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HIGH-FREQUENCY PERIODIC PROCESSES IN TWO-WINDING POWER TRANSFORMERS

Purpose. Mathematical modeling of high-frequency periodic processes in winding power transformers to improve the technology of their design and operation.

Methodology. The methods of the formation of mathematical models for the research of high-frequency periodic processes in transformers and methods of solving systems of partial differential equations are applied.

Findings. The mathematical model for the research of high-frequency periodic processes in two-winding transformers, with adequate considering of electromagnetic connections of windings and structural parameters of transformers, is created.

Originality. To form a mathematical model for the research of high-frequency periodic processes, a substitute scheme of a two-windings transformer, taking into account the parameters of the electric and magnetic circuits of windings and electromagnetic connections between them, is proposed.

Practical value. The mathematical model, which allows analyzing the voltage distribution in the transformer windings for high-frequency periodic processes in windings, and adjusting their insulating abilities, is created.

Keywords: *high-frequency periodic process, mathematical modeling, transformer, boundary value problem, ordinary differential equations*

Introduction. High-frequency periodic processes in the windings of power transformers are crucial for the choice of insulation of transformers during their design and depend on the proper coordination of insulation.

The need to take into account the electromagnetic connections between the transformer windings, taking into account the main magnetic flux, ensures the proper adequacy of the results of mathematical modeling. The development of the mathematical model for the research of high-frequency periodic processes in the windings of power transformers, taking into account these factors, is relevant.

Literature review. The creation of efficient methods for the analysis of high-frequency periodic processes in transformers is relevant, as studies were performed to analyze high-frequency wave processes in one transformer winding during pulse overvoltage without taking into account electrical and magnetic connections between windings [1–3]. A mathematical model and method of nodal voltages for the analysis of voltage distribution in the transformer winding in the frequency domain are proposed in [4, 5].

To calculate the parameters of the substitution scheme of the transformer based on its geometric dimensions, the software complex EMTR is used in [6], and the frequency spectrum in the winding during the action of an overvoltage pulse is analyzed.

Nowadays, mathematical models for the research on wave processes in the windings of transformers during pulse overvoltage are created by taking into account the electrical and magnetic connections between the windings, as well as the magnetic connection between the turns of the windings [7, 8]. The mathematical models obtained in these works for the study of wave processes in the transformer windings are suitable for the analysis of high-frequency periodic processes in the windings of power transformers [9, 10].

The basic principles of formation of mathematical models of the elements of the power system, taking into account all the parameters of the substitution circuit, which allows researching its internal processes, are given in [11, 12].

Main material and mathematical model of wave processes in two-winding transformer. A mathematical model for the research on high-frequency periodic processes in transformers with two-windings, taking into account the electromagnetic connections between the windings and turns of the windings, is created basing on the subschema given in Fig. 1 [7, 8].

The equation of change in currents flowing through the windings is written basing on Kirchhoff's current law (*I^st^* law).

$$\frac{-\partial i_1(x,t)}{\partial x} = g_{10}u_1(x,t) + (C_{10}+C_{120})\frac{\partial u_1(x,t)}{\partial t} - C_{120}\frac{\partial u_2(x,t)}{\partial t} - C_{M10}\frac{\partial^3 u_1(x,t)}{(\partial x^2 \partial t)}; \quad (1)$$

$$\frac{-\partial i_2(x,t)}{\partial x} = g_{20}u_2(x,t) + (C_{20}+C_{120})\frac{\partial u_2(x,t)}{\partial t} - C_{120}\frac{\partial u_1(x,t)}{\partial t} - C_{M20}\frac{\partial^3 u_2(x,t)}{(\partial x^2 \partial t)}. \quad (2)$$

The equation of voltage fall per unit length of windings is written basing on Kirchhoff's voltage law (*2^st^* law).

