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APPROXIMATION OF BLADES OF RADIAL MACHINES WITH MULTIPARAMETER FAMILY OF SMOOTH SURFACES

Purpose. Development of a mathematical model for creating the spatial forms of blade devices of rotating radial dynamic blade machines.

Methodology. An approach to the development of a mathematical model of blade profiling of radial dynamic blade machines, as parts of power plants, air-jet engines and fuel component supply systems of rocket engines, has been suggested. The approach is based on a physical model of the working body flow over helical surfaces.

Findings. A system of equations for describing the blade of a radial dynamic blade machine of any purpose as a family of smooth surfaces has been obtained. A multi-parameter correction of the shape of the smooth surface of the blade, accounting for the change in geometric data, based on input and output parameters of the blade, has been developed. Based on a review of modern technology samples, possible configurations of the spatial shape of the blade of the radial and radial-axial type, geometric factors affecting the surface of the blade being created are taken into account. The possibility of obtaining a multi-level blade apparatus by changing the conditions of the geometric parameters at the entrance is shown.

Originality. As part of the developed approach, in relation to the conditions for ensuring the calculated geometric parameters and the working process conditions of the blade machine, blade machines operating on compressible and non-compressible working bodies are considered. In particular, the possibility is shown of ensuring the construction of the spatial surface of the blade of the impellers of radial blade machines with a wide range of angles of the blades at the entrance and exit using smooth surfaces.

Practical value. The use of a developed mathematical method allows you to perform the profile of rotating vane devices for radial vane dynamic machines of various purposes, such as centrifugal pumps and compressors, centrifugal radial turbines, as well as diagonal type vane machines. The practical significance of the obtained results is determined by the use of dynamic radial vane machines in aviation and rocket technology, aggregates of the mining industry, and technological devices of chemical industry enterprises.

Keywords: *helical surface, vane, free edge, disk, profile, movement, coordinate system, equations*

Introduction. The wide use of bladed machines for various purposes in engineering determines the relevance of their design methods, which include profiling methods of impeller blades. In the modern theory of radial-type dynamic vane machines, three main methods of vane profiling are used: 1) direct or geometric; 2) reverse, or hydro-gas dynamic; 3) optimization.

In the direct method, the shape of the vane from the inlet to the outlet of the impeller is defined by some class of surfaces with free parameters. The flow of the working body, under the imposed physical boundary conditions, is thus completely parametrically determined; therefore, by changing the values of the free parameters, it is possible to select the determining indicators of the impellers to ensure the work process in them.

Similar methods for designing technical devices are widespread, for example; let us point to the description of the surfaces of gear wheels of mechanical transmissions by multiparameter families, which is presented in the work by Litvin F. L., Fuentes A., Gear Geometry and Applied Theory *2nd* ed., 2004.

In the inverse method, the spatial shape of the vane is given as the current surface of a known flow. Classes of exact solutions of model equations (Euler, Navier-Stokes, etc.), as a rule, depend on free parameters, and are quite numerous. In this way, it is possible to choose the movement of the working body and, accordingly, the shape of the blades for the required indicators of the impellers.

In the optimization method, the movement of the working body and the shape of the blades, which provide the specified indicators of the impellers, are found together. As an example, this is shown in the works by Yershov S. V., Yakovlev V. A. Aerodynamic Optimization of Spatial Shape of Steam and Gas Turbine Blading, 2008, and Lampart P., Ershov S. Direct Constraint Computational Fluid Dynamics Based Optimization of Three-Dimensional Blading for the Exit Stage of a Larger Power Steam Turbine, 2003.

Note that in fact optimization is nothing more than a combination of direct and inverse methods, as shown, for example, in the work by Kvasha Yu. A., Melashich S. V., On the joint solution of the direct and inverse problem of gas dynamics of compressor grates, 2008.

Unsolved aspects of the problem. For dynamic, radial blade machines operating on compressible and incompressible fluids, different methods for constructing blade profiles are used, but all these methods have a common part, which consists in the proper description (approximation) of the surface of the blade. Even in the inverse method, which allows you to take the known surface of the flow for the surface of the vane, the task of approximation in one form or another is present. Indeed, the flow surface, as a component of the exact or approximate solution of the model equations (Euler, Navier-Stokes, etc.), can be written in a very complicated way, for example, with the help of special functions, which will cause great difficulties in the manufacture of the vane, if only not approximate the latter by some family of smooth surfaces.

These circumstances determine the need to develop a generalized method of describing the surface of the blade of the impeller of radial blade machines.

The object of research is smooth helical surfaces.

The subject of the study is blade machines of the radial dynamic type.

The purpose of the work is to describe the spatial shape of the blade of a radial dynamic blade machine by a multiparameter family of smooth surfaces.

Literature review. In work [1], an overview of the methods for profiling blades of radial vane pumps when designed is given. It is indicated that even now there is a rather limited number of publications regarding the explanation of the design procedure of the blade profile of the radial type. Thus, there are certain difficulties in the actual process of obtaining the spatial shape of the blade, and there is a need for secondary design to ensure the required performance characteristics. In this article, an attempt is made to give a step-by-step methodology for designing the profile of a radial type blade.

The geometric method for creating a blade profile of a radial turbomachine has been developed and is quite widely presented in the work by, for example, Dr. Ing. B. Eckert Axialkompressoren und Radialkompressoren. Anwendung. Theory. Berechnung. 1959. In this work, the meridional section of the impeller is calculated based on the determination of the parameters of the averaged flow, which is calculated for the average values of the parameters at the intersections of the flow meridional part, for which the presence of expansion and contraction zones is excluded in order to minimize energy losses in it. At the same time, the profile is created in such a way as to ensure the known law of change in relative and meridional velocities in the interscapular channel. For the geometric method, the most common is the use of an arc of a circle to create the meridional contour of the radial blade.

