varphi) - F(x_1, y_2, \varphi) - F(x_2, y_1, \varphi).$$

Similarly, the dependence of the interference of a three-well battery with the depth of the wellbore l_1, l_2, l_3 and the angles of their projections on the soil of the reservoir has the form (Fig. 4). Here α is an angle between L_1 and L_2 , γ – an angle between L_1 and L_3 , β – an angle between L_2 and L_3 .

$$P_c - P_w = \frac{\mu Q}{2\pi kh} \left\{ 1.5 + \frac{1}{3L} \left(l_1 \ln \frac{r_c}{l_1} + l_2 \ln \frac{r_c}{l_2} + l_3 \ln \frac{r_c}{l_3} \right) + \left[\ln \frac{h}{2\pi r_w} - \frac{1}{3} \ln \left(\sin \frac{\pi h_1}{2\pi r_w} \sin \frac{\pi h_2}{h} \sin \frac{\pi h_3}{h} \right) \right] + \frac{1}{6l_3} \left[\left(\frac{1}{l_1} + \frac{1}{l_2} \right) F(x, z, \alpha) \oint_{x_1, z_1}^{x_2, z_2} 0 + \left(\frac{1}{l_2} + \frac{1}{l_3} \right) F(y, z, \gamma) \oint_{y_1, z_1}^{y_2, z_2} 0 + \left(\frac{1}{l_1} + \frac{1}{l_3} \right) F(x, z, \beta) \oint_{x_1, z_1}^{x_2, z_2} 0 \right] \right\}; \quad (16)$$

$$L_3 = l_1 + l_2 + l_3; \quad x_1 = r_1; \quad x_2 = r_1 + l_1; \quad y_1 = r_2; \\ y_2 = r_2 + l_2; \quad z_1 = r_3 + l_3; \\ \alpha + \beta + \gamma = 2\pi.$$

When operating the n^{th} number of branched-horizontal wells with arbitrary length, different distances from the soil and different angles of their projections, the noted problem is solved similarly.

Difference between $P_c - P_w$, that is, pressure changes from source to drain, the calculation formula has the form

$$P_c - P_w = \frac{\mu Q}{2\pi kh} \left\{ \frac{3}{2} + \frac{1}{nL_n} \left(\sum_{i=0}^n l_i \ln \frac{r_c}{l_i} \right) + \frac{h}{L_n} \left[\ln \frac{h}{2\pi r_w} - \ln \left(\prod_{i=1}^n \sin \frac{\pi h_i}{h} \right) \right] + \frac{1}{2\pi L_n} + \frac{1}{2\pi L_n} \sum_{i=1}^{n-1} \sum_{j=i+1}^n \left(\frac{1}{l_i} + \frac{1}{l_j} \right) F(x_i, x_j, \varphi_{ij}) \oint_{x_i, x_j}^0 \right\}, \quad (17)$$

where

$$x_{i1} = r_i; \quad x_{i2} = r_i + l_i; \quad x_{j1} = r_j; \quad x_{j2} = r_j + l_j; \\ L_n = l_1 + l_2 + \dots + l_n,$$

where φ_{ij} is an angle between projections of trunks L_i and L_j , on the plane of the bottom of the formation. The rest of the designations of the parameters are like the previous ones.

For the case, with the same angles between their trunks, equal to $\frac{2\pi}{n}$, the following approximate formula for the pressure drop was obtained

$$P_c - P_w = \frac{1}{\sigma} \left[1 - \sqrt{\frac{2\sigma\Omega}{a} + 1} \right], \quad (18)$$

where

$$\Omega = \frac{\mu Q_r}{2\pi k_0 h L_n} \left[\sum_{i=0}^n l_i \ln \frac{a_i R_c}{l_i} + h F_0 \right], \quad (19)$$

here

$$F_0 = \ln \left[\frac{h}{2\pi R_w} \frac{1}{\sin \pi z_0/n} \right]. \quad (20)$$

From (13) it follows that the mass flow rate in a horizontal well is equal to

$$Q_r = \frac{2\pi k_0 \rho h}{\mu} \frac{F_c - F_w}{\ln \frac{3.1422 R_c}{l} + \frac{h}{l} \ln \left(\frac{h}{2\pi R_w} \frac{1}{\sin \frac{\pi z_0}{h}} \right)}. \quad (21)$$

