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APPLICATION OF THE WAVELET TRANSFORMATION THEORY IN THE ALGORITHM FOR CONSTRUCTING A QUASIGEOID MODEL

Purpose. To investigate the interaction of geodesic and normal altitude indicators according to quasigeoid data, the joint use of space measurements and those performed on the Earth's surface in the implementation of geodetic tasks. In this article, the task is to create a calculation algorithm for further research on the quasigeoid model and the application of the model in solving geodetic problems.

Methodology. Reliable determination of the height anomaly requires great accuracy, therefore, the theory of wavelet-transformation was used in the model of the variant of space technologies as an alternative to the laborious leveling of the Earth's surface, which characterizes the actual fluctuations from the normal of the Earth's gravitational field, when calculating the mean square deviations of the plumb line is an urgent task.

Findings. A block diagram of the calculation algorithm has been compiled using a software package to solve the boundary problem of physical geodesy, in which the Earth's surface is subject to modern space measurements.

Originality. The use of wavelet analysis for processing information from satellite data in geodesy improves the results of image classification, and the coefficients of the wavelet transformation can be used as indicators for recognizing the coordinates of points with high accuracy.

Practical value. Application of the theory of wavelet transformations as a powerful mathematical tool for solving problems of geodetic information, data compression and recovery, increasing computing performance, encoding information.

Keywords: *physical geodesy, coordinate systems, gravitational field, geoid, quasigeoid, gravimetric height, modeling, coordinate transformation*

Introduction. At the end of the *20th* century, a new and important direction in the theory and technology of signal processing emerged and is successfully developing, which means "splash" or "small wave". Graphs with splash functions have become more frequent. They can be used to decompose signals instead of harmonic waves when solving problems of physical geodesy. The term wavelet was introduced in their article by Grossmann and Morlet in connection with the analysis of the properties of seismic and acoustic signals. These studies served as the beginning of an intensive study on wavelets in the next decade by a number of scientists such as Dobechies, Meyer, Mallat, Farge, Chui and others. Wavelet theory is a powerful complement to Fourier analysis and provides a more flexible technique for signal processing for the entire time period of its observation. The main advantage of wavelet analysis is its ability to detect highly localized changes in signals, whereas the discrete Fourier transform does not give this, because its coefficients reflect the behavior of the signal. Due to the completeness property of this system, it is possible to restore the process by means of an inverse wavelet transform to solve physical geodesy problems.

For centuries, for geodetic measurements and determination of the shapes and sizes of the geoid and quasigeoid, mechanical, later optical instruments were used [1].

Fundamental research on the parameters of the Earth continues to the present time and is relevant in science. On the main stages of solving this problem, the following can be noted:

- a spheroid close to an ellipsoid of revolution was considered from the *17th* century to the second half of the *19th* century;
- a triaxial ellipsoid, which is a model of a more complex form of the Earth – a geoid (quasigeoid) from the second half of the nineteenth century [2];
- from the 40s of the *20th* century to the present, the figure of the Earth is considered to be a body limited by the physical surface of the Earth.

Improvement of geodetic information has changed the quality of measurements, made it possible to develop geodesy, gravimetry as a science, using space technology [3].

The modernization of space methods allows high-precision networks to be carried out without complex instrumental observation with high accuracy subsequent approximations of the height anomaly.

The application of space measurement methods using global systems that receive a signal from satellites with a certain constancy allowed it to be distributed, which confirms the known surface of the Earth and the geoid, quasigeoid.

The geoid differs from the selected ellipsoid (for example, GRS 1980) by less than 100 m. Geocentric positions can now be determined using GPS with an accuracy of better than 1 dm in a purely geometric way.

In modern studies, it is possible to carry out mathematical calculations with greater accuracy in the field of pure gravity anomalies than mixed ones, from the solution of the boundary problem of physical geodesy, in which the Earth's surface is subject to modern cosmic measurements.

Methods for determining the external gravitational field of the Earth are based on the need to know the external gravitational field of the Earth. This problem can be solved in the form of a digital model of the average values of gravity anomalies or in the form of a system of coefficients in the decomposition of the gravity potential by spherical functions and the solution of integral equations. In this case, it is possible to increase the accuracy of the final results versatility and theoretical unlimited.

The calculations of the gravitational field using pure gravity anomalies are proved by the integrals performed by V. V. Brovar and reduce the errors of the height anomaly and the general components of the plumb deviation by several times, relative to mathematical calculations, using mixed anomalies.

The main point in improving the accuracy of transformant calculations is mathematical solutions that allow using pure gravity anomalies, latitude, longitude and geodetic height functions. The Neumann integral is used to determine the height anomaly, and the transformed Wening-Meins integral is used to calculate the components of the plumb deviation.

The modern paradigm based on the use of discrete linear transformations proves the sequence of the classical series of solutions to the problems of Molodensky, Neumann, Stokes, and the modified Vening-Meins integral. Modern methods of window Fourier transforms or discrete Hartley transformations have their drawbacks, despite the significant advantage of this approach over classical methods for determining coordinates and elevation.

Currently, an optimal wavelet transform methodology has been obtained for solving spectral analysis problems, which is used to calculate integrals of proper Fourier transforms and discrete Hartley transformations with a slight disadvantage, which is solved by mathematical wavelet analysis.

