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MATHEMATICAL 3D-MODELING IN THE FORMATION PROCESS OF THE OVERHAULED SURFACES IN THE REPAIR INDUSTRY

Purpose. Mathematical representation of the geometry of the overhauled valve seat surfaces in the shape-forming process in the auto repair industry.

Methodology. The research methods were based on: the theory of cutting and metalworking scientific positions, theoretical foundations of the technology of mechanical engineering and overhaul repairs; the basics of technical operation and vehicle structure; provisions of standardization, technical measurements; means of mathematical modelling, statistics and programming.

Findings. The geometric 3D-model of the processing surfaces of valve seats has been presented, which is used in the overhaul process of these parts, which is basic in the general model of the shape-forming system, which allows controlling the shaping parameters with the required precision due to the structure of the model itself and introducing the results of processing particular models: interpolation (to assess the worn surfaces state) and optimization (for determining the volume of cut material and setting cutting conditions).

Originality. For the first time a particular mathematical 3D-model of the general shape-forming system has been developed, consisting of other particular mathematical models (interpolation and optimization), which gives a geometric representation of the processed valve seat surfaces within their overhaul in the auto repair industry.

Practical value. The proposed mathematical 3D-model allows:

- implementing the shape-forming process adequately within the simultaneous boring of three inner conical surfaces by copying with a profile cutting plate, which provides the required parameters of precision and quality of processing during the valve seats overhaul;
- implementing the results of the flaw detection process of worn surfaces of valve seats with a complex-variable topography due to usage of the of roundness- and profile-diagrams and method based on an interpolation geometric 3D-model;
- introducing an optimization model to determine the optimal volumes of cut material and the rational cutting modes select.

The MathCAD software package has been developed, which allows having a graphical performance of the formed surface based on the interpolation model.

Keywords: *processed valve seat surfaces, shape-forming process and function, cutting modes in metalworking, precision and quality parameters of processing, valve seat overhaul*

Introduction. At present, the problem of overhaul of valve seats [1, 2] working surfaces [3, 4] of valve timing gear in compression machines for air, gases and their mixtures remains quite relevant in the repair industry [5, 6]. Besides, this is not an easily solved issue in terms of ensuring the necessary precision and quality in the shape-forming of the processing surfaces [7, 8]. The valve seat performance restoration of the valve timing gear of the internal combustion engine in the repair industry is especially noteworthy [5, 9]. The causes of difficulties are the following:

1) strict parameters of precision and quality of repaired surfaces (which must be similar to the values of new factory products) in the absence of reliable methods for their ensuring;

2) imperfection of existing technologies and equipment used in the flaw detection process and metal-cutting processing (as research shows, there are mainly organoleptic methods for assessing worn surfaces, or analytical ones, which have no practical implementation, as well as various repair equipment for restoring such parts manual) [10, 11];

3) complex and diverse design of modern cylinder heads, which does not allow processing on universal equipment and requires the usage of special technological equipment (for orientation of cylinder heads in space and cutting conical surfaces) [12, 13];

4) economic inexpediency [14, 15] in the use of metal-cutting equipment for main production when trying to operate it as repair equipment [16].

The last three issues mentioned above were researched and solved in detail and addressed in the relevant scientific publications [1, 2], so they are not considered in this article. The topic of the first question after an in-depth study on relevant sources of information [8, 9] pointed to the need for a mathematical approach by developing a mathematical apparatus to ensure the required quality [3, 4] and accuracy of the shape-forming process during valve seats overhaul in auto repair industry [7, 9].

Purpose of this work is the geometrization process of worn surfaces through the development of a special mathematical model that is part of the general mathematical apparatus, and appropriates to restore the valve seats at the specified parameters of precision and quality.

Literature review and methods. In [8, 9], a dimensional analysis of the bush-valve-seat conjugation in the valve timing gear was performed (Fig. 1).

The main parameters of precision and quality were revealed, which are responsible for the sealing degree of the compression chamber during landing of the valve plate on the seat, such as concentricity of surfaces D_k and D_c, their roughness and run-out of the valve plate at the time of landing on the seat, which depends on the coaxiality of these surfaces (parameter A).