$$-\frac{\partial u_1(x,t)}{\partial x} = r_{10}i_1(x,t) + L_{10}\frac{\partial i_1(x,t)}{\partial t} + M_0\frac{\partial i_2(x,t)}{\partial t}; \quad (3)$$

$$-\frac{\partial u_2(x,t)}{\partial x} = r_{20} i_2(x,t) + L_{20}\frac{\partial i_2(x,t)}{\partial t} + M_0\frac{\partial i_1(x,t)}{\partial t}, \tag{4}$$

where $L_{01} = L_{\mu 0} + L_{\sigma 01}$; $M_0 = \frac{L_{\mu 0}}{k} + M_{\sigma 0}$; $L_{02} = \frac{L_{\mu 0}}{k^2} + L_{\sigma 02}$; $L_{\sigma 10}$, $L_{\sigma 20}$, $M_{\sigma 0}$ are self and mutual scattering inductances of the primary and secondary windings and between them; $L_{\mu 0}$ is inductance of the magnetic system of the transformer; k is transformer transformation ratio.

The system of differential equations in partial derivatives (1–4) in the given form, directly unsuitable for the analysis of periodic processes, is focused on the calculation of fleeting transient processes.

In this regard, it is necessary to transform this system of equations into a form that will provide the research on periodic sinusoidal wave processes (steady-state) of high-frequency transformers.The problem for a practically important case – sinusoidal periodic processes – is solved by applying the symbolic Steinmetz transformation (the method of complex amplitudes). To analyze the steady-state to equations (1–4), there is a symbolic transformation by introducing complex amplitudes of currents and voltages applied, namely [7].

$$\begin{aligned} &i_1(x,t) = \dot{I}_1(x)e^{j\omega t};\quad i_2(x,t) = \dot{I}_2(x)e^{j\omega t};\\ &u_1(x,t) = \dot{U}_1(x)e^{j\omega t};\quad u_2(x,t) = \dot{U}_2(x)e^{j\omega t}, \end{aligned} \tag{5}$$

where $\dot{I}_1(x), \dot{I}_2(x), \dot{U}_1(x), \dot{U}_2(x)$ are the complex amplitudes of currents and voltages of the transformer windings; ω is the angular frequency of the periodic process.

After simplification, equations (3–4) have the following form

$$-\frac{\partial u_1(x,t)}{\partial x} = r_{10} i_1(x,t) + L_{10}\frac{\partial i_1(x,t)}{\partial t} + M_0\frac{\partial i_2(x,t)}{\partial t}; \tag{6}$$

$$-\frac{\partial u_2(x,t)}{\partial x} = r_{20} i_2(x,t) + M_0\frac{\partial i_1(x,t)}{\partial t} + L_{20}\frac{\partial i_2(x,t)}{\partial t}. \tag{7}$$

Expressions (5) are substituted in equations (1–2) and (6–7), the result is a system of ordinary differential equations in complex form

$$\begin{aligned} -\frac{d\dot{I}_1(x)}{dx} &= (g_{10} + j\omega(C_{120} + C_{10}))\dot{U}_1(x) -\\ &- j\omega C_{M10}\frac{d^2\dot{U}_1(x)}{dx^2} - j\omega C_{120}\dot{U}_2(x); \end{aligned} \tag{8}$$

$$\begin{aligned} -\frac{d\dot{I}_2(x)}{dx} &= (g_{20} + j\omega(C_{120} + C_{20}))\dot{U}_2(x) -\\ &- j\omega C_{M20}\frac{d^2\dot{U}_2(x)}{dx^2} - j\omega C_{120}\dot{U}_1(x); \end{aligned} \tag{9}$$

$$-\frac{d\dot{U}_1(x)}{dx} = (r_{10} + j\omega L_{10})\dot{I}_1(x) + j\omega M_0\dot{I}_2(x); \tag{10}$$

$$-\frac{d\dot{U}_2(x)}{dx} = j\omega M_0\dot{I}_1(x) + (r_{20} + j\omega L_{20})\dot{I}_2(x). \tag{11}$$