Using the known analytical dependence $\bar{K}(p)$ from effective pressure

$$\bar{K}(p) = a \left[1 - \delta (P_c - P_w) \right], \quad (22)$$

we represent (21) in the form

$$Q_r = \frac{2\pi k_0 \rho h}{\mu} \frac{a(P_c - P_w) \left[1 - \frac{\delta}{2} (P_c - P_w) \right]}{\ln \frac{3.1422 R_c}{l} + \frac{h}{l} \ln \left(\frac{h}{2\pi R_w} \frac{1}{\sin \frac{\pi z_0}{h}} \right)}, \quad (23)$$

where

$$a = A + B P_c - C P_r; \quad \delta = \frac{B}{a},$$

where A, B, C are known coefficients determined by the least squares method using a standard program according to experimental data for three classes of rocks that differ in the elastic properties of the skeletal component of the solid phase; P_r is mountain pressure.

Mass inflow to a vertical well under identical conditions is determined by the formula

$$Q_B = \frac{2\pi k_0 \rho h}{\mu} \frac{a(P_c - P_w) \left[1 - \frac{\beta}{2(P_c - P_w)} \right]}{\ln \frac{R_c}{R_w}}. \quad (24)$$

The relative difference between the inflows to a horizontal well, considering the change in rock permeability depending on the effective pressure and without considering this change, is the value

$$\frac{Q_r|_{\beta=0} + Q_r}{Q_r|_{\beta=0}} = \frac{\beta}{2} (P_c - P_w) \cdot 100\%. \quad (25)$$

To estimate the length of the horizontal wellbore, starting from which the inflow to the horizontal well exceeds the inflow to the vertical well, equating the right parts of formulas (24 and 25) at $z_0 = 0.5h$ we get

$$\ln \frac{2}{x} = (x-1) \ln \frac{h}{2\pi R_w}, \quad (26)$$

where

$$x = \frac{h}{l}.$$

Having found the roots of (26) with respect to x at different values of h , we determine the values of the length of the horizontal wellbore given in Table 1 (for $R_w = 0.1m$).

The results of numerous calculations are presented in Figs. 5–7.

To establish the pressure distribution in the reservoir, in the laboratory for research and production of oil and gas of the Atyrau University of Oil and Gas named after Safi Utebaev, experimental studies were carried out to characterize the pressure distribution along the axis of the oil-bearing reservoir segment under changing boundary conditions.

The experimental setup provides the ability to change the sequence of connection of reservoir models: serial, parallel and mixed connection.

Coefficient k , m^2 is calculated based on Darcy's law by the formula

$$k = \frac{Q \cdot \mu \cdot L}{\Delta p \cdot F},$$

Table 1

Determined lengths of a horizontal wellbore

h, m	10	20	30	40	50	70	85	100
l, m	8.42	17.27	26.21	35.21	44.21	62.36	75.99	89.66
$1/h$	0.842	0.864	0.874	0.880	0.884	0.891	0.894	0.897

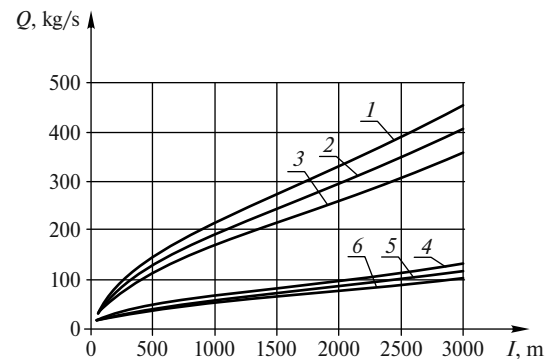


Fig. 5. Inflow to a horizontal well as a function of wellbore length: 1–3 – at $h = 120 m$ and δ respectively 0; 0.0085; 0.02044; 4–6 – at $h = 30 m$ and δ respectively 0; 0.0085; 0.02044