For this engine, the tolerances on the accuracy parameters of the bush-valve-seat conjugation surfaces are the following: for the valve seat, the diameter is 30.5 + 0.15 mm, the roundness is 0.01 mm, the taper angle corresponds to 45° ± 5′, the roughness R_a is 1. 25 microns, the width of a working chamfer is 2–0.2 mm; for the guide bush the diameter is 8.03 + 0.02 mm, the roundness is 0.005 mm, the roughness R_a is 2.5 μm; for the valve plate the diameter is 31.5–0.015 mm, the roundness is 0.01 mm, the taper angle is 45° ± 5′, the roughness R_a is 1.25 μm, the width of the working chamfer is 5 + + 0.2 mm.

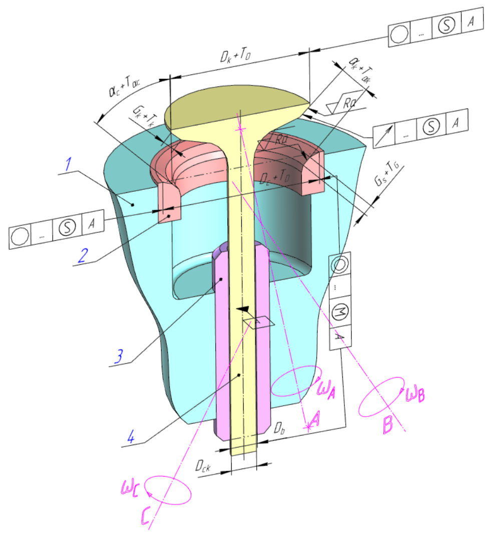

Fig. 1. Parameters of conjugation of the assembly unit surfaces:

1 – the body of valve timing gear; 2 – the valve seat; 3 – the guide bush; 4 – the valve stem

Deviations from the coaxiality of the contact surfaces in the connection of bush-valve-saddle are as follows: for the seat and guide bush, the radial and angular alignment is 0.05 mm and 0.028°, respectively; for the guide bush and the valve stem, the radial alignment is 0.05 mm; for the seat and the valve plate, the radial and angular alignments are 0.05 and 0.035 mm, respectively.

The research was carried out on the example of the upper-valve valve timing gear of the car internal combustion engine.

Analysis of the dimensional chain showed that to achieve the accuracy of parameter A (shown in Fig. 1), it is necessary to ensure alignment of the valve plate and its rod (parameter *B*), valve rod and cylindrical sleeve hole D_b (parameter *C*), seat chamfer D_c and hole D_b parameter *E*). Here are D_b – errors of links of technological dimensional chains of turns [9].

In [7, 8] the method of processing is defined and substantiated – fine boring by copying three conical surfaces oriented by a profile plate, as well as schemes of basing the cutting tool and devices on the air bag.

Among the parameters used for further analysis of errors and control of processing accuracy, there are the following:

1) R_2 – the radius of the cone of the seat working surface (working chamfer), which is affected by errors in processing and basing of the device;

2) α_2 – the angle of conicity of the seat working surface;

3) parameter *A* is the radial concentricity of the working chamfer relative to the axis of the hole of the bush 3.

Moreover, *A* is affected by the following parameters: *C* – radial error of the cutting edge of the tool relative to the bush *E* – radial error of the cutting edge of the tool and the pilot; *B* – radial error of the pilot's position relative to the bush; *N* – the radial error of the location of the pilot and the working surface;

4) angular parameter σ – concentricity of the working chamfer relative to the axis of the guide bush hole 3.

The parameter σ is affected by the angular positional errors: γ – of the cutting edge relative to the axis of the bush; ε – of the instrument pilot; δ – of the pilot relative to the bush; λ – of the working surface and the pilot location;

5) G_S – width of the seat working chamfer;

6) error of the working chamfer shape of the machined seat $F_{B\delta}$, which is a function of the spindle speed and parameters *B* and δ;

7) R_a – roughness of the treated surface of the working chamfer of the saddle.

The formation procedure of a shape-forming geometric scheme, which describes the complete trajectory of the working edges points of the cutting tool relative to the workpiece, uses a mathematical apparatus for converting coordinates through matrices of generalized displacements.