Let us write equations (8–9) as follows

$$\begin{aligned} -j\omega C_{M10}\frac{d^2\dot{U}_1(x)}{dx^2} - \frac{d\dot{I}_1(x)}{dx} &= \omega C_{120}\dot{U}_2(x) -\\ &-(g_{10} + j\omega(C_{120} + C_{10})\dot{U}_1(x); \end{aligned} \tag{12}$$

$$\begin{aligned} -j\omega C_{M20}\frac{d^2\dot{U}_2(x)}{dx^2} - \frac{d\dot{I}_2(x)}{dx} &= j\omega C_{120}\dot{U}_1(x) -\\ &-(g_{20} + j\omega(C_{120} + C_{20}))\dot{U}_2(x). \end{aligned} \tag{13}$$

The obtained differential equations (10–13) are equations with complex coefficients, the search for a solution of which is a difficult task. From a physical point of view, if the next conditions $g_{10} = g_{20} = 0$ and $r_{10} = r_{20} = 0$ are accepted, the complex coefficients from these equations can be removed. Such an assumption of the adequacy of the result of the calculation of periodic processes in the windings of power transformers is not a significant influence. Given this assumption, equations (12, 13) are written in a matrix-vector form

$$\begin{Vmatrix} j\omega L_{10} & j\omega M_0 \\ j\omega M_0 & j\omega L_{20} \end{Vmatrix} \times \begin{vmatrix} \frac{d\dot{I}_1(x)}{dx} \\ \frac{d\dot{I}_2(x)}{dx} \end{vmatrix} = \begin{vmatrix} \frac{d^2\dot{U}_1(x)}{dx^2} \\ \frac{d^2\dot{U}_2(x)}{dx^2} \end{vmatrix}. \tag{14}$$

For the obtained system of equations (14) the determination of the matrix is found, namely $\Delta = \omega^2(L_{10}L_{20} - M_0^2)$.

Derivatives of currents from equation (14) are found and have the following form

$$\frac{d\dot{I}_1(x)}{dx} = \frac{jL_{20}}{\Delta}\frac{d^2\dot{U}_1(x)}{dx^2} - \frac{jM_0}{\Delta}\frac{d^2\dot{U}_2(x)}{dx^2}; \tag{15}$$

$$\frac{d\dot{I}_2(x)}{dx} = \frac{jL_{10}}{\Delta}\frac{d^2\dot{U}_2(x)}{dx^2} - \frac{jM_0}{\Delta}\frac{d^2\dot{U}_1(x)}{dx^2}. \tag{16}$$

The system of equations (8–11) reduction to one variable by substituting (15–16) into (12–13). Entered designation $a = L_{10}L_{20} - M_0^2$ is obtained

$$\begin{aligned} &\left(j\omega C_{M10} - \frac{jL_{20}}{\omega a}\right)\frac{d^2\dot{U}_1(x)}{dx^2} + \frac{jM_0}{\omega a}\frac{d^2\dot{U}_2(x)}{dx^2} +\\ &+ j\omega C_{120}\dot{U}_2(x) - j\omega(C_{120} + C_{10})\dot{U}_1(x) = 0; \end{aligned} \tag{17}$$

$$\begin{aligned} &\left(j\omega C_{M20} - \frac{jL_{01}}{\omega a}\right)\frac{d^2\dot{U}_2(x)}{dx^2} + \frac{jM_0}{\omega a}\frac{d^2\dot{U}_1(x)}{dx^2} +\\ &+ j\omega C_{120}\dot{U}_1(x) - j\omega(C_{120} + C_{20})\dot{U}_2(x) = 0. \end{aligned} \tag{18}$$