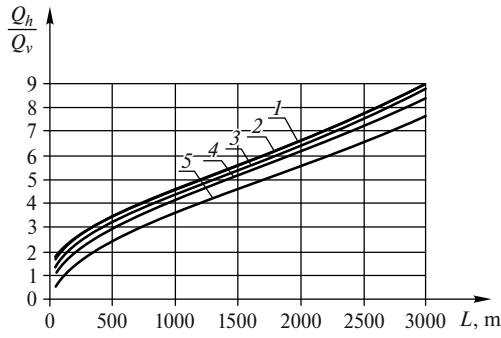


Fig. 6. Dependence of the ratio of inflows to horizontal and vertical wells on the S length of the horizontal wellbore:
1–5, respectively, with a reservoir thickness of 7.5; 15; 30; 60 and 120 m

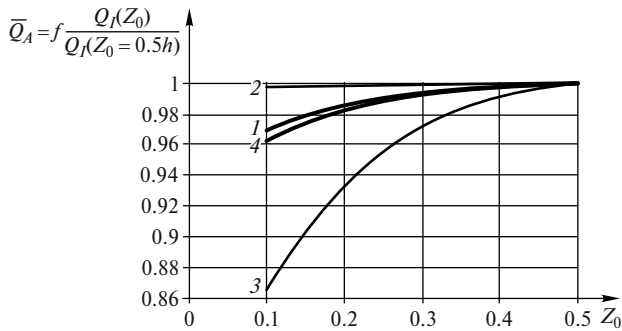


Fig. 7. Dependences of the change in inflow to a horizontal well on its position relative to the roof and bottom of the reservoir:
1 and 2 – $h = 7.5$ m; l – respectively 50 and 2000 m; 3 and 4 – $h = 120$ m, l – respectively 50 and 2000 m

where μ is the coefficient of dynamic viscosity of the liquid, for water it is $\mu \approx 10^{-3}$ Pa · s; F is a reservoir cross-sectional area, $F = 0.00196$ m²; L is the length of the control section of the formation, $L = 1$ m; Δp is fluid pressure drops along the reservoir length L , Pa; Q is volume flow of liquid, m³/s.

Data on permeability, depending on the fractions of sand, are shown in Table 2.

To characterize the pressure distribution along the axis of the oil-bearing formation segment under changing boundary conditions, the task was to obtain the parameters of the pressure distribution along the formation and study the influence of the boundary conditions on the value of this pressure; Fig. 8 shows the hydraulic scheme of the installation.

Based on the obtained values, the patterns of pressure distribution to the pressure sensor throughout the reservoir through the Q_{cp} and the dependencies of the Q_{cp} output and pressure were established (Figs. 9, 10).

The third layer of the stand was modified by filling core samples.

The core is pre-washed and cleaned of oil residues. The core was taken from the depths of 830–871.8 m from the Neokomsky II layer. Core samples are 1.0 m high (100 %), samples from the Neokom-I formation are 0.4–0.5 m high. The core is uniformly saturated with oil 0.0, 0.20, 0.40, 0.65, 1.0 m. Blu-

Table 2

Permeability of quartz sands

Sand fraction	Sand permeability (mcm ²)
From 0.4 to 0.8 mm	0.101–0.102
from 0.6 to 1.2 mm	0.135
from 2.5 to 3 mm	0.230

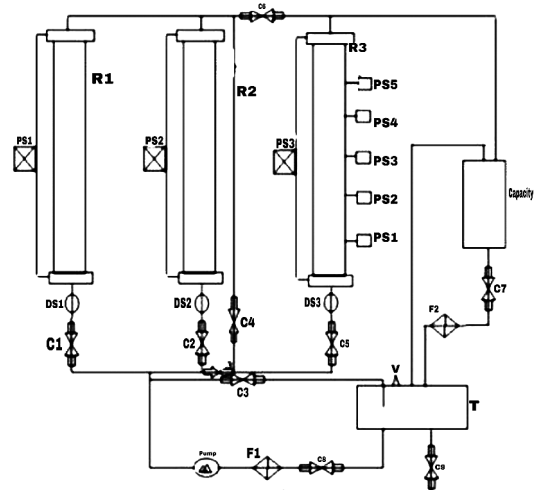


Fig. 8. Hydraulic scheme of the installation for the experiment on segment No. 1–3