Materials and methods. Object of the study. All existing methods for determining transformants in modern studies on the gravitational field can be divided into classical and modern. Classical methods are based on integral formulas that require continuous gravimetric information, which is almost impossible.

In practice, the initial information is discrete, burdened by measurement errors and is not known on the entire surface of the Earth.

The advent of satellites and new opportunities for studying the gravitational field has significantly expanded the range of tasks of the theory of the Earth's figure. In recent decades, very high-precision global high-resolution geo-potential models have appeared [4]. This situation became possible due to the introduction of new measurement methods and techniques.

Due to the indeterminacy of the geoid figure, a quasigeoid acts as an auxiliary surface when studying the physical surface of the Earth. Its figure, unlike a geoid, is unambiguously determined by the measurement results, coincides with the geoid on the territory of the World Ocean and very close to it on land, deviating no more than 2 meters in high mountains and a few centimeters on flat terrain [5].

Global models of the Earth's gravitational field play an important role in constructing theories of motion of artificial Earth satellites, in modeling geodynamic processes and the internal structure of the Earth, in the study on natural resources, in oceanography, in marine and aviation navigation, and in solving defense problems, as well as for highly accurate figure determination Earth needed to establish a common Earth coordinate system [6].

In the world there are two second generation global navigation satellite systems (GNSS): GPS (USA) and GLONASS (Russia). At different stages of deployment, there are two more global positioning systems – European Galileo and Chinese BeiDou-2 (international name Compass), as well as two regional satellite navigation systems, Indian IRNSS and Japanese QZSS. GPS is fully operational: as of February 2016, 31 GPS satellites are operating in orbit [6]. The constellation of GLONASS satellites currently has 27 satellites. The Galileo system is being developed by the European GNSS Agency. The first Galileo system services for demonstration purposes are expected to be provided in the coming years. For test tests in 2011–2012, 4 experienced Galileo system satellites were launched into orbit. The full deployment of the Chinese GNSS BeiDou/Compass was planned by 2020. The satellite constellation of this system is to include 35 navigation satellites (5 geostationary and 30 non-geostationary). The Indian IRNSS system will provide regional navigation using 7 satellites launched in geosynchronous orbits. The Japanese regional satellite system QZSS will include a constellation of 3 satellites, expanding GPS capabilities for mobile devices, providing more accurate positioning and data transmission in the Asia-Pacific region [7].

The accuracy of measuring the signal from the satellite to the receiver should not be less than (0.25 m + 1mm/km). Achieving such accuracy is possible on the basis of the atomic time scale and special time support of the satellite system. The satellite uses atomic clocks, one second of atomic time in metrology is equal to 9,192,631,770 periods of oscillation, corresponding to the transition between two ultrathin levels of the cesium atom 133. The time scales of navigational artificial Earth satellites (NESS) and receivers are synchronized with the GNSS system time scale. Each GNSS has its own system time, which is atomic time and is based on one of the international time scales, as a rule, on the UTC (Universal Time Coordinated) scale (Mayer-Guerr T. ITG-Grace03s, 2015). Satellite observations to solve the scientific and practical problems of geodesy require the registration of time instants with very high accuracy in determining heights.

Global navigation satellite systems occupy a special place in the space infrastructure, providing continuous access to navigation services to consumers on the Earth's surface, in the air and near-Earth space. The most widespread in the world are the American and Russian satellite radio navigation systems GPS (NAVSTAR) and GLONASS (Global Navigation Satellite System).

Table 1 shows a list of STS stations in the Republic of Kazakhstan, this information is not classified, it is publicly available.

Fig. 1 presents the map of the international network of gas stations; Fig. 2 shows the scheme of the network of gas stations in Kazakhstan.

Using the GNSS coordinate data, it is possible to obtain accurate information about the transmission of signals to ground stations, using the wavelet transform of wave data to solve scientific problems of physical geodesy in seismic exploration.

The main type of source information for calculating heights remains the data of areal gravity surveys. To accurately determine the geoid heights, it is necessary to know the internal structure of the Earth. As a result, the approach to determining the shape of the Earth through the height of the geoid does not seem sufficiently strict, since the distribution of the mass density inside the Earth is not known with the necessary accuracy.

At present, a lot of modern methods have been developed for determining the transformants of the gravitational field, in which the main efforts are aimed at taking into account the specifics of real data. Such methods include: variational method, collocation method, convolution method based on linear discrete transforms (for example, fast Fourier transform). Naturally, they all have their advantages and disadvantages.

For example, the collocation method, from a mathematical point of view, determines functions by selecting an analytical approximation to a certain number of given linear functionals [9].

This method plays an important role in solving interpolation problems, with a further generalization of the collocation theory associated with its application to stochastic objects, when "collocation" is understood as a generalization of the least squares method to the case of infinite-dimensional Hilbert spaces [10].

The practical implementation of collocation models is based on the connection of the theory of Hilbert spaces with the reproducing kernel with the covariance theory of random processes.

The covariance functions (autocovariance and mutual covariance) of the random processes under study, as well as the reproducing core in the functional approach, play a fundamental role in collocation models.

The collocation method, as applied to the problems of physical geodesy, was developed in foreign works [8, 9].