In this case, the equation of the relation between the coordinates of the cutting tool individual points and the workpiece (obtained by implementing the interpolation model by roundgram and profilogram) in general will have the right part, which is a function of formation

$$\bar{r}_o = A_{baz} \cdot \bar{r}_t, \tag{1}$$

where $\bar{r}_o, \bar{r}_t$ are radius-vectors of points (machined surface and cutting tool); A_{baz} is a transformation matrix related to the conditions of basing technological equipment (as a product of a few matrices of generalized displacements, Table 1).

The shape-forming function contains the radius-vector $\bar{r}_t$ of the working edge points of the cutting tool. We use a single-blade tool in which the cutting part has a broken profile. Given that each point of the cutting edge is in the same system with the known coordinates, the radius-vector of such a tool can be represented as a vector function (1) of one independent variable *u*

$$\bar{r}_t = \bar{r}_t(u), \tag{2}$$

Table 1

Matrices of generalized displacements

Type of movement	Matrices modeling movement on the *Y* axis
Progressive along axis	$A_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Rotation around axis	$A_5 = \begin{pmatrix} \cos\psi & 0 & \sin\psi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\psi & 0 & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Type of movement	**Matrices modeling movement on the *Z* axis**
Progressive along axis	$A_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Rotation around axis	$A_6 = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Progressive along axis	$A_1 = \begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Rotation around axis	$A_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi & 0 \\ 0 & \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Note: *Generalized coordinates φ, ψ, θ – angles of rotation relative to the axes (Fig. 2); *x*, *y*, *z* – linear displacements along the axes; the index in the notation of the matrix **A** is the number of the generalized coordinate (symbol of the freedom degree). All coordinate systems are right-handed, i.e. rotation from the *X*-axis to the *Y*-axis when viewed from the end of the *Z*-axis must be counterclockwise where $\overline{r_t} = (x_t(u), y_t(u), z_t(u), 1)^\tau$; u_1, u_2 are limit values of u ($u_1 \le u \le u_2$).

In the coordinate system of the cutting tool, equation (2) describes the spatial curve.

In the future, the radius-vector of the tool will be represented as

$$\overline{r_t} = A_t(u) \cdot \overline{e}_4, \qquad (3)$$

where $\overline{e}_4 = \left(0,0,0,1\right)^\tau$ is the radius-vector of the origin; $A_t(u)$ is a matrix of transformations of the tool starting point, which can always be represented as a product of a few matrices of generalized displacements (Table 1).

Equation (3), which concentrates data on the freedom degree of the cutting tool and the conditions of basing technological equipment, can be constructed as a complete mathematical model of shaping. However, in equation (3) it should be used to estimate the errors and quality of processing, and in the development of the constituent elements of the model, you can use (1, 3), followed by combining into form (4).

Then the model of the forming system, which connects the coordinates of the cutting tool points (2) with the coordinates of the workpiece (1), in matrix form has the form

$$\overline{r_o} = A_{baz} \cdot A_t \cdot \overline{e}_4. \qquad (4)$$

Due to the fact that the cutting tool and the workpiece are the end links of the forming chain, in this research we will use a consistent approach to forming a model of shaping, starting with the links of technological equipment that provide the main movement of the cutting tool relative to the part and set the main direction.

To do this, let us consider the general scheme of processing the valve seat by copying with a description of the basic elements of technological equipment and parameters that affect the quality of processing.

The quality of forming surfaces is determined by angular parameters α_1, α_2, α_3, diameters of cones D_1, D_2, D_3, their roundness, concentric with the bush hole and depends on the total processing error [17]

$$\omega_o = \omega_a + \omega_b + \omega_c,$$

where ω_a, ω_b, ω_c are errors of the machining method (fine boring), the base of the device (pilot 3 along the guide bush 1) and the errors of the device (tool holder) assembled with the cutting tool, respectively.

Errors ω_b, ω_c are formed by the links of the linear technological chain B, C, E and N, as well as elements of the chain of angular rotations γ, τ, σ and β.