Let us divide equations (17, 18) by a coefficient j, as a result we obtain an equation with real coefficients in this form

$$\begin{aligned} &(\omega^2 aC_{M10} - L_{20})\frac{d^2\dot{U}_1(x)}{dx^2} + M_0\frac{d^2\dot{U}_2(x)}{dx^2} -\\ &-\omega^2 a((C_{120} + C_{10})\dot{U}_1(x) + C_{120}\dot{U}_2(x)) = 0; \end{aligned} \tag{19}$$

$$\begin{aligned} &(\omega^2 aC_{M20} - L_{10})\frac{d^2\dot{U}_2(x)}{dx^2} + M_0\frac{d^2\dot{U}_1(x)}{dx^2} -\\ &-\omega^2 a((C_{120} + C_{10})\dot{U}_2(x) - C_{120}\dot{U}_1(x)) = 0. \end{aligned} \tag{20}$$

The coefficients in (19) are divided by the coefficient $\omega^2 aC_{M10}$, and in equation (20) – by $\omega^2 aC_{M20}$. Then equations (19–20) have the following form

$$\begin{aligned} &\left(1 - \frac{L_{20}}{\omega^2 aC_{M10}}\right)\frac{d^2\dot{U}_1(x)}{dx^2}\frac{M_0}{\omega^2 aC_{M10}}\frac{d^2\dot{U}_2(x)}{dx^2} -\\ &-\frac{C_{120} + C_{10}}{C_{M20}}\dot{U}_1(x) + \frac{C_{10}}{C_{M10}}\dot{U}_2(x) = 0; \end{aligned} \tag{21}$$

$$\begin{aligned} &\frac{M_0}{\omega^2 aC_{M20}}\frac{d^2\dot{U}_1(x)}{dx^2} + \left(1 - \frac{L_{10}}{\omega^2 aC_{M20}}\right)\frac{d^2\dot{U}_2(x)}{dx^2} +\\ &+\frac{C_{20}}{C_{M20}}\dot{U}_1(x) - \frac{C_{120} + C_{20}}{C_{M20}}\dot{U}_2(x) = 0. \end{aligned} \tag{22}$$

For equations (21, 22) the expressions for the coefficients is simplified, namely

$$\begin{aligned} a &= (L_{10}L_{20} - M_0^2) =\\ &= (L_{\sigma 10} + L_{\mu 10})(L_{\sigma 20} + L_{\mu 10}/k^2) - (M_{\sigma 0} + L_{\mu 10}/k)^2 =\\ &= L_{\mu 10}(k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)/L_{\mu 10} + L_{k10})/k^2, \end{aligned} \tag{23}$$

where $L_{k10} = L_{\sigma 10} + k^2 L_{\sigma 20} - 2kM_{\sigma 0}$ is reduced to the primary winding longitudinal scattering inductance of the short-circuit experiment of the transformer.

If this inductance is reduced to the secondary winding, then the coefficient is calculated by the following formula

$$a = L_{m10}(k^2(L_{s10}L_{s20} - M_{s0}^2)\big/L_{m10} + L_{k20}). \quad (24)$$

In the following transformations, to calculate the coefficient a the expression (23) is used. To obtain formulas for determining the coefficients of equations, find expressions for the ratio of the components of these coefficients

$$\frac{L_{20}}{a} = \frac{L_{\sigma 20} + L_{\mu 10}\big/k^2}{L_{\mu 10}(k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)\big/L_{\mu 10} + L_{k10})\big/k^2} = \frac{1 + k^2 L_{\sigma 20} / L_{\mu 10}}{k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)\big/L_{\mu 10} + L_{k10}}; \quad (25)$$

$$\frac{L_{10}}{a} = \frac{L_{s10} + L_{m10}}{L_{m10}(k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)\big/L_{m10} + L_{k10})\big/k^2} = \frac{k^2(1 + L_{s10}\big/L_{m10})}{k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)\big/L_{m10} + L_{k10}}; \quad (26)$$

$$\frac{M_0}{a} = \frac{M_{\sigma 0} + L_{m10}\big/k}{L_{m10}(k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)\big/L_{m10} + L_{k10})\big/k^2} = \frac{k(1 + kM_{\sigma 0}\big/L_{m10})}{k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)\big/L_{m10} + L_{k10}}. \quad (27)$$