Digital signal processing uses, as a rule, a discrete representation of signals, discrete linear transformations, the mathematics of discrete transformations originated in the depths of analog mathematics in the *18th* century, mainly in series theory and their application for approximating functions. But it became widespread and developed only in the *20th* century with the advent of computers. In principle, in its basic provisions, the mathematical apparatus of discrete transformations is similar to the transformations of analog signals and systems.

Table 1

Satellite coordinates of GNSS base stations in Kazakhstan [8]

No	Code	Locality	Position	Satellite groups	Status
1	ASTN	Astana	Latitude: 51° 09' 09,46359» N Longitude: 71° 32' 55,82708» E Altitude: 323.814 m	GPS/GLONASS/COMPASS/ GALILEO/QZSS	Working
2	ALM3	Almaty	Latitude: 43° 14' 15,17850» N Longitude: 76° 53' 05,11209» E Altitude: 835.088 m	GPS/GLONASS/COMPASS/ GALILEO/QZSS	Working
3	TLDK	Taldykorgan	Latitude: 45° 01' 15,39073» N Longitude: 78° 23' 20,03287» E Altitude: 555.2 m	GPS/GLONASS	Working
4	TARZ	Taraz	Latitude: 42° 54' 25.82496» N Longitude: 71° 22' 27.87429» E Altitude: 583.9 m	GPS/GLONASS	Working
5	SHMK	Shymkent	Latitude: 42° 19' 06.31780» N Longitude: 69° 36' 04.16680» E Altitude: 483.421 m	GPS/GLONASS	Working
6	KZLR	Kyzylorda	Latitude: 44° 48' 56,94865» N Longitude: 65° 32' 52,03476» E Altitude: 100.564 m	GPS/GLONASS	Working
7	AKTA	Aktau	Latitude: 43° 39' 03.33972» N Longitude: 51° 10' 16.47160» E Altitude: -12.090 m	GPS/GLONASS/COMPASS/ GALILEO/QZSS	Working
8	ATRU2	Atyrau	Latitude: 47° 05' 14,65744» N Longitude: 51° 54' 41,77845E Altitude: -17.855 m	GPS/GLONASS	Working
9	URLS	Uralsk	Latitude: 51° 12' 55,11656» N Longitude: 51° 21' 53,60423» E Altitude: 41.461 m	GPS/GLONASS	Working
10	AKSA	Aksay	Latitude: 51° 10' 03.93590» N Longitude: 53° 01' 01.61270» E Altitude: 64.498 m	GPS/GLONASS	Working
11	AKTB	Aktobe	Latitude: 50° 17' 11,35790» N Longitude: 57° 12' 10,50263» E Altitude: 203.616 m	GPS/GLONASS	Working
12	KKSH	Kokshetau	Latitude: 53° 16' 55,21732» N Longitude: 69° 22' 58,25710» E Altitude: 218.310 m	GPS/GLONASS	Working
13	KSTN	Kostanay	Latitude: 53° 13' 11,64296» N Longitude: 63° 37' 21,06674» E Altitude: 165.071 m	GPS/GLONASS	Working
14	KRGD	Karagandy	Latitude: 49° 48' 05,68260» N Longitude: 73° 05' 25,46637» E Altitude: 525.528 m	GPS/GLONASS	Working
15	PVLD	Pavlodar	Latitude: 52° 17' 04,60152» N Longitude: 76° 56' 42,33345» E Altitude: 113,260 m	GPS/GLONASS	Working
16	EKBS	Ekibastuz	Latitude: 51°42'42.97447" N Longitude: 75°19'59.59637" E Altitude: 186.831 m	GPS/GLONASS	Temporarily not working
17	KRCH	Kurchatov	Latitude: 50° 43' 30.38924» N Longitude: 78° 35' 49.95628» E Altitude: 130.149 m	GPS/GLONASS	Working
18	SMSK	Semey	Latitude: 50° 24' 10,22716» N Longitude: 80° 13' 36,72725» E Altitude: 158.855 m	GPS/GLONASS	Working
19	USTK	Ust-Kamenogorsk	Latitude: 49° 58' 26.17766» N Longitude: 82° 34' 12.52766» E Altitude: 244.847 m	GPS/GLONASS/COMPASS/ GALILEO/QZSS	Working
20	PTRP	Petropavlovsk	Latitude: 54° 51' 28.73754» N Longitude: 69° 10' 00.05604» E Altitude: 123.88 m	GPS/GLONASS	Working

Fig. 1. GNSS Station International Network Map

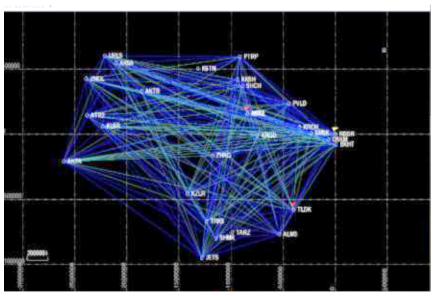

Fig. 2. GNSS station network diagram of Almaty region

However, the discreteness of the data introduces its specificity in the processing and requires consideration of this factor.

Ignoring discreteness can lead to significant errors. In addition, a number of discrete mathematics methods have no analogues in analytical mathematics.