Then, using equation (1), we can write the function of formation in the matrices of generalized displacements for the i^{th} processing step

$$\overline{r_o} = A_6\left(\theta\right) \cdot A_3\left(-Z_i\right) \cdot A_1\left(X_{Ai}\right) \cdot A_2\left(Y_{Ai}\right) \times$$
$$\times A_4\left(\alpha_{Xi}\right) \cdot A_5\left(\alpha_{Yi}\right) \cdot A_6\left(\alpha_{Zi}\right) \cdot \overline{r_t},$$

where $A_6(\theta)$ is the matrix of rotation of the coordinate system relative to the OZ axis by the angle $\theta (0 \le \theta \le 2\pi)$ (main motion); $A_3(-Z_i)$ is the matrix of the current point offset of the cutter along the axis OZ, taking into account the established modes of processing of the part in the i^{th} processing step (feed); $A_1(X_{Ai})$, $A_2(Y_{Ai})$ are matrices of radial shifts of current point coordinates of the cutter along the axes OX and OY, taking into account the position and base of the cutting tool; $A_4(\alpha_{Xi})$, $A_5(\alpha_{Yi})$, $A_6(\alpha_{Zi})$ are matrices of angular displacements of current point coordinates of the cutter relative to the axes OX, OY, OZ during the orientation of the cutting tool.

To describe the 3D-model of the forming system in matrix form, which connects the forming points coordinates of the tool with the processed seat coordinates, it is necessary to consider the scheme of the cutting tool (Fig. 2).

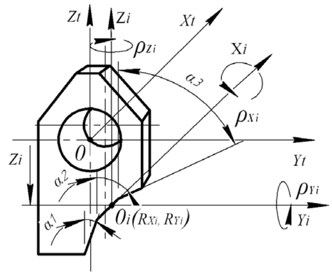

Fig. 2. Cutting tool scheme

The parameters that must be included in the matrix of generalized movements of the cutting tool in the i^{th} step of processing are: ρ_{Xi}, ρ_{Yi}, ρ_{Zi} – the cutting edge angles of rotation relative to the axes OX, OY, OZ; R_{Xi} – the radius of the cutting edge current point of the cutter relative to the OX; R_{Yi} – the radius of the cutting edge current point of the cutter relative to the axis OY.

Then the radius vector $\overline{r_t}$ of the forming points of the cutting tool is

$$\overline{r_t} = A_6(\psi) \cdot A_1(R_{Xi}) \cdot A_2(R_{Yi}) \cdot A_4(\rho_{Xi}) \cdot A_5(\rho_{Yi}) \cdot A_6(\rho_{Zi}) \cdot \overline{e}_4,$$

where $A_6(\psi)$ is the matrix of the cutter edge coordinates rotation relative to the axis OZ by the angle ψ ($0 \le \psi \le 2\pi$); $A_1(R_{Xi})$, $A_2(R_{Yi})$ are coordinate transfer matrices along the axes OX, OY to the current radius of the cutter; $A_4(\rho_{Xi})$, $A_5(\rho_{Yi})$, $A_6(\rho_{Zi})$ are matrices of cutter coordinates angular displacements of current point relative to the axes OX, OY, OZ.

Insofar as the generalized coordinates of the polar angles of the main motion and the cutter coincide (hidden relation), we can put $\varphi = \theta + \psi$. Angle φ is a parameter that depends on the output parameters (cutting machine) and set modes of the processing part (speed n, feed S_o) and in the process of shaping is directly related to the current value of the parameter Z_i.

Given the hidden connection, the general 3D-model of the shaping scheme is the following

$$\overline{r_o} = A_6(\psi) \cdot A_3(-Z_i) \cdot A_1(X_{Ai}) \cdot A_2(Y_{Ai}) \cdot A_4(\alpha_{Xi}) \cdot A_5(\alpha_{Yi}) \times$$
$$\times A_6(\alpha_{Zi}) \cdot A_1(R_{Xi}) \cdot A_2(R_{Yi}) \cdot A_4(\rho_{Xi}) \cdot A_5(\rho_{Yi}) \cdot A_6(\rho_{Zi}) \cdot \overline{e}_4. \qquad (5)$$

The right part of equation (5) contains twelve variables, but the 3D-model of a particular workpiece surface must contain two independent variables [18, 19]. To obtain the expression of all variables through two independent (x and y) connections of different types are superimposed – envelope connections and functional ones.