The work [2], which is essentially an example of an optimization design method, is devoted to the study on the characteristics of the impeller by developing a blade profile using an arc of a circle to form its edges, as well as a point method performed using CFD analysis of the blade profile of the impeller for direct and reverse. The impeller blade profile was designed and analyzed using SolidWorks Flow Simulation. Numerous studies have been conducted using the k-ε model of turbulence.

In work [3], the authors used a parametric equation to describe the angles of inclination of the blade, after which the parameters were optimized, taking into account the operating conditions of the unit. In this work, the inverse problem of designing a centrifugal pump is solved.

The work [4] presents a method for optimizing the blade profile of a radial blade machine. This method includes blade profile parameterization, experimental design, and computational fluid dynamics algorithms. In particular, a non-uniform cubic B-spline curve was used to parameterize the blade profile. An analysis of the effectiveness of each point of the work process was carried out based on methods of computational fluid dynamics, and the problem of blade profile optimization was solved.

Work [5] is also an example of an optimization method for designing a radial vane dynamic machine. The influence of the shape of the blade on the performance of the centrifugal pump is analyzed and shown by designing the blade profile using the concentric arc method and the step-by-step design method. For both methods, the design parameters are kept the same and the CFD analysis is performed to obtain the highest pump performance under different conditions and flow parameters.

In [6], a method for creating a blade profile of a radial turbine is proposed, which consists in using fourth-order Bezier curves to determine the sleeve, body, and blade. The thickness of the scapula is assumed to vary linearly along the meridional coordinate and along the height.

In [7], the methods for creating a blade profile, which are proposed by various authors, are considered, and their critical analysis is also given. The problems that arise during the construction of the blade profile are noted. The use of the geometric method is indicated, since the basic relationship for calculating the radius of curvature, the method of fixing the center of curvature and the starting point of the arc is not clearly defined in the profiling procedure. The principle of constructing a blade profile is to draw an arc of a circle with one radius, using the calculated angles of the blade at the entrance and exit from the wheel β_1, β_2 and the corresponding radius R_1, R_2. Next, the procedure for obtaining the key geometric data of the blade profile, which is obtained using the arc of a circle, is shown.

In [8], an approach to the formation of the design of the blades of the impeller is considered, which is based on the analytical choice of the dependence of the flow change along the flow line. This approach is implemented in the form of custom software that allows the user to build a linear drawing based on the given parameters in the normal mode by designing the blade and calculate the geometric dimensions of the impeller. It is indicated that the method used has been tested and confirmed by a physical experiment. The meridional shape of the impeller is defined by several Bézier curves or B-spline curves for both blade and shroud contours. The contours are divided into three main curves: for the entrance, for the blade and for the diffuser. Coordinates of control points, and geometric parameters that can be changed, modify the meridional direction.

In [1], it is noted that the geometric methods available for designing impeller blades with radial flow are as follows: arc method, double arc method, concentric circle arc method, and point method. Each of these methods is analyzed in detail, and blade design calculations are performed using the concentric circle arc method and the point-by-point method. Based on the vane design methods considered, calculations are given for the concentric circle arc method and the point-by-point method for the pump parameters that provide the greatest efficiency.

In [9], the meridional shape of the impeller is determined by several Bézier curves or B-spline curves for both contours of the bushing and casing. The geometric parameters could be changed by the optimization program to modify the meridional passage of the interscapular channel. In total, the meridional shape is controlled by 11 parameters, and upper and lower limits are defined for each parameter.

We will show an example of an analytical method for creating a spatial part of a radial turbomachine, for example, in work [10]. The detailed aerodynamic design of the rotor blades uses a general pipeline design system using a polynomial Bezier curve, which is also used to estimate the imported sleeve and shroud curve. It is claimed that this curve is well suited for creating a smooth curve of the blade surfaces in the gas pipeline design process. The suction and pressure sides of the rotor blade in the sleeve and casing can be calculated using a system of equations, which are the equations of the helical surface. By defining the sleeve and casing contours on the suction and pressure sides, the intermediate surface coordinates are determined by connecting the sleeve and casing surface points along a quasi-normal line.

Thus, it can be stated that the optimization design method is more widespread in research practice than others. The relevance of conducting scientific research on the creation and improvement of geometric methods for designing the spatial shape of the blade of a radial dynamic blade machine is confirmed by the insufficiency of research in this field, the ongoing search for the optimal design method. In combination with the methods of mathematical modeling of hydrodynamic and gas-dynamic processes in radial vane machines, geometric methods for creating the spatial configuration of radial vane machines constitute a significant part of their design. In addition, the creation of a universal geometric method for obtaining the spatial shape of the blade of the impeller of a radial vane machine, which takes into account the peculiarities of the parameters of the blade of the impeller, is an important scientific and technical task, the solution of which will greatly facilitate the design process.

Methodology. To construct the blade profile, the results of the thermogas-dynamic calculation of the impeller parameters of a centrifugal compressor, radial turbine, or the hydrodynamic calculation of the centrifugal pump stage are applied and the following geometric parameters are used: D_1 – the peripheral diameter of the blades at the inlet section to the wheel; D_2 – the peripheral diameter of the wheel; D_{H1} – the diameter hubs of the wheel at the inlet section; Z – the number of blades; b_1 – the height of the blades at the inlet section to the wheel; b_2 – the height of the blades at the exit from the wheel; h_1 – the thickness of the blades at the inlet section to the wheel; h_2 – the thickness of the blades at the exit from the wheel; β_{1B} – an angle of blades at the inlet section to the wheel; β_{2B} – an angle of blades at the exit out the wheel. In addition, the type of the impeller is selected, as well as the thickness of the hub h_H on the diameter D_2 – if the impeller is semi-open, or the thickness of the cover disk is also specified, if the impeller is of a closed type.