An important way to analyze discrete sequences is by z-transform. The Z-transformation was first introduced by P. Laplace in 1779 and repeatedly "discovered" by V. Gurevich in 1947 with a change of symbolism to $z \sim k$; however, these methods have their drawbacks.

Therefore, the researchers were on the path to finding a transformation devoid of these shortcomings. Currently, in the field of solving spectral analysis problems, the wavelet transform is actively used, which also obeys the Heisenberg uncertainty principle, but has multiscale properties. The wavelet transform has good resolution in the time domain and poor performance at high frequencies in the time range.

Currently, the most efficient method for calculating the heights of a quasigeoid is based on the fast Fourier transform. Such an approach makes it possible to practically solve the problem using rigorous formulas of the Molodenskii theory with an accuracy of not only zero, but also the first and subsequent approximations. Finding the spectral components of a discrete complex signal directly by the formula of the differential Fourier transform requires complex multiplications and complex additions. Since the number of calculations, and, consequently, the calculation time is approximately proportional, the number of arithmetic operations is very large for large data arrays. Therefore, finding the spectrum in real time, even for modern computer technology, is a difficult task. For this reason, computational procedures that reduce the number of multiplications and additions are of considerable interest. At present, computational methods and algorithms are known that can significantly improve the speed of calculations due to the efficient use of the uniformity property of calculated points along meridians and parallels. Under certain conditions, the most productive algorithm is based on the use of the fast Fourier transform. At the same time, the cost of computer time is significantly reduced, and the calculation results are obtained in the nodes of the regular grid, which greatly facilitates their further use.

Results and discussion. To apply the general methodology for using and compiling a software package and creating an algorithm for calculating and studying a quasigeoid model, we have proposed a flowchart that clearly shows the necessary data processing levels for creating a quasigeoid model (Fig. 3). The exact definition of a quasigeoid for solving geodetic problems is carried out with the recalculation of (ellipsoidal) heights directly measured by GPS receivers, then with the input of satellite and ground data with the determination of their standard error. Further, all the information is mathematically calculated taking into account the data of the gravitational anomaly of the earth and through multiscale wavelet expansions and the introduction of corrections, the data is transformed to obtain a preliminary quasigeoid model.

The result of the Fourier transform is the amplitude-frequency spectrum, by which it is possible to determine the presence of a certain frequency in the signal under study.

In the case when there is no question of localizing the temporal position of frequencies, the Fourier method gives good results. But if it is necessary to determine the time interval for the presence of frequency, other methods have to be applied [9].

One of these methods is the generalized Fourier method (local Fourier transform). This method consists of the following steps:

In the function under study, a "window" is created – the time interval, outside which the function $f(x) = 0$.

For this "window", the Fourier transform is calculated.

The "window" is shifted, and the Fourier transform is also calculated for it. By "passing" such a "window" along the entire signal, a certain three-dimensional function is obtained, depending on the position of the "window" and frequency.

This approach allows us to determine the presence of any frequency in the signal interval. This greatly expands the capabilities of the method compared to the classical Fourier transform, but there are certain disadvantages. According to the consequences of the Heisenberg uncertainty principle, in this case it is impossible to state the fact that the frequency ω_0 is present in the signal at time t_0, it can only be determined that the frequency spectrum (ω_1, ω_2) is present in the interval (t_1, t_2) and the frequency resolution (in time) remains constant regardless of areas of frequencies (times) in which the study is performed. Therefore, if, for example, only the high-frequency component is significant in the signal, then you can increase the resolution only by changing the parameters of the method. As a method that does not have such shortcomings, the apparatus of wavelet analysis was proposed.

Wavelet bases can be well localized both in frequency and in time, in contrast to the Fourier transform, have a lot of diverse basic functions whose properties are oriented to solving various problems.

Wavelet transform eliminates methodological errors of the Fourier transform and gives a more accurate result.

It is believed that the connection of the gravimetric model of a quasigeoid with GPS leveling data using second-generation wavelets provides the best conversion of GPS ellipsoidal heights to normal heights. Since GPS leveling data is irregular in the space domain, and the classical wavelet transform is related to the Fourier theory, which cannot be applied to irregular data sets without first building a grid, the classical wavelet transform is not directly applicable. Instead, second-generation wavelets and associated lift patterns that do not require regular data intervals are used to combine gravimetric quasigeoid models and GPS leveling data, and the results can be cross-checked. Cross-evaluation means that GPS alignment points are used to evaluate the simulation results when checking the combined surface, which is repeated for all points in the dataset. Wavelet transform based results are also compared with least squares results. This comparison shows that the second generation wavelet method can be used instead of the least squares method with similar results, but the stationarity assumption for the least squares method is not required in the wavelet method. In particular, there is no need to (arbitrarily) reorient the data deviations before applying the Wavelet transform method, as is the case for the least squares method. In our opinion, the wavelet transform method is better suited for reducing the maximum and minimum differences between the combined geoid and cross-checking data to determine the level of GPS alignment.