The obtained expressions (25–27) are substituted into the corresponding coefficients of equations (21–22), which have the following form

$$a_{11}\frac{d^2\dot U_1(x)}{dx^2} + a_{12}\frac{d^2\dot U_2(x)}{dx^2} - b_1\dot U_1(x) + b_2\dot U_2(x) = 0; \quad (28)$$

$$a_{21}\frac{d^2\dot U_1(x)}{dx^2} + a_{22}\frac{d^2\dot U_2(x)}{dx^2} + b_3\dot U_1(x) - b_4\dot U_2(x) = 0, \quad (29)$$

where $a_{11} = 1 - \dfrac{1 + k^2 L_{\sigma 20}\big/L_{\mu 10}}{\omega^2 C_{M10}(k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)\big/L_{m10} + L_{k10})}$;

$$a_{12} = \frac{k(1 + kM_{\sigma 0}\big/L_{\mu 10})}{\omega^2 C_{M10}(k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)\big/L_{m10} + L_{k10})};$$

$$b_1 = \frac{C_{120} + C_{10}}{C_{M10}}; \quad b_2 = \frac{C_{10}}{C_{M10}};$$

$$a_{21} = \frac{k^2(1 + kM_{\sigma 0}\big/L_{\mu 10})}{\omega^2 C_{M20}(k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)\big/L_{m10} + L_{k10})};$$

$$a_{22} == 1 - \frac{k^2(1 + L_{\sigma 10}\big/L_{\mu 10})}{\omega^2 C_{M20}(k^2(L_{\sigma 10}L_{\sigma 20} - M_{\sigma 0}^2)\big/L_{m10} + L_{k10})};$$

$$b_3 = \frac{C_{20}}{C_{M20}}; \quad b_4 = \frac{C_{120} + C_{20}}{C_{M20}}.$$

Note that for an optimally designed high-frequency transformer in the operating frequency range, the magnetization inductance is much larger than the inductances and mutual inductances of the scattering of the windings; thus, we can consider that $L_{\mu 10} \to \infty$. Then the expressions for calculating the coefficients of equations (28–29) are simplified and take the form

$$a_{11} = 1 - \frac{1}{\omega^2 C_{M10}L_{k10}}; \quad a_{12} = \frac{k}{\omega^2 C_{M10}L_{k10}};$$

$$a_{21} = \frac{k^2}{\omega^2 C_{M20}L_{k10}}; \quad a_{22} = 1 - \frac{k^2}{\omega^2 C_{M20}L_{k10}}.$$

Equations (28–29) are written in operator form with zero initial conditions

$$(a_{11}\lambda^2 - b_1)\dot U_1(\lambda) + (a_{12}\lambda^2 + b_2)\dot U_2(\lambda) = 0; \quad (30)$$

$$(a_{21}\lambda^2 + b_3)\dot U_1(\lambda) + (a_{22}\lambda^2 - b_4)\dot U_2(\lambda) = 0. \quad (31)$$

The characteristic equation of the system of differential equations (30–31) has the form

$$a_4\lambda^4 - a_2\lambda^2 + a_0 = 0, \quad (32)$$

where $a_4 = a_{11}a_{22} - a_{12}a_{21}$; $a_0 = b_1b_4 - b_2b_3$; $a_2 = a_{11}b_4 + a_{12}b_{3a} + a_{21}b_2 + a_{22}b_1$.

The roots of the characteristic biquadratic equation (32) are found by the following formulas

$$\lambda_{1,2} = \pm\sqrt{\frac{a_2 + \sqrt{a_2^2 - 4a_0a_4}}{2a_4}}; \quad (33)$$

$$\lambda_{3,4} = \pm\sqrt{\frac{a_2 - \sqrt{a_2^2 - 4a_0a_4}}{2a_4}}. \quad (34)$$

Solution of equations (30–31) in the form of a superposition of standing waves occurs when the roots (33–34) of the characteristic equation (32) are imaginary. This condition is satisfied if the coefficient of equation is (32) $a_4 < 0$. It follows that the distribution of voltage along the windings in the form of a superposition of standing waves is possible only in a certain frequency range, which depends on the ratios of their length parameters.