In the Cartesian coordinate system, in the plane (y, z), the leading edge of the blade and the forming part of the hub, which forms the flow part in the axial direction, can be described by different curves, as shown in Fig. 1.

A quarter of circle or ellipse, hyperbola, or sine curve can be used to describe the shape of the curve. We will use further the equation of the ellipse to determine the coordinates in the meridional plane

$$\left[\frac{(z-z_0)}{a}\right]^2+\left[\frac{(y-y_0)}{b}\right]^2=1,$$

or its particular case, the equation of the circle

$$\frac{\left(z-z_0\right)^2+(y-y_0)^2}{a^2}=1,$$

where y, z are coordinates; a is the semi-minor axis of the ellipse; b is the semi-major axis of the ellipse, as shown in Fig. 2.

Fig. 3 shows the coordinate system chosen to build the impeller blade model.

Let us take z as the independent variable, then for the dependent variable y we will have

$$y=y_0+b\cdot\sqrt{1-\left(\frac{z-z_0}{a}\right)^2}$$

in case of the ellipse and

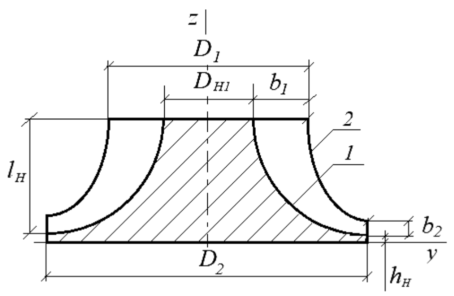


Fig. 1. Scheme of the impeller:
1 – hub contour; 2 – shroud contour

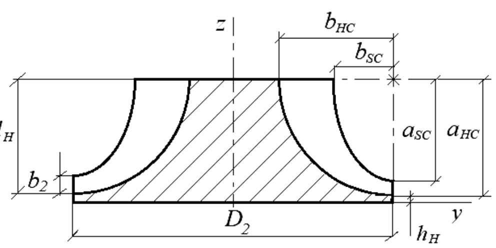


Fig. 2. Schematic of the formation of the flow part of the impeller

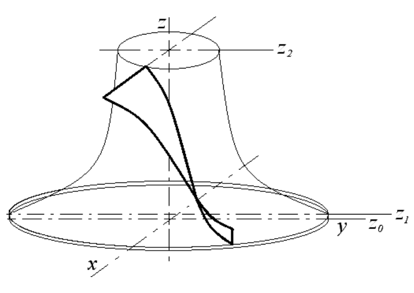


Fig. 3. Impeller in the coordinate system (x, y, z)

$$y=y_0+\sqrt{(a^2-(z-z_0)^2)}$$

in case of the circle.

Let us make the following assumptions:

1) the inlet part of the leading edge of the blade is a part of the helical surface;

2) the output part of the blade is connected with the input part along a curve, the coordinate representation of which is performed below;

3) the blade edge at the inlet is a straight line;

4) the spatial part of the blade is a smooth curve;

5) the blade thickness is taken constant.

In the plane z = Const, as shown in Fig. 2, we assume that

$$z_1=h_H;\quad z_2=l_H+h_H;$$
$$y_1=D_2/2;\quad y_2=D_{H1}/2,$$

and we will use the known relations between Cartesian and polar coordinates as follows

$$\begin{cases}x=r\cos\varphi\\ y=r\sin\varphi\\ z=h_H+l_H\dfrac{\varphi}{\varphi'}\end{cases},\qquad(1)$$

where x is a coordinate; r is the radius; h is thickness; φ is an angle, to describe the helical line along which the working body moves when the impeller rotates. In (1), the value of r depends on the angle , which characterizes the amount of rotation when forming the element during its motion along the helical surface. For impellers of radial turbines, the angle β_{1B} = = 90° can be accepted, and for compressor and pump impellers, as is known, it varies in wide ranges as – 120° > β_{2B} > 60°.

Consider the line of intersection of the helical surface of the blade with the bearing disk, in the plane z = Const. The trace of this line in the plane $z = z_1 = h_B$ is shown in Fig. 4.

Let us write the equation of this line in the form

$$y(x)=\begin{cases}0,\quad 0\le x\le r_1\\ y_1(x),\quad r_1\le x\le R\cos\varphi_1\end{cases},\qquad(2)$$

where $y_1(x)$ describes the curvilinear part of the line in the form of a second-order polynomial

$$y_1(x)=A_0+A_1\cdot x+A_2\cdot x^2,\qquad(3)$$

and r_1 is the coordinate of the point of smooth connection of straight and curved sections.