In general terms, wavelet analysis is based on two main functions: the scaling function φ_x and wavelet function ψ_x. The classic wavelet system contains an endless set of translated and scaled versions φ_x and ψ_x

$$\varphi_{j,k}(x) = 2^{\frac{j}{2}}\varphi(2^j x - k), \quad j,k \in Z;$$

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^j x - k), \quad j,k \in Z.$$

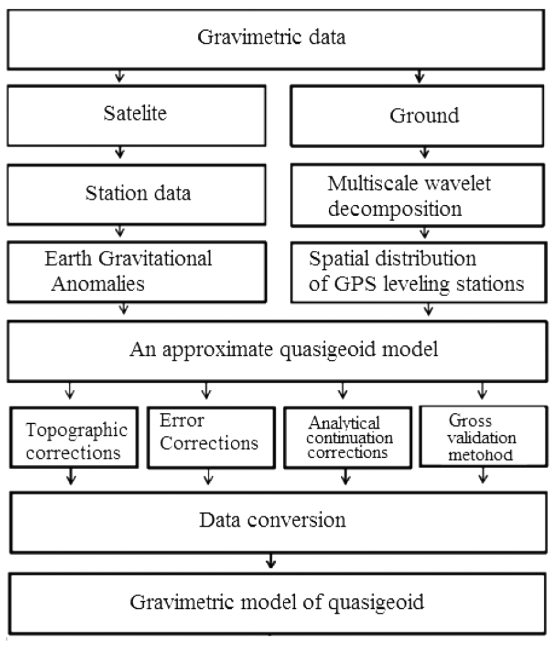

Fig. 3. The block diagram of the algorithm for constructing a quasigeoid model

Considering function $f(x)$ and mother wavelet $\psi(x)$, the function $f(x)$ can be expressed as a linear combination of basis functions $\psi_{j,k}(x)$ as

$$f(x)=\sum_{j,k}a_{j,k}\psi_{j,k}(x),$$

where $a_{j,k}$ is a particular or wavelet coefficient.

This property represents wavelet analysis as a powerful tool for processing signals whose spectral content varies in space (or time). Essentially high frequency at $x0_0$ only affects the coefficient $\psi_{j,k}$, corresponding to location and frequency at a point $x0_0$.

Reviewing analysis $\{V_j\}_{j\geq 0}$ multiple resolution in L_2 sequence of subspaces is defined as

$$V_j\subset V_{j+1} \quad\text{and}\quad \bigcup_{j=0}^{\infty}V_j=L_2,$$

where V is the resolution of the waves; U is wave propagation length.

In addition, there are additional fields W_j such that $V_{j+1} = V_j \oplus W_j$. Therefore, the field V_j with exact resolution can be expanded into coarser field and additional fields

$$V_j=V_0\oplus\bigoplus_{i=0}^{j-1}W_i.$$

This is called multiscale decomposition.

Fields V_j and W_j stretch in scale $\psi_{j,k}$ and wavelet function $\varphi_{j,k}$ respectively. Functions of scaling and wavelets at a coarser level are calculated by scale functions at a finer level using some refinement coefficients h_{jlk} and g_{jlm} such as

$$\varphi_{j,k}=\sum_l h_{jlk}\varphi_{j+1,k} \quad\text{and}\quad \psi_{j,m}=\sum_l g_{glm}\varphi_{j+1,l}, \qquad (1)$$

or

$$\varphi_j=\varphi_{j+1}H_j \quad\text{and}\quad \psi_j=\varphi_{j+1}G_j, \qquad (2)$$

where φ_j and ψ_j are line vectors containing functions $\varphi_{j,k}$ and $\psi_{j,m}$ respectively; H_j and G_j are refinement matrices.

In biorthogonal cases, various basic functions are used to decompose and reconstruct the signal. At the reconstruction stage there are used φ_j and ψ_j, and their duality, $\tilde{\phi}_j$ and $\tilde{\psi}_j$, for decomposition. Any function $f(x)$ can now be expressed by wavelet-based functions as

$$f(x)=\sum_k s_{j0,k}\varphi_{j0,k}(x)+\sum_{j,k}d_{j,k}\psi_{j,k}(x),$$

where

$$s_{j0,k}=f,\tilde{\varphi}_{j0,k};$$
$$d_{j,k}=f,\tilde{\psi}_{j,k}.$$

In this case, the biorthogonal condition in the refined matrix and its duality (i. e. filters) should look like

$$\tilde{H}_j^*H_j=I,\tilde{G}_j^*H_j=0;$$
$$\tilde{H}_j^*G_j=0,\tilde{G}_j^*H_j=I.$$

Considering the two initial pairs of biorthogonal filters as H_j^0,G_j^0 and $\tilde{H}_j^0,\tilde{G}_j^0$, their properties can be improved with a lifting scheme. The lifting diagram indicates that for any operator P_j a new pair of biorthogonal filters can be found as

$$(H_j=H_j^0+G_j^0P_j,\quad G_j=G_j^0);$$
$$(\tilde{H}_j=\tilde{H}_j^0,\quad \tilde{G}_j=\tilde{G}_j^0-\tilde{H}_j^0P_j),$$

which change the role of the original and double filters, as well as for any operator of U_j

$$(H_j=H_j^0,\quad G_j=G_j^0-H_j^0U_j);$$
$$\tilde{H}_j=\tilde{H}_j^0+\tilde{G}_j^0U_j,\quad \tilde{G}_j=\tilde{G}_j^0.$$

Now we can consider the model

$$e_i=m(x_i,y_i)+n_i,$$

where e_i is from (1) localized in two-dimensional space (x_i, y_i) and these are noise waves in e_i. A wavelet transform adapted to unevenly distributed two-dimensional data is used to evaluate the function $m(x_i, y_i)$. Some compactly supported scaling functions $\{\varphi_j, \varphi_j\}$ identified at the highest level (where J is the observation level), a larger scale and wavelet functions $j = J - 1$, $J - 2$ are obtained from equation (1) or (2).