Homogeneous equations (28–29) have the following solution

$$\dot U_1(\omega, x) = \dot C_1(e^{j\lambda_1 x} + e^{-j\lambda_1 x}) + \dot C_2(e^{j\lambda_2 x} + e^{-j\lambda_2 x}); \quad (35)$$

$$\dot U_2(\omega, x) = \dot C_3(e^{j\lambda_1 x} + e^{-j\lambda_1 x}) + \dot C_4(e^{j\lambda_2 x} + e^{-j\lambda_2 x}), \quad (36)$$

where $\dot C_1, \dot C_2, \dot C_3, \dot C_4$ are constants of integration.

Constants of integration are determined by the known boundary conditions for the voltage of the beginning and end of the windings, namely

$$\dot U_1(0,\omega) = \dot U_{10}(\omega); \dot U_1(l,\omega) = \dot U_{l1}(\omega); \quad (37)$$

$$\dot U_2(0,\omega) = \dot U_{20}(\omega); \dot U_2(l,\omega) = \dot U_{2l}(\omega); \quad (38)$$

$$0 \le x \le l,$$

where $\dot U_{10}(\omega), \dot U_{l1}(\omega)$ are complex voltage amplitudes of the beginning and end of the primary windings of the transformer; $\dot U_{20}(\omega), \dot U_{l2}(\omega)$ are complex voltage amplitudes of the beginning and end of the secondary windings of the transformer.

Substitute equations (37–38) in equations (35–36), respectively by entering the notation $d_1 = (e^{j\lambda_1 x} + e^{-j\lambda_1 x})$ and $d_2 = (e^{j\lambda_2 x} + e^{-j\lambda_2 x})$ and find the expression of constant integrations.

$$\dot C_1 = \frac{\dot U_1(\omega,0)d_2 - \dot U_1(\omega,l)}{d_2 - d_1}; \quad (39)$$

$$\dot C_2 = \frac{\dot U_1(\omega,l) - \dot U_1(\omega,0)d_1}{d_2 - d_1}; \quad (40)$$

$$\dot C_3 = \frac{\dot U_2(\omega,0)d_2 - \dot U_2(\omega,l)}{d_2 - d_1}; \quad (41)$$

$$\dot C_4 = \frac{\dot U_2(\omega,l) - \dot U_2(\omega,0)d_1}{d_2 - d_1}. \quad (42)$$

Constants of integration (39–42) are substituted in equations (37–38); as a result, we obtain formulas for calculating the voltage distribution of the primary and secondary windings along their axes

$$\dot U_1(\omega,x) = \frac{\dot U_1(\omega,0)d_2 - \dot U_1(\omega,l)}{d_2 - d_1}d_1 + \frac{\dot U_1(\omega,l) - \dot U_1(\omega,0)d_1}{d_2 - d_1}d_2; \quad (43)$$

$$\dot U_2(\omega,x) = \frac{\dot U_2(\omega,0)d_2 - \dot U_2(\omega,l)}{d_2 - d_1}d_1 + \frac{\dot U_2(\omega,l) - \dot U_2(\omega,0)d_1}{d_2 - d_1}d_2. \quad (44)$$

For writing equations (43–44) of the distribution of winding voltage along their axes in instantaneous values, the complex voltage amplitudes of the boundary conditions (37, 38) have the form

$$u_{10}(\omega t,\,0)=U_{m10}(\omega t+\psi_{10});\qquad u_{11}(\omega t,\,l)=U_{1lm}(\omega t+\psi_{11});\tag{45}$$

$$u_{20}(\omega t,\,0)=U_{m20}(\omega t+\psi_{20});\qquad u_{21}(\omega t,\,l)=U_{m2l}(\omega t+\psi_{21}).\tag{46}$$