Let us introduce the angles describing the properties of the curvilinear part of the line (3) at the exit from the impeller and shown in Fig. 4, then the system of equations with respect to the coefficients of (3) will take the form

$$\begin{cases}y_1(x=r_1)=A_0+A_1r_1+A_2r_1^2=0\\ y_1'(x=r_1)=A_1+2A_2r_1=0\\ y_1'(R\cos\varphi_1)=A_1+2A_2R\cos\varphi_1=-\text{ctg}(\varphi_1-\varphi_2)\end{cases}.\qquad(4)$$

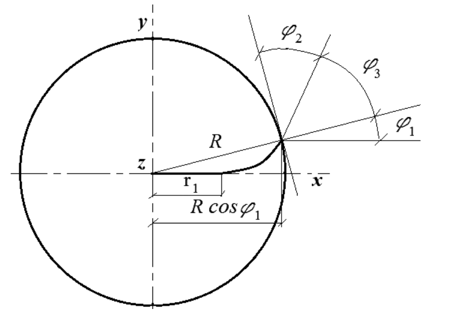


Fig. 4. Trace of the forming curve of the vane feather at the exit from the impeller

In Fig. 4, the angles are: φ_1 – the angle between the plane passing through the axis of rotation and the point of exit of the blade edge, and the *x* axis; φ_2 – angle β_{2B}; φ_3 – is an angle that complements φ_2 to a right angle.

Having solved the system of equations (4), after evident transformations we obtain

$$A_0 = \frac{r_1^2 \operatorname{ctg}(\varphi_1 - \varphi_2)}{2R\left(\frac{r_1}{R} - \cos\varphi_1\right)}; \quad A_1 = -\frac{r_1}{R}\frac{\operatorname{ctg}(\varphi_1 - \varphi_2)}{\left(\frac{r_1}{R} - \cos\varphi_1\right)};$$

$$A_2 = \frac{\operatorname{ctg}(\varphi_1 - \varphi_2)}{2R\left(\frac{r_1}{R} - \cos\varphi_1\right)}.$$

Then, substituting the obtained values of the required coefficients (3), we will have the resulting equation of the curvilinear part

$$y_1(x) = \frac{\operatorname{ctg}(\varphi_1 - \varphi_2)}{R\left(\frac{r_1}{R} - \cos\varphi_1\right)}\left[\frac{r_1^2}{2} - r_1 x - \frac{X^2}{2}\right] =$$

$$= (r_1 - x)^2 \frac{\operatorname{ctg}(\varphi_1 - \varphi_2)}{2R\left(\frac{r_1}{R} - \cos\varphi_1\right)}.$$

In polar coordinates, according to (2), the dependence of the angle on *x* is determined from the ratio

$$\operatorname{tg}(\varphi) = \frac{y}{x} = \frac{\operatorname{ctg}(\varphi_1 - \varphi_2)}{2R\left(\frac{r_1}{R} - \cos\varphi_1\right)}\frac{(r_1 - x)^2}{x},$$

from which it follows

$$\varphi(x) = \operatorname{arctg}\left(\frac{\operatorname{ctg}(\varphi_1 - \varphi_2)}{2R\left(\frac{r_1}{R} - \cos\varphi_1\right)}\frac{(r_1 - x)^2}{x}\right).$$

The equations given above can be attributed to the rise along the helical surface along the axes *x, y, z* of the curve described by the system of (2) to the point $x(r_1)$, $y(r_1)$, $z(r_1)$. After passing this point, further ascent can be considered as an ascent of a straight line, because part of the curve is cut off by the surface of the free edges of the blades, on one side, and the carrier disk on the other side. The system of equations for describing the section $z_1 < z < z_2$ of the helical surface, which is presented in Fig. 2, can be written like this

$$\begin{cases} x = r\cos(\varphi(r) - \varphi_h(r_1)) \\ y = r\sin(\varphi(r) - \varphi_h(r_1)) \\ \varphi_h(z) = \varphi_4 \dfrac{e^{\left(\alpha \cdot \left[\frac{Z - Z_1}{Z_2 - Z_1}\right]\right)} - 1}{e^{\alpha} - 1} \end{cases}, \tag{5}$$

where α is a parameter.

The parameters and φ_4 are entered in the equation for $\varphi_h(z)$ of system (5). Parameter varies within $0 \le \alpha \le 90$, characterizes the slope of the blade's leading edge on diameter D_2. Parameter φ_4 determines the magnitude of the rotation angle during movement along the helical surface from z_1 to z_2 in the x–y plane, as shown in Fig. 5.

The angle of rotation forming around the axis of rotation and the determining bend of the feather of the vane relative to the initial position can vary in the range from 20 to 60°. When the surface of the feather blade is being constructed, this angle can be changed, for example, from 0 to the calculated value. Let us consider the impellers of radial turbines, when the value of the angle $\varphi_4 = 0$ is kept constant with the coordinate *z* being changed to $z = b_1$, and with $z > b_1$ the angle φ_4 changes from the value 0 to the calculated value. In this case the surface of the vane feather is obtained, which is shown in Figs. 6, *a, b*, and corresponds to the impeller with $\beta_{1B} = 90°$.

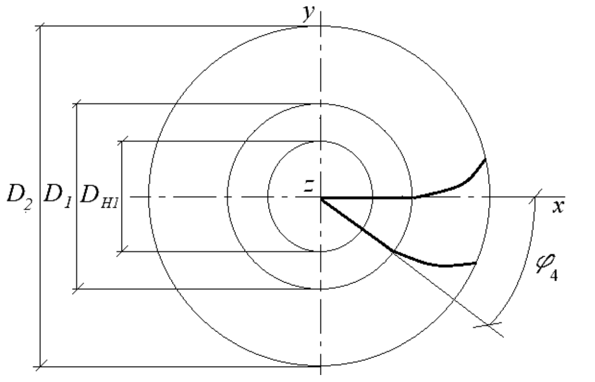


Fig. 5. The angle of rotation of the forming φ_4 with respect to the z axis

The same procedure is valid for impellers of centrifugal compressors with $\beta_{2B} = 90°$.