The second generation wavelet transform is based on the so-called uplift pattern and starts with a “lazy wavelet” that reduces the signal to even and odd samples. Odd samples are then used to predict even ones. Detail γ_{j-1} – is the predicted value subtracted from the even sample. The parts are then used to update the odd samples to keep the average value of the signal unchanged.

Let $\lambda_{0,k} = f(x)$, $k \in Z$ source signal. The first approximation based on the application of a lazy wavelet is

$$\lambda_{-1,k}=\lambda_{0,2},\quad k\in Z.$$

And the wavelet coefficients are expressed as

$$\gamma_{-1,k}=\lambda_{0,2k+1}-\frac{1}{2}(\lambda_{-1,k}+\lambda_{-1,k+1}),\quad k\in Z.$$

If the signal is correlated, wavelet coefficients $(\gamma_{i,j})$ are small and values below a certain threshold can be ignored (described below). To maintain an average approximation value (λ), approximated values $(\lambda_{-1,k})$ must be updated using parts or wavelet coefficients. Thus, (11) is modified into the following equation

$$\tilde{\lambda}_{-1,k}=\lambda_{-1,k}+\frac{1}{4}(\gamma_{-1,k-1}+\gamma_{-1,k}),\quad k\in Z.$$

The above calculations are shown schematically in Fig. 5. You can use a higher order scheme to predict odd-indexed values from even ones. For example, $\lambda_{j,2k+1}$ can be predicted based on cubic polynomial interpolation through values $\lambda_{j,2k-1}$, $\lambda_{j,2k}$, $\lambda_{j,2k+2}$ and $\lambda_{j,2k+4}$. Thus, there is some interpolation built into the second generation wavelet method, but this applies only to the processing of wavelet coefficients, and not to the original data.

The second generation reverse wavelet transform is simply applied by changing the update and prediction steps [11].

When threshold values are applied to second generation wavelets, any coefficients that are less than a given threshold value are replaced by zeros.

Finding the optimal threshold value is an important part of smoothing or filtering.

The following data model (distorted by noise, n) and its wavelet transform are considered

$$y=f+n;$$
$$\omega=\tilde{W}y,$$

where $\tilde{W}$ is direct wavelet transform; ω is the vector of wavelet coefficients. Coefficients lower than limit values λ are replaced by zero, and the rest remain unchanged (stable limit value) or are compressed by a factor λ (flexible limit value). Applying the inverse wavelet transform to threshold coefficients yields filtered data

$$y_\lambda=W\omega_\lambda.$$

Wavelets must be localized both in the time and in the frequency domain of the representation. When designing such functions, one inevitably has to deal with the uncertainty principle that relates the effective values of the duration of the functions and the width of their spectrum. The more accurately the localization of the temporary position of the function is carried out, the wider its spectrum becomes, and vice versa.

Using the MatLAB program, some universal code examples for data definition can be presented.

```
Script to calculate anomalies in free air- dg_free_air:
%%%% Turekhanova Venera
```

```
clc; clear; format long g
dg = load('bouguer.xyz'); %% bouguer gravity anomaly (here you can specify from which resource information is taken)
DEM= load('dem.xyz'); %% DEM - resource
Ha = DEM(:,3); %% hights
for i =1:length(dg)
B=0.1119*Ha(i);
dg(i,3)=dg(i,3)+B;
```

Script for computing anomalies of Molodensky – dgA_Molodensky:

```
clc; clear; format long g
DEM = load('dem.xyz'); %% DEM – information from the received resource
Ha = DEM(:,3); %% hights
dgA = load('bouguer.xyz'); %% bouguer gravity anomaly
sinf= sind(DEM(:,2)); %% sin of the latitude Ellipsoid GRS80
a =q; %% semi-major axis [m]
b = q; %% semi-minor axis [m]
e2 = q; %% first eccentricity squared
f_ = q; %% geometrical flattening
ye = q; %% normal gravity at the equator [m/s^2]
yp = q; %% normal gravity at the poles [m/s^2]
m = q;
// q does not equal const, this is a variable number, depending on the data received.
k =(b*yp - a*ye)/(a*ye);
for i = 1: length(dgA)
yq= ye*((1 + k*sinf(i)^2)/sqrt(1 - e2*sinf(i)^2)); yB(i,1) = yq -(2*ye/a)*(1+f_+m+(-3*f_+5/2*m)*sinf(i)^2)*Ha(i)+(3*ye/a^2)*Ha(i)^2;
dgA(:,3)= (dgA(:,3)-yB); %% gravity anomaly of Molodensky [m/s^2]
dgA(:,1)= DEM(:,2); %% geodetic lat [deg]
dgA(:,2)= DEM(:,1); %% geodetic lon [deg]
```