Given that the inner conical surface of the seat working chamfer (in the first approximation) has a direct functional relation $z = f(x, y)$ and can be obtained by one-parameter bending during processing, the mixed product of three vectors of partial derivatives of radius vector $\overline{r_0}$ must be equal to zero

$$\overline{r}_{0x} \cdot \overline{r}_{0y} \cdot \overline{r}_{0\varphi} = 0,$$

or in a converted form

$$\frac{\partial \overline{r_0}}{\partial \varphi} \cdot \left(\frac{\partial \overline{r_0}}{\partial x} + z'_x \frac{\partial \overline{r_0}}{\partial z} \right) \cdot \left(\frac{\partial \overline{r_0}}{\partial y} + z'_y \frac{\partial \overline{r_0}}{\partial z} \right) = 0.$$

For other parameters, the formation system of connections and conditions is presented in the following form

$$\varphi = \text{const};\ R_1 = \text{const};\ \alpha_2 = \text{const};\ \gamma = \text{const};\ \lambda = \text{const};$$

$$\varepsilon = \text{const};\ \sigma = \text{const};$$

$$Z_i = Z_i(n, S_0);\quad X_{Ci} = X_{Ci}(x);\quad X_{Ei} = X_{Ei}(x);\quad X_{Bi} = X_{Bi}(x);$$

$$Y_{Ci} = Y_{Ci}(y);\quad Y_{Ei} = Y_{Ei}(y);\quad Y_{Bi} = Y_{Bi}(y);$$

$$X_{Ai} = X_{Ci} + X_{Ei} + X_{Bi}; \quad Y_{Ai} = Y_{Ci} + Y_{Ei} + Y_{Bi}; \tag{6}$$

$$\alpha_{Xi} = \alpha_{Xi}(\varepsilon, \sigma); \quad \alpha_{Yi} = \alpha_{Yi}(\varepsilon, \sigma); \quad \alpha_{Zi} = \alpha_{Zi}(\varepsilon, \sigma);$$

$$R_{Xi} = R_1 + Z_i \cdot \mathrm{ctg}(\alpha_2 \pm \alpha_{Xi}); \quad R_{Yi} = R_1 + Z_i \cdot \mathrm{ctg}(\alpha_2 \pm \alpha_{Yi});$$

$$\rho_{Xi} = \rho_{Xi}(\gamma, \lambda); \quad \rho_{Yi} = \rho_{Yi}(\gamma, \lambda); \quad \rho_{Zi} = \rho_{Zi}(\gamma, \lambda).$$

To construct a model of shape errors of the machined surface using matrix methods [18], we should consider a complete variation of the shaping function (5), taking into account the relations between the arguments of the matrices included in it (6). Then, taking into account the geometric meaning of the matrices of particular rotation and particular transfer, the initial error of the system can be expressed in the general vector form

$$\Delta \overline{r}_0 = \sum_{i=0}^{12} \sum_{j=1}^{6} (A_{i,j} \cdot D_{i,j}) \cdot \delta \overline{q}_{i,j}, \tag{7}$$

where $A_{i,j}$ is the matrix of generalized displacements (3) for the i^{th} node of the shaping scheme by the jth generalized coordinate; $D_{i,j}$ is the error matrices $\delta \overline{q}_{i,j}$ for the i^{th} node in the j^{th} generalized coordinate (Table 2).

If in (7) we enter the notation for the multiplier in parentheses, which is the transfer coefficient at the error variation

$$\overline{d}_{i,j} = A_{i,j} \cdot D_{i,j} \cdot \overline{e}_4,$$

then the initial error of the forming system can be represented as

$$\Delta \overline{r}_0 = \sum_{i=0}^{12} \sum_{j=1}^{6} \overline{d}_{i,j} \cdot \delta q_{i,j}. \tag{8}$$

In metrological estimates of precision, the calculation by formula (8) can be simplified by using the representation of accuracy in scalar form, for example, the projection of the vector $\Delta \overline{r}_0$ in the direction normal to the surface

$$\Delta r_n = (\Delta \overline{r}_0 \cdot \overline{n}) = \sum_{i=0}^{12} \sum_{j=1}^{6} \delta q_{i,j} \cdot (\overline{d}_{i,j} \cdot \overline{n}). \tag{9}$$

The presented model (8, 9) by computer is used to further analyze the initial error of the forming system and assess the quality of processing worn valve seat; however, it is multi-parameter.