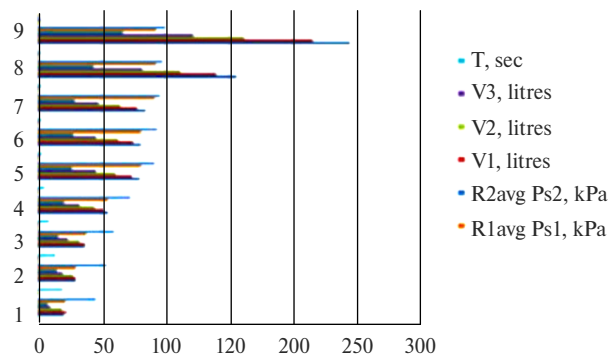


Fig. 9. Pressure distribution in reservoir Nos. 1, 2, 3 P_{ps3} average, kPa

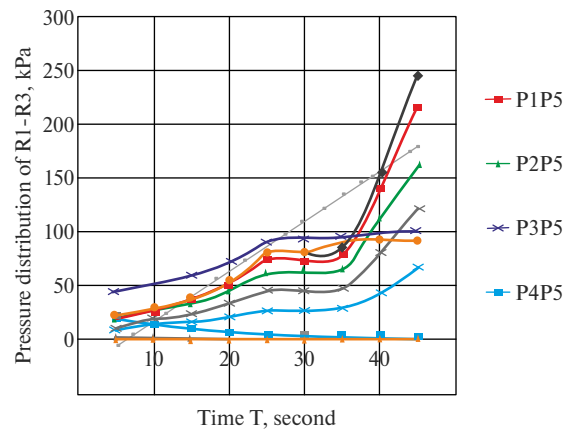


Fig. 10. Pressure distributions in reservoir No. 3 P_{ps3} medium, kPa

ish-gray, silty-like, argillite-like, layered, thin-lamellar, uneven, roughly fractured, calcareous clays. On all spans, the seams are separated by coal residues within 1–2 mm.

Pressure sensors have been placed below. 3 sensors are placed along section F of the area. The sensor readings are presented in Table 3, according to which the graph is built, i. e., the purpose of the experiment is to determine the change in pressure by the angle of rotation of the radius vector. The experiment showed that the pressure changes to three different values at three points. This confirms the theoretical conclusion about the patterns of pressure distribution.

As an example, the parameters of the field were taken, and the deflection angle of 120° was calculated (Fig. 12). The ob-

Table 3

Calculation of pressure distribution parameters in the injection well

Angle radius vector, φ	120	80	145	240
Pressure, P , MPa	2.98	3	2.9	3

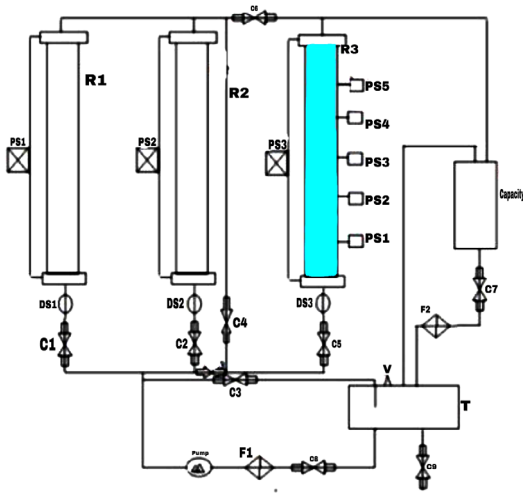


Fig. 11. Hydraulic scheme of the experiment reservoir No. 3

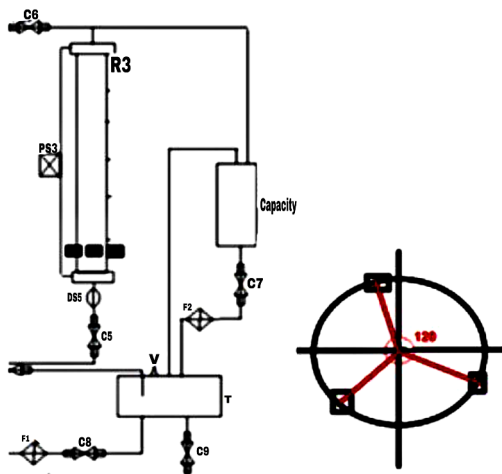


Fig. 12. Scheme of the location of the sensors on the area of the reservoir

tained research results and calculated values confirm the established regularities of pressure distribution in reservoir conditions, show the correspondence of the model to the field development processes, thereby allowing one to improve the field development methods.