The function of converting coordinates from geodesic to geocentric - Geodetic_2_geocentric:

```
function [f_,l,r] = Geodetic_2_geocentric (f,l,h,a,e2)
ellipsoid =[a,sqrt(e2)];
[x, y, z] = geodetic2ecef(f, l, h, ellipsoid);
for i=1:length (f)
f_(i,1) = atan(z(i)/sqrt(x(i)^2+y(i)^2));
r (i,1) = sqrt(x(i)^2+y(i)^2+z(i)^2);
```

The script for preparing the source matrix for calculations - Sp_coord_and_gQ:

```
coord=load('KZ_grid.xyz'); %% coordinates of the computation points
f=coord(:,1)*pi/180; %% geodetic latitude [rad]
l=coord(:,2)*pi/180; %% geodetic longitude [rad]
h=coord(:,3); %% height
%% Ellipsoid GRS80
a=6378137; %% semi-major axis [m]
b=6356752.3141; %% semi-minor axis [m]
R=6371000; %% mean Earth's radius [m]
ye=9.7803267715; %% normal gravity at the equator [m/s^2]
yp=9.8321863685; %% normal gravity at the poles [m/s^2]
e2 = 0.006694380023; %% first eccentricity squared
% a. convert geodetic coordinates to geocentric
[f_,l,r] = Geodetic_2_geocentric (f,l,h,a,e2);
SP_gQ=f_; %% geocentric latitude [rad]
SP_gQ(:,2)=l; %% geocentric longitude [rad]
SP_gQ(:,3)=r; %%
% b. normal gravity on the surface of the ellipsoid using
k =(b*yp - a*ye)/(a*ye);
for i = 1:length(f)
sinf= sin(f(i));
SP_gQ(i,4)= ye*((1 + k*sinf^2)/sqrt(1 - e2*sinf^2));
dlmwrite('Sp_coord_gQ.xyz',SP_gQ,'delimiter', '\t','precision', '%.8f')
```

Calculation of a preliminary model of quasigeoid heights in the MATLAB program

The script for calculating the short-wave component of the preliminary model of the heights of the quasigeoid - Approx_N1:

```
sn=load('Sn_unb.prn'); %% The sn coefficients for the optimum modification
CP=load('KZ_grid.xyz'); %% The computation points (f,l,h)
GD=load('dg_free_air.xyz'); %% The grid gravity database (f,l,dgA)
SP=load('Sp_coord_gQ.xyz'); %% Spherical coordinates and normal gravity of the computation points (f_,l,r,yq)
Mmax=360; %% Upper limit of the GGM
L=Mmax; %% Upper bound of the harmonics to be modified in Stokes's function
Mmax_exp=2000;
pso=3*pi/180;
a=6378137; %% semi-major axis [m]
b=6356752.3141; %% semi-minor axis [m]
R=6371000; %% mean Earth's radius [m]
[indfp,indlp]= grid_index(CP); %% [rows,colums] in CP grid
[indfi,indli]= grid_index(GD); %% [rows,colums] in GDATA grid
fp_min= min(CP(:,1))*(pi/180); %% min lat of computation points [rad]
lp_min= min(CP(:,2))*(pi/180); %% min lon of computation points [rad]
fmin = min(GD(:,1))*(pi/180); %% min lat [rad]
lmin = min(GD(:,2))*(pi/180); %% min lon [rad]
d =(5/60)*(pi/180); %% Block sizes [rad]
%% Computation
p=1;
lim=cos(pso); %% cos of the radius of the truncation cap
N1=CP;
for s = 1:indfp
fp= fp_min+(s-1)*d;
gQ=(SP(p,4)*1000000); %% normal gravity on the surface of the ellipsoid
for v = 1:indlp
lp= lp_min+(v-1)*d;
r=1;
SUM=0;
for i=1:indfi
for j=1:indli
fd(i,j)=fmin+(i-1)*d;
ld(i,j)=lmin+(j-1)*d;
if abs(fp-fd(i,j))<=(pso)
% a. Compute the cos of the spherical distance psi ,t=cos(psi)
t(i,j)=sin(fp)*sin(fd(i,j))+cos(fp)*cos(fd(i,j))*cos(ld(i,j)-lp);
if t(i,j)>=lim & t(i,j)<1
pso2(r)=acos(t(i,j))*(180/pi);
% b. Compute the area Aijof block yij
A(i,j)=2*d*sin(d/2)*cos(fd(i,j));
% c. Compute the original Stokes function S(psi)
S1(i,j)=sqrt(2/(1-t(i,j)))-6*(sqrt((1-t(i,j))/2))+1-5*t(i,j)-3*t(i,j)*log(sqrt((1-t(i,j))/2)+((1-t(i,j))/2));
% d. Compute the second term of the modified Stokes function S(psi)
% Compute the Legendre polynomials Pn of degree n
Pn(1,1)=1; Pn(2,1)=t(i,j); sum1=0;
for k=3:(L+1)
q=k-1;
Pn(k,1)=(-(q-1)/q)*Pn((k-2),1)+((2*q-1)/q)*t(i,j)*Pn((k-1),1);
S2(q,1)=((2*q+1)/2)*sn((q-1),1)*Pn(k,1);
sum1=sum1+S2(q,1);
sum2(i,j)=sum1;
SUM=SUM+(S1(i,j)-sum2(i,j))*((GD(r,3)))*A(i,j);
r=r+1;
N1(p,3)=((R/(4*pi*gQ))*(SUM)); %% approximate geoid height [m]
p=p+1
dlmwrite('KZGGM_N1.xyz',N1,'delimiter', '\t','precision', '%.8f')
```