Thus, we will use the general model of formation (5) and to simplify it we will use one of the specific properties of generalized matrices, when the matrix of the sum of arguments is equal to the product of matrices of the same type from summands. Therefore, the product of matrices of the same name can be represented as a generalized matrix of the sum of arguments.

Due to the fact that all input parameters of the model (5) are divided into four groups according to the nature of the impact on output errors, we choose the most general variant of this allocation for this model, assuming that input parameters always affect output errors, even when they are constant for the entire working volume.

Then, using the rule of adding the same type of parameters (cutter current point radial and angular shifts of the coordinates), the model is simplified, and after grouping the same type of parameters and transforming equation (5), we obtain the function of forming in a modified form, which connects the cutting tool current point coordinate with the coordinate of the machined seat.

$$\overline{r}_o = A_6(\theta) \cdot A_3(-Z_{nS}) \cdot A_1(x) \cdot A_2(y) \cdot A_4(\beta_X) \cdot A_5(\beta_Y) \cdot \overline{e}_4, \tag{10}$$

where $\overline{r}_o, \overline{e}_4$ are four-dimensional radius-vectors $\overline{r}_0 = (x_0, y_0, z_0, 1)^\tau$; $\overline{e}_4 = (0,0,0,1)^\tau$.

Performing the transformation in (10) and taking into account the values of the matrices $\mathbf{A}_j$ ($j = 1, \ldots, 6$), the system of relation between the parameters (Table 3) and their general description, we obtain for the working chamfer vector equation of the machined surface (expected)

$$\overline{r}_o = \begin{pmatrix} x \cdot \cos\theta - y \cdot \sin\theta \\ x \cdot \sin\theta + y \cdot \cos\theta \\ z - Z_{nS} \end{pmatrix}, \tag{11}$$

where $z - Z_{nS}$ is the coordinate of cutting tool current point along the Z axis (depending on the feed and the spindle rotation frequency).

Given the set processing modes ($\theta = \varphi + \beta_Z$, $R_X - X_A \le x \le R_X + X_A$, $R_Y - Y_A \le y \le R_Y + Y_A$, $h_1 \le z \le h_2$), equation (11) allows constructing trajectory of the points of the cutter forming edge (as a 3D-model) and the profiles of the machined surface [18] using MathCAD.

The obtained results can be used during the production and repair of other vehicle units and aggregates [18, 19], as well as in joint production areas [20].

Table 2

Matrix of input errors

$D_{i,1}$	$D_{i,2}$	$D_{i,3}$
$\begin{pmatrix} 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{pmatrix}$	$\begin{pmatrix} 0&0&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \end{pmatrix}$	$\begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix}$
$D_{i,4}$	$D_{i,5}$	$D_{i,6}$
$\begin{pmatrix} 0&0&0&0 \\ 0&0&-1&0 \\ 0&1&0&0 \\ 0&0&0&0 \end{pmatrix}$	$\begin{pmatrix} 0&0&1&0 \\ 0&0&0&0 \\ 0&-1&0&0 \\ 0&0&0&0 \end{pmatrix}$	$\begin{pmatrix} 0&-1&0&0 \\ 1&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{pmatrix}$