In order to reduce the angle of attack when the flow of the working medium hits the edge of the blade, the latter can have a curved inlet section, as shown in Fig. 6, *c*. To obtain such a surface of the blade when raising the forming one along the *z* axis, changes in the values of the angle φ_4 first take negative values up to the point of maximum concavity of the profile along the height of the blade b_1, after which they increase to the calculated value.

The value of the parameter can be determined by conditions

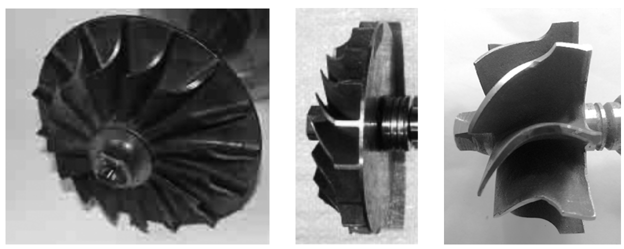
$$\varphi_h(z) = \frac{d\varphi_h}{dz} = \varphi_4 \frac{\alpha e^{\left[\alpha \frac{(z - z_1)}{(z_2 - z_1)}\right]}}{e^{\alpha} - 1}; \quad \varphi_h(z = z_1) = \varphi_4 \frac{\alpha}{e^{\alpha} - 1}.$$

The angle of inclination of the blade to the plane of the hub, as shown in Fig. 7, is determined from the ratio

$$\operatorname{tg}(\varphi_5) = \frac{dz}{Rd\varphi_h} = \frac{1}{R}\frac{dz}{d\varphi_h} = \frac{1}{R}\frac{1}{\varphi_4}\frac{(e^{\alpha} - 1)}{\alpha}. \tag{6}$$

An example of the design of the working wheels of radial vane compressor machines with different angles of inclination of the vanes is shown in Fig. 8.

Equation (6) with respect to parameter can be solved by expanding the function into a power series



a *b* *c*

Fig. 6. Impellers of centripetal turbines profiled with different angles of rotation φ_4 forming relative to the z axis

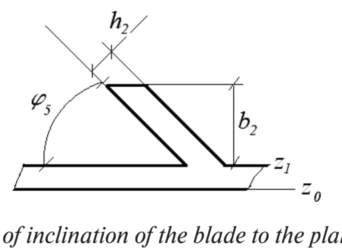
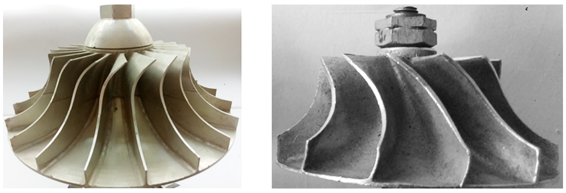


Fig. 7. Angle of inclination of the blade to the plane of the hub



a *b*

Fig. 8. Impellers of centrifugal compressors with different angles of inclination of the blade to the plane of the carrier disk φ_5: $a - \varphi_5 = 90°$; $b - \varphi_5 < 90°$

$$e^{\alpha} = 1 + \alpha + \frac{\alpha^2}{2} + \frac{\alpha^3}{6} + \frac{\alpha^4}{24} \ldots,$$

where from

$$\frac{e^{\alpha} - 1}{\alpha} = 1 + \frac{\alpha}{2} + \frac{\alpha^2}{6} + \ldots = \varphi_4 R \,\mathrm{tg}\, \varphi_5.$$

The turning angle of the vane feather relative to the axis of rotation in the meridional plane has small values, therefore, to find the parameter , we limit ourselves to taking into account the first three or four terms of the expansion, then the solution (6) will be reduced to the usual quadratic equation

$$\alpha^2 + 3\alpha + 6 \cdot (1 - R\varphi_4 \,\mathrm{tg}\, \varphi_5) = 0.$$

Deciding on α, and highlighting only a positive solution, we will get

$$\alpha = \frac{\sqrt{9 - 24\left(1 - R\varphi_4 \,\mathrm{tg}\,\varphi_5\right)}}{2} - 1.5.$$

The resulting system of equations for describing the spatial configuration of the blade of the impeller of the radial blade machine of the compressor and turbine types will have the following form

$$\begin{cases} x = r\cos(\varphi(r) - \varphi_h(r_1)) \\ y = r\sin(\varphi(r) - \varphi_h(r_1)) \\ \varphi_h(z) = \varphi_4 \dfrac{e^{\left(\alpha \cdot \left[\frac{z - z_1}{z_2 - z_1}\right]\right)} - 1}{e^{\alpha} - 1} \\ \varphi(x_1) = \begin{cases} 0, \quad 0 \le r \le r_1 \\ \mathrm{arctg}\left(\dfrac{\mathrm{ctg}(\varphi_1 - \varphi_2)}{2R(r_1/R - \cos\varphi_1)} \dfrac{(r_1 - r)^2}{r}\right), \end{cases} \\ r_1 \le r \le R\cos\varphi_1 \end{cases}, \qquad (7)$$

The resulting system of equations makes it possible to design the spatial shape of the blade of a radial-type turbomachine.

For example, taking into account the peculiarities of design, manufacture and testing of impellers, in order to reduce the cluttering of the inlet of the impeller of the pump, and to improve the hydraulic conditions of the flow of the working fluid, the impellers are shortened in the meridional direction.

Reducing the length of the blade in the meridional direction improves the value of the hydraulic efficiency of the impeller due to the reduction of hydraulic losses at the entrance. Which shape of the vane will provide the greatest increase in the hydraulic and overall efficiency of the pump, and will be rational, depends on a large number of parameters and features of the working process of the pump unit, namely: geometric parameters of the vane on the inlet D_1 and outlet D_2 diameters; presence and parameters of the input rectifier or axial stage; adjustment conditions, for example, if the input straightening device can change the flow angle by adjusting the position of the trailing edge; physical properties of liquid or gas, and others.