The function of counting information from a file given in the format .grd - grid_index:

```
%%%% Turekhanova Venera
% grid_index - computes how many rows and colums consist GRID file
% which is given in xyz format
%%%% last updated 2015.04.19
function [rows,colums]=grid_index(Grid_xy)
[n,m]=size(Grid_xy);
rows = 1;
colums= 1;
for i=2:n
if Grid_xy(i,1)~=Grid_xy((i-1),1)
```

```
rows = rows+1;
for i=2:n
if Grid_xy(i,1)==Grid_xy((i-1),1)
if Grid_xy(i,2)~=Grid_xy((i-1),2)
colums= colums+1;
Script for calculating topographic effect - dN_Topo:
% dN_Topo - computes the combined topographic effect in the
   KTH
approach
CP=load('KZ_grid.xyz'); %% the computation points (f,l,h)
SP=load('Sp_coord_gQ.xyz'); %% the computation points
   (f_,l,r,yq)
Ha=CP(:,3); %% heights [m]
yq=(SP(:,4)); %% normal gravity on the surface of the earth [m/
   s^2]
a=q; %% semi-major axis [m]
b=q; %% semi-minor axis [m]
R=q; %% mean Earth's radius [m]
ye=q; %% normal gravity at the equator [Gal]
yp=q; %% normal gravity at the poles [Gal]
G=6.673*(10^-11); %% Newtonian gravitational constant [m^3
   * kg^-1* s^-2]
po=2.67*(10^3); %% topographic density at sea level kgr/m^3
dN=CP;
for i=1:length(CP)
if Ha(i)<0
Ha(i)=0;
dN(i,3)=-((2*pi*G*po)/yq(i))*((Ha(i))^2);
dlmwrite('dN_topo.xyz',dN,'delimiter',
'\t','precision', '%.8f')
```

Remark. In this code fragment, program scripts codes are found, such as account calculations, or the completion of an operation, iteration, and the start of a count, to save space on the paper, as well as to make reading easier for human perception. The main scripts are given in lines that show the calculation of data related directly to the topic, in connection with this the program is shortened, understandable, but when working directly in the program of this programming language, it will be necessary to fully expand the fragment to the full one with the presence of all the functions mentioned above!

Conclusion. To increase the accuracy of determining the normal height from the local vertical coordinate system using GPS, second-generation wavelets based on the lifting scheme, together with a limit coefficient, on the differences between gravimetric quasigeoid models and discrete GPS-leveling data were introduced and implemented.

Unlike the classical wavelet transform, the second generation wavelet can be applied directly to irregular data sets. The second-generation wavelet coefficients were softly fixed by the verified optimal limit value by the global threshold. It is important to note that this method is applicable to non-stationary data. Thus, the removal of the a priori deviation necessary for the fusion on the base of least squares method and is not required for the second generation wavelet method.

The flowchart created by us will ensure the correct construction of the quasigeoid model when determining the exact coordinates on the ground. And second-generation wavelets are another alternative method that can be used to combine gravimetric models of quasigeoids/geoids and GPS leveling data with specific altitude data.

Gratitude. *This article is written based on the results of research on the topic: "Development of a prototype of radar stations of continuous radiation of the meter wave range" No. BR109009-0221.*

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Застосування теорії вейвлет-перетворення в алгоритмі побудови моделі квазігеоїду

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Мета. Дослідити взаємодію геодезичних і нормальних показників висот за даними квазігеоїду, спільне використання космічних вимірювань і виконаних на поверхні Землі за реалізації геодезичних завдань. У цій роботі поставлене завдання створити алгоритм обчислень для подальших досліджень моделі квазігеоїду й застосування моделі у вирішенні геодезичних задач.

Методика. Надійне визначення аномалії висот вимагає великої точності, тому була застосована теорія вейвлет-перетворення в моделюванні з використанням космічних технологій як альтернатива трудомісткому нівелюванню земної поверхні, що характеризує фактичні коливання від нормалі гравітаційного поля Землі при розрахунку середньоквадратичних відхилень лінії відвісу.

Результати. Складена блок-схема алгоритму розрахунку з використанням програмного комплексу для вирішення граничної проблеми фізичної геодезії, в якій поверхня Землі підлягає сучасним методам космічних вимірів.

Наукова новизна. Використання вейвлет-аналізу для обробки інформації за супутниковими даними в геодезії покращує результати класифікації знімків, а коефіцієнти вейвлет-перетворення можна застосовувати як індикатори при розпізнаванні координат точок із високою точністю.

Практична значимість. Застосування теорії вейвлет-перетворень є потужним математичним інструментом для вирішення задач геодезії, стискання та відновлення даних, збільшення продуктивності обчислень, кодування інформації.

Ключові слова: *фізична геодезія, системи координат, гравітаційне поле, геоїд, квазігеоїд, гравіметрична висота, перетворення координат*

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