Table 3

Elements of the formative system

Displacement matrices and their function	
$A_6(\varphi)$	The rotation matrix of the coordinate system relative to the *OZ* axis at an angle φ (main motion)
$A_3(-Z)$	The offset matrix of the cutter current point along the axis *OZ*, taking into account the set-up modes of the part processing (feed)
$A_1(X_A)$, $A_2(Y_A)$	The matrices of radial shifts of the cutter current point coordinates along the axes *OX* and *OY* due to the location and base of the cutting tool
$A_1(R_X)$, $A_2(R_Y)$	The matrix of radial shifts of the cutter current point coordinates along the axes *OX* and *OY* in the process of machining
$A_1(X) = A_1(X_A) \cdot A_1(R_X) = A_1(X_A + R_X)$	
$A_2(Y) = A_2(Y_A) \cdot A_2(R_Y) = A_2(Y_A + R_Y)$	
$A_4(\alpha_X)$, $A_5(\alpha_Y)$, $A_6(\alpha_Z)$	The matrices of radial shifts of the cutter current point coordinates of the axes *OX*, *OY*, *OZ* due to the location of the pilot relative to the cutter holder and the bush
$A_4(\rho_X)$, $A_5(\rho_Y)$, $A_6(\rho_Z)$	The matrices of radial shifts of the cutter current point coordinates of the axes *OX*, *OY*, *OZ* due to the location of the cutting edge relative to the axis of the bush and the pilot
$A_4(\beta_X) = A_4(\alpha_X) \cdot A_4(\rho_X) = A_4(\alpha_X + \rho_X)$	
$A_5(\beta_Y) = A_5(\alpha_Y) \cdot A_5(\rho_Y) = A_5(\alpha_Y + \rho_Y)$	
$A_6(\beta_Z) = A_6(\alpha_Z) \cdot A_6(\rho_Z) = A_6(\alpha_Z + \rho_Z)$	

Conclusions.

1. The special mathematical 3D-model of the general forming system was developed, which is intended for geometrization of machined surfaces of the valve seats during their repair.

2. On the example of the real upper valve timing gear of the car's internal combustion engine, the proposed mathematical model is numerically solved in MathCAD and trajectories are constructed for points of the cutter forming edge, which performs surface processing of valve seats during their overhaul.

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Математичне 3D-моделювання процесу формоутворення відновлюваних поверхонь у ремонтному виробництві

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Мета. Математичне подання геометрії відновлюваних поверхонь сідла клапана у процесі їх формоутворення в авторемонтному виробництві.

Методика. Дослідження базувалися на: наукових положеннях теорії різання й металообробки, теоретичних засадах технології машинобудування та відновлювальних ремонтів; засадах технічної експлуатації та конструкції автомобілів; положеннях стандартизації, технічних вимірювань; засобах математичного моделювання, статистики та програмування.

Результати. Представлена геометрична 3D-модель оброблюваних поверхонь сідел клапанів, що використовується у процесі відновлення таких деталей і є базовою в загальній моделі системи формоутворення, яка дозволяє з необхідною точністю контролювати параметри формоутворення за рахунок структури самої моделі та впровадження результатів обробки окремих моделей інтерполяції (для оцінки стану зношених поверхонь) та оптимізації (для визначення обсягів зрізаного матеріалу й налаштування режимів різання).

Наукова новизна. Уперше розроблена окрема математична 3D-модель загальної системи формоутворення, що складається з інших окремих математичних моделей (інтерполяційної та оптимізаційної), яка надає геометричне уявлення оброблюваних поверхонь сідел клапанів під час їх відновлення в авторемонтному виробництві.

Практична значимість. Запропонована математична 3D-модель дозволяє:

- адекватно реалізувати процес формоутворення одночасного розточування трьох внутрішніх конічних поверхонь методом копіювання профільною ріжучою пластиною, що забезпечує необхідні параметри точності та якості обробки при відновлюваному ремонті сідел клапанів;

- реалізувати результати процесу дефектації зношених поверхонь сідел клапанів зі складно-змінною топографією за рахунок використання методики круглограм і профілограм на базі інтерполяційної геометричної 3D-моделі;

- впровадити модель оптимізації для визначення оптимальних обсягів зрізаного матеріалу й вибору раціональних режимів різання.

Розроблено пакет програмного забезпечення в середовищі MathCAD, що дозволяє мати графічне уявлення сформованої поверхні на базі інтерполяційної моделі.

Ключові слова: *оброблювані поверхні сідла клапана, процес і функція формоутворення, режими різання при металообробці, параметри точності і якості обробки, відновлювальний ремонт сідел клапанів*

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