Practice shows that when shortening the blade length to improve the speed and pressure profile, the edge of the blade can be either a straight line or a curve, in accordance with the calculated flow conditions. Moreover, the cutting of the blade can be performed both on a conical surface and on a cylindrical surface, which is determined by the constant value of the diameter D_1 to the intersection with the bearing disk, which is the limiting reduction in the length of the blade.

Thus, it is necessary to find the line of intersection of the vane feather constructed according to equations (7) and the forming cone, either a straight one, or with a concave side surface, or with a convex side surface, whose top is on the z axis within the length of the impeller l_H. The cross-section of the cone and blade of the impeller is shown in Fig. 9.

Let us note some features of the definition of the surface of the intersection of the spatial shape of the feather blade and the cone. As you know, the lateral surface of the cone can have a forming that provides a convex, concave or no curvature – which is the case of a normal straight cone.

The base of the cone may not coincide with the plane defined by the coordinate z_2, but be between the planes $z = z_1$ and $z = z_2$, and behind the plane defined by the coordinate z_2. The top of the cone can occupy any position on the $0 - z$ axis.

As an example, let us assume that the cone with which the blades intersect is straight, its base coincides with the plane $z = z_2$, the top is on the z axis and belongs to the plane $z = z_0$. Fig. 10 shows the intersection of the cone and the impeller in the plane (y, z), according to the accepted assumptions.

The algorithm for finding the intersection of the cone and the spatial feather of the blade of the impeller of the radial blade machine consists in the following sequence of actions.

1. Let us set the height of the impeller $E = h_H + l_H$ and the diameter of the base of the cone, which is equal to $D = D_1$.

2. Find the position of point G (Fig. 8) as the point of intersection of the forming cone in the plane of intersection of the elliptical flow part, bounded by the surface of the carrier disk, which belongs to the carrier disk, forming the cone and the vane blade from equation (2)

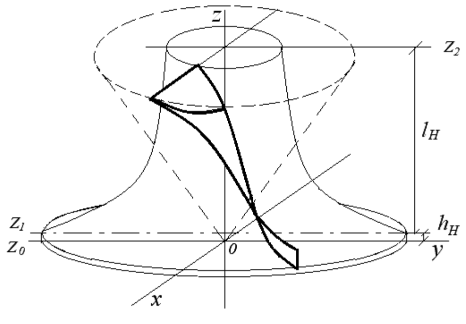


Fig. 9. Trace of the intersection of the vane blade with the cone in the x, y, z coordinate system

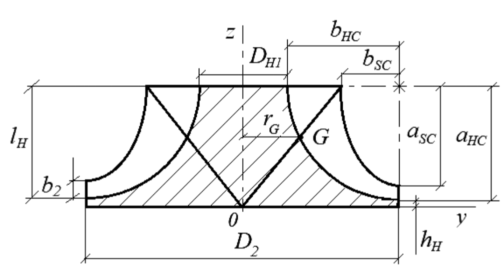


Fig. 10. Trace of the intersection of the vane blade with the cone in the y–z plane

$$\left[\frac{(z_G - z_2)}{a}\right]^2 + \left[\frac{(y_G - y_2)}{b}\right]^2 = 1,$$

from which the coordinate y_G is determined by the ratio

$$y_G = y_2 + b\cdot\sqrt{1-\left(\frac{z_G - z_2}{a}\right)^2} = \frac{D_1}{2}\frac{z_G}{h_{Ä} + l_{Ê}}.$$

The z_G parameter can be found from the following quadratic equation

$$\left(\frac{D_1}{2}\right)^2\frac{z_G^2}{E^2} - 2\frac{D_1}{2}\frac{z_G}{E}y_2 + y_2^2 - b^2\left[1-\left(\frac{z_G^2 - 2z_G z_2 + z_2^2}{a^2}\right)\right] = 0.$$

Given that $z_2 = E = h_H + l_H,\ y_2 = \frac{D_2}{2},\ a = l_H,\ b = \frac{D_2 - D_{H1}}{2}$, let us convert the equation to the canonical form

$$z^2 + pz + q = 0,$$

where

$$p = -\frac{2\left[\frac{D_1}{2}\frac{D_2}{(h_H + l_H)} + (h_H + l_H)\left(\frac{D_2 - D_{H1}}{2l_H}\right)^2\right]}{\left(\frac{D_1}{2}\right)^2\frac{1}{(h_H + l_H)^2} + \left(\frac{D_2 - D_{H1}}{2l_H}\right)^2};$$

$$q = \frac{\left(\frac{D_1}{2}\right)^2 + \left(\frac{D_2 - D_{H1}}{2l_H}\right)^2\left[\left(1+\frac{h_H}{l_H}\right)^2 - 1\right]}{\left(\frac{D_1}{2}\right)^2\frac{1}{(h_H + l_H)^2} + \left(\frac{D_2 - D_{H1}}{2l_H}\right)^2}.$$