Substituting (45–46), respectively, in (43–44), we obtain the equation of distribution of voltages along their axes in instantaneous values, namely

$$u_1(\omega t,x)=\frac{U_{m10}(\omega t+\psi_{10})d_2-U_{1lm}(\omega t+\psi_{1l})}{d_2-d_1}d_1+\frac{U_{1lm}(\omega t+\psi_{1l})-U_{m10}(\omega t+\psi_{10})d_1}{d_2-d_1}d_2;\tag{47}$$

$$u_2(\omega t,x)=\frac{U_{m20}(\omega t+\psi_{20})d_2-U_{m2l}(\omega t+\psi_{2l})}{d_2-d_1}d_1+\frac{U_{m2l}(\omega t+\psi_{2l})-U_{m20}(\omega t+\psi_{20})d_1}{d_2-d_1}d_2.\tag{48}$$

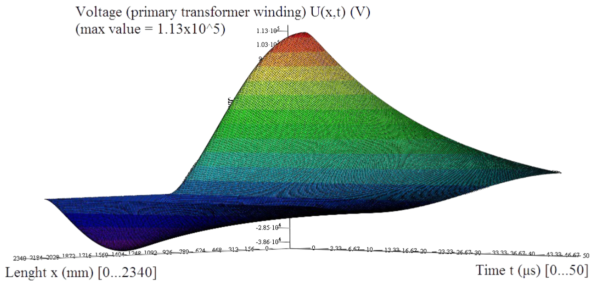

Fig. 1. Voltage distribution in the primary transformer winding at a frequency of 10 kHz depending on the length and time

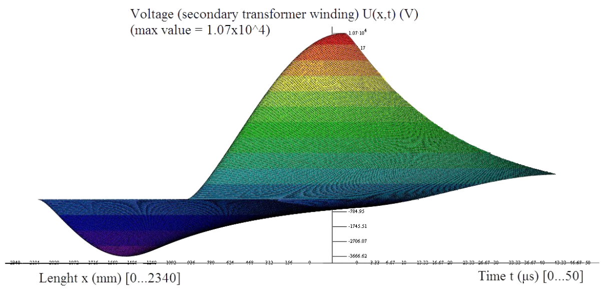

Fig. 2. Voltage distribution in the secondary transformer winding at a frequency of 10 kHz depending on the length and time

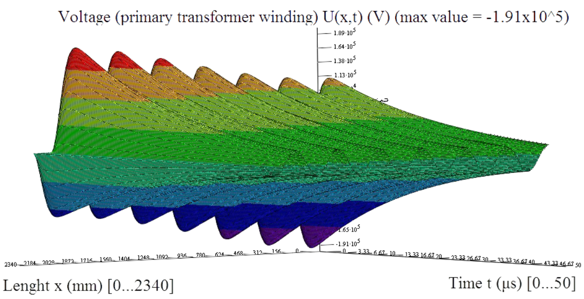

Fig. 3. Voltage distribution in the primary transformer winding at a frequency of 100 kHz depending on the length and time

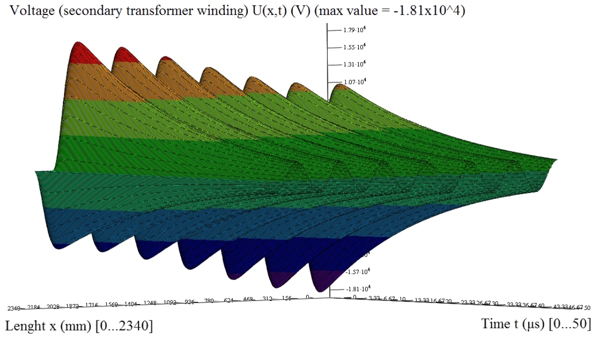

Fig. 4. Voltage distribution in the secondary transformer winding at a frequency of 100 kHz depending on the length and time