Comparing the theoretical data and the results of experimental work, it was found that in addition to the physical properties of the reservoir, in which the regularity of pressure distribution in an inhomogeneous medium was revealed, the rate of its distribution also depends on the angle of change.

The results of experimental studies aimed at studying the pressure distribution in the reservoir make it possible to measure the pressure distribution parameters along the axis of the oil-bearing reservoir segment under changing boundary conditions.

Thus, as a result of experimental work, the regularity of pressure distribution with the growth of a layer filled with quartz sand was established, and the experiment gave a positive result.

Conclusions. The critical analysis of literary sources and studies on the application of the tertiary method of enhanced

oil recovery for the conditions of the fields of Western Kazakhstan shows the effectiveness of the polymer flooding technology, which is determined by the properties of the reagents used, the choice of which is carried out taking into account the individual characteristics of the physical and capacitive properties of productive formations and the state of development and operation of a particular Place of Birth.

A two-dimensional mathematical model of the dynamics of pressure distribution in a circular deposit has been built. Considering circular deposits as a source and sink, the calculation of the dynamics of pressure distribution inside the circle and outside the circle, taking a given value at the boundary of the circle, reduces to solving the internal and external Dirichlet problem and is shown for any value $r < R$ and $-\pi \leq \varphi \leq \pi$.

The conducted experimental studies aimed at studying the pressure distribution in the reservoir made it possible to measure the pressure distribution parameters along the axis of the oil-bearing reservoir segment under changing boundary conditions. To carry out the necessary studies, an experimental setup was designed, consisting of three reservoirs with different physical characteristics. The main formation was equipped with special pressure sensors, which made it possible to record pressures along the segment axis. In general, by selecting the necessary porous formation media, close parameters of the pressure distribution in the formation were obtained under changing boundary conditions.

Analysis of the results of the calculations performed showed that:

- in isotropic formations with single-phase filtration, the flow rate of a horizontal well exceeds the flow rate of a vertical well that has completely penetrated the formation with a thickness h , if the length l of the horizontal wellbore is greater than the values given in Table 1;
- the ratio of the length of the horizontal wellbore to the thickness of the formation penetrated by a vertical well, which ensures the equality of the flow rates of both wells, increases with the growth of the penetrated formation thickness;
- the relative value of the increase in the production rate of a horizontal well compared to the production rate of a vertical well decreases with increasing reservoir thickness;
- the intensity of the increase in the production rate of a horizontal well decreases with an increase in the length of the horizontal wellbore;
- ceteris paribus, the highest flow rate of a horizontal well corresponds to the location of the horizontal wellbore at equal distances from the roof and bottom of the formation.

Thus, the problem of determining the inflow to a horizontal well in a deformable finite circular formation with stationary fluid filtration has been mathematically formalized and solved.

The influence of formation deformability, well location relative to its impermeable top and bottom, horizontal wellbore length and thickness of the penetrated formation on the magnitude and intensity of inflow to the horizontal well is assessed.

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Розподіл тиску в нафтопласті у двовимірній площині

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Мета. Встановлення закономірностей мінливості динаміки тиску у пласті та розробка на цій основі методики контролю й регулювання видобутку вуглеводнів.

Методика. Для досягнення поставленої мети були проведені експериментальні дослідження та узагальнені результати експериментів.

Результати. Установлені функції розподілу тиску при стаціонарному притоці рідини у двох площинах, що дозволяють вести контроль і управління видобувними роботами, особливо на пізніх стадіях розробки.

Наукова новизна. На основі встановлених закономірностей створена модель розподілу тиску у пласті у двовимірній площині. Проведене експериментальне дослідження розподілу тиску у пласті, що дозволило зняти характеристики розподілу тиску вздовж осі сегмента нафтоносного пласта при змінюваних граничних умовах

Практична значимість. Запропонована математична модель процесів розподілу тиску за кутом нахилу, що дозволяє визначити ефективність заводнення. Дана оцінка впливу деформованості пласта, місця розташування свердловини відносно непрониких покрівлі й підшви пласта, протяжності горизонтального ствола й потужності розкритого пласта на величину та інтенсивність припливу до горизонтальної свердловини.

Ключові слова: родовище, тиск у нафтопласті, управління видобутком

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