The roots of the equation can be found using the known formula

$$z = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q},$$

and choosing the smallest of them

$$z_G = \frac{\left[\frac{D_1}{2}\frac{D_2}{(h_H + l_H)} + (h_H + l_H)\left(\frac{D_2 - D_{H1}}{2l_H}\right)^2\right]}{\left(\frac{D_1}{2}\right)^2\frac{1}{(h_H + l_H)^2} + \left(\frac{D_2 - D_{H1}}{2l_H}\right)^2} - \sqrt{\left(\frac{\left[\frac{D_1}{2}\frac{D_2}{(h_H + l_H)} + (h_H + l_H)\left(\frac{D_2 - D_{H1}}{2l_H}\right)^2\right]}{\left(\frac{D_1}{2}\right)^2\frac{1}{(h_H + l_H)^2} + \left(\frac{D_2 - D_{H1}}{2l_H}\right)^2}\right)^2 - \frac{\left(\frac{D_1}{2}\right)^2 + \left(\frac{D_2 - D_{H1}}{2l_H}\right)^2\left[\left(1+\frac{h_H}{l_H}\right)^2 - 1\right]}{\left(\frac{D_1}{2}\right)^2\frac{1}{(h_H + l_H)^2} + \left(\frac{D_2 - D_{H1}}{2l_H}\right)^2}}.$$

3. Substitute the dependence of the radius of the cone on the z coordinate

$$r_S(z) = \frac{D_1}{2}\frac{z}{h_H + l_H},$$

into the system of equations of the helical surface (7), from which we obtain the equation of the intersection of the conical surface with the surface of the blade of the impeller, which is a helical surface

$$\begin{cases} x(z) = r_{S.}(z)\cos(\varphi(r) - \varphi_{\mathrm{h}}(r_1)) \\ y(z) = r_{S.}(z)\sin(\varphi(r) - \varphi_h(r_1)), \quad z_1 \le z \le z_2 \\ \varphi_h(z) = \varphi_4\dfrac{e^{\left(\alpha\cdot\left[\frac{z - z_1}{z_2 - z_1}\right]\right)} - 1}{e^{\alpha} - 1} \\ \varphi(x_1) = \begin{cases} 0, \quad 0 \le r \le r_1 \\ \mathrm{arctg}\left(\dfrac{\mathrm{ctg}(\varphi_1 - \varphi_2)}{2R(r_1/R - \cos\varphi_1)}\dfrac{(r_1 - r)^2}{r}\right), \end{cases} \\ r_1 \le r \le R\cos\varphi_1 \end{cases} \quad (8)$$

The equation of the line of intersection of the conical surface with the surface of the impeller blade, which is a helical surface obtained by rotating around the z – axis by an angle of φ_4 and simultaneously rising relative to the y – axis with respect to its combined line, which is described by (2).

The spatial shape of the vane feather itself will be limited, as previously accepted, by the surface of the hub and the surface that will define the free edge of the vane, or in the case of using a closed-type wheel, by the inner surface of the cover hub. The corresponding radiuses of the outer surface of the hub and the free edge of the blade, which is a function of the z coordinate, can be written as

$$r_{íC}(z) = \frac{D_2}{2} - b_{íC}\cdot\sqrt{1-\left(\frac{z - h_H - l_H}{l_H}\right)^2};$$

$$r_{SC}(z) = \frac{D_2}{2} - b_{SC}\cdot\sqrt{1-\left(\frac{z - h_H - l_H}{l_H - b_2}\right)^2}.$$

4. Let us write the equation of the blade of the impeller, referring to the equation of the helical surface (8), in which we consider two areas of change in the z coordinate:

a) if $z_1 \le z \le z_G$, then in (8) the values of r defined by the following limits should be used

$$r_{HC}(z) \le r \le r_{SC}(z);$$

b) if $z_G \le z \le z_2$, then in (21) the values of r determined by the following limits should be used

$$r_S(z) \le r \le r_{SC}(z).$$

Discussion. In the method being proposed for constructing the blade surface of a radial turbomachine, as an example, the intersection of the surface of the blade feather with the surface of a straight cone was considered, in order to cut the inlet edge of the blade for reducing the clutter of the inlet section of the blade machine, as shown in Fig. 11.

It is possible to use other surfaces other than the surface of a straight cone. These can be the surfaces of single-cavity paraboloids and hyperboloids or ellipsoids of rotation.

Performing trimming on different surfaces is dictated by the need and desire to obtain a flow with as uniform a pattern of relative velocities of the working body as possible along the height of the vane device in the inlet section of the impeller.

Fig. 11. Centrifugal pump impeller with a tapered inlet edge

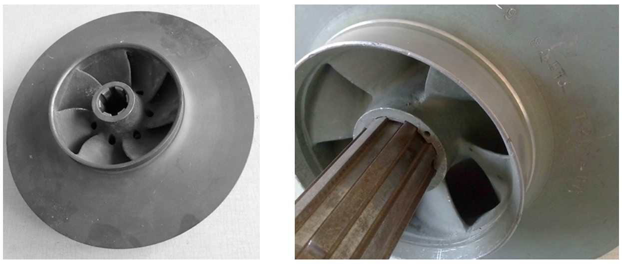
Fig. 12 shows the impeller of a centrifugal pump with an undercut inlet edge on the surface of a single-cavity hyperboloid of rotation.

The blades of the impellers of centrifugal compressors can have a spatial shape such that for certain design parameters, such as angles β_{1B} and β_{2B}, the flow part was limited to blades that are straight in relative motion, as shown in Fig. 13.

This type of vane profiling is known for centrifugal pumps, and was used for pumps first stage of the liquid rocket engines, as shown in Fig. 14.

This organization of the flow part and blade profiles is due to the fact that the flow of the working body in relative motion makes fewer turns in the flow part than in the wheel of a traditional design, as shown in Fig. 13.