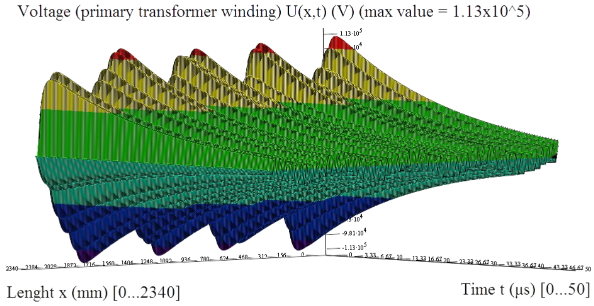

Fig. 5. Voltage distribution in the primary transformer winding at a frequency of 800 kHz depending on the length and time

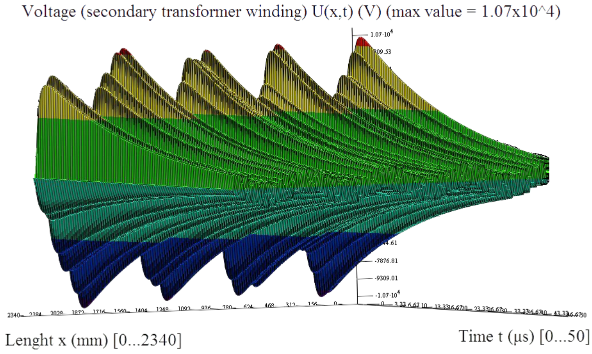

Fig. 6. Voltage distribution in the secondary transformer winding at a frequency of 800 kHz depending on the length and time

Figs. 1 and 2 show the voltage distribution in the primary and secondary transformer windings at a frequency of 10 kHz depending on the length and time.

Figs. 3 and 4 show the voltage distribution in the primary and secondary transformer windings at a frequency of 100 kHz depending on the length and time.

Figs. 5 and 6 show the voltage distribution in the primary and secondary transformer windings at a frequency of 800 kHz depending on the length and time.

Conclusion. The mathematical model for the research on high-frequency periodic processes in the windings of power two-winding transformer is created. It allows optimizing the insulation of the windings during the design of transformers, thereby reducing their size, cost and weight. The model allows us to significantly expand the possibilities of researching high-frequency periodic processes in the windings of power two-winding transformers in solving specific problems of classical and technical electrodynamics.

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Високочастотні періодичні процеси в силових двообиткових трансформаторах

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Мета. Математичне моделювання високочастотних періодичних процесів в обвитках силових трансформаторів для вдосконалення технології їхнього проектування й експлуатації.

Методика. Запропоновано підхід до формування математичної моделі для дослідження високочастотних періодичних процесів в обвитках силових трансформаторів з урахуванням електромагнітних зав'язків між обвитками та витками обвиток.

Результати. Розроблена математична модель розрахунку високочастотних періодичних процесів в обвитках силових трансформаторів на підставі диференційних рівнянь у часткових похідних, що використовуються для розрахунку швидкоплинних перехідних процесів, звівши їх до рівнянь у повних похідних, придатних для дослідження високочастотних періодичних процесів в обвитках трансформаторів.

Наукова новизна. Створена математична модель для дослідження високочастотних періодичних процесів в обвитках силових трансформаторів на підставі запропонованої заступної схеми двообиткового трансформатора з урахуванням електромагнітних зв'язків між обвитками та основним магнітним потоком, шляхом зведення диференційних рівнянь у часткових похідних до рівнянь у повних похідних, застосувавши символічне перетворення Штейнмеца, а також для цих рівнянь сформована крайова задача.

Практична значимість. Запропоновано метод аналізу розподілу напруги в обвитках трансформаторів у вигляді суперпозиції стоячих хвиль у певному діапазоні частот, що залежить від співвідношень їхніх параметрів для високочастотних періодичних процесів в обвитках. Це дозволяє удосконалити технологію проектування силових трансформаторів, тобто правильно координувати ізоляцію обвиток трансформаторів.

Ключові слова: *високочастотний періодичний процес, математичне моделювання, трансформатор, крайова задача, звичайні диференційні рівняння*

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