The application of this type of impellers is extremely limited, because with the equality of the relative velocities at the inlet and outlet of the impeller, the blade device is affected by the flow of the working medium at the highest values of M_{W1}. Such impellers usually have a kinematic degree of reactivity close to 1, and as a result, the theoretical pressure coefficient is close to 0. When calculating the surface of the profile, in this case, the forming one, which is described by the equation of a straight line, carries out an ascent by the z coordinate while maintaining the value of the angle $\varphi_4 = 0$.



a *b*

Fig. 12. Impellers of centrifugal pumps with an undercut inlet edge on the surface of a single-cavity hyperboloid of rotation

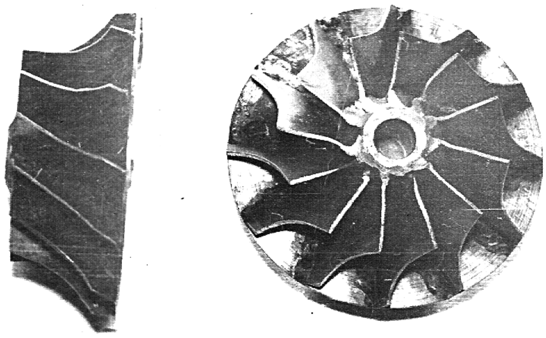


Fig. 13. Centrifugal compressor impeller with a straight blade in relative motion

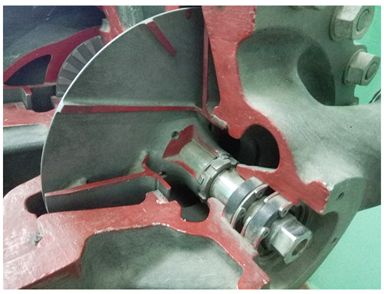


Fig. 14. Centrifugal pump impeller with straight blade in relative motion

In the case when it is necessary to obtain the spatial shape of the impeller blade of a centrifugal pump or compressor with $\beta_{2B} > 90°$, the forming element must have a bend in the direction of rotation, unlike the case with $\beta_{2B} < 90°$, as shown in Fig. 4.

Thus, the proposed multiparameter family of smooth surfaces allows describing the blades of radial dynamic machines as helical surfaces for all possible cases of construction of the flow part of the impellers of radial dynamic machines, such as compressors, pumps, and turbines.

Conclusions. As a result of the conducted theoretical research, the following conclusions can be formulated:

1. A multi-parameter family of smooth surfaces is proposed, which allows approximating the helical surfaces of blades of radial dynamic machines.

2. A feature of the construction of the blade profile of radial turbines is that at the value of the angle $\beta_{1B} = 90°$, rotation of the forming feather blade along the z axis to the angle of rotation of the curve φ_4, the turn starts from the coordinate $z = b_1$, and within the limits of the change $z_0 \le z \le z = b_1$, the raising of the forming part is carried out without turning relative to the z axis. This will also be true for the impellers of centrifugal pumps and compressors, in which $\beta_{2B} = 90°$. At the same time, in (2), the curvilinear part of the component will be missing, and a straight line will be used.

3. When constructing the profiles of the blades of the multi-tiered impeller of the radial blade machine, you should use the z coordinate change within $z_0 \le z < l_H$. At the same time, the length of the layer of shortened blades in relation to the main layer can be taken within $l'_H = (0.6-0.8)l_H$.

4. Reduction of the length of the blades at the entrance is possible not only on the conical surface, but also on the cylindrical surface, which is considered as a conical one with an infinite height, which is characteristic of some impellers of pumping units.

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Опис лопаток радіальних машин багатопараметричним сімейством гладких поверхонь

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Мета. Формування математичної моделі, що дозволяє створювати просторові форми лопаткових апаратів радіальних динамічних лопаткових машин, які обертаються.

Методика. Розвинуто підхід до формування теоретичної математичної моделі профілювання лопаток радіальних динамічних лопаткових машин, як агрегатів енергетичних установок, повітряно-реактивних двигунів і систем постачання компонентів палива ракетних двигунів. Розвинений підхід базується на фізичній моделі, що передбачає рух робочого тіла у проточній частині міжлопаткового апарату по гвинтовій поверхні.

Результати. Отримана система рівнянь, що дозволяє описати лопатку радіальної динамічної лопаткової машини будь якого призначення сімейством гладких поверхонь. Розроблена й використана система багатопараметричного корегування форми гладкої поверхні лопатки з урахуванням особливостей зміни геометричних даних, що базується на сталих вхідних і вихідних параметрах лопатки. На основі огляду сучасних зразків техніки враховані можливі конфігурації просторової форми лопатки радіального й радіально-осьового типу, геометричні фактори, що впливають на поверхню лопатки, яка створюється. Показана можливість отримання багатоярусного лопаткового апарату шляхом зміни умов геометричних параметрів на вході.

Наукова новизна. У рамках розвиненого підходу, стосовно умов забезпечення розрахункових геометричних параметрів і умов робочого процесу лопаткової машини, розглядаються лопаткові машини, які працюють на робочих тілах, що стискаються й не стискаються. Зокрема, показана можливість забезпечення побудови просторової поверхні лопатки робочих коліс радіальних лопаткових машин із широким діапазоном кутів лопаток на вході й виході з використанням гладких поверхонь.

Практична значимість. Використання розвиненого математичного методу дозволяє виконувати профіль обертових лопаткових апаратів для радіальних лопаткових динамічних машин різного призначення, таких як відцентрові насоси й компресори, доцентрові радіальні турбіни, а також лопаткові машини діагонального типу. Практична значимість отриманих результатів обумовлюється застосуванням динамічних радіальних лопаткових машин в авіаційній і ракетній техніці, агрегатах гірничої промисловості, технологічних пристроях підприємств хімічної промисловості.

Ключові слова: *гвинтова поверхня, лопатка, вільна кромка, диск, профіль, рух, система координат, рівняння*

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