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DETERMINATION OF STRESS CONCENTRATION NEAR THE HOLES UNDER DYNAMIC LOADINGS

Purpose. To develop an approach for determining the stress state of plate structural elements with holes under dynamic loads with controlled accuracy.

Methodology. The study was carried out on the basis of the Laplace transform and the method of integral equations.

Findings. An approach to determining the dynamic stresses at the holes in the plates is proposed, which includes: the Laplace transform in the time coordinate; a numerical method for determining transformants of displacements and stresses based on the method of integral equations; finding originals on the basis of Prudnikov's formula adapted to dynamic problems of elasticity theory. The problem of determining the Laplace images for displacements is reduced to solving singular integral equations. Integral equations were solved numerically based on the approaches developed in the boundary element method. To find displacements and stresses, the Laplace transform inversion formulas proposed by Prudnikov are adapted to dynamic problems. The study on dynamic stresses at holes of various shapes was carried out.

Originality. A new approach to the regularization of the Prudnikov formula for inverting the Laplace transform as applied to dynamic problems of the theory of elasticity has been developed. For its implementation: convergence of Fourier series based on pre-set stresses at the initial time is improved; the remainder is taken into account in the conversion formula.

Practical value. A method has been developed for calculating the stress concentration at holes of arbitrary shape in lamellar structural elements under dynamic loads. The proposed approach makes it possible to determine stresses with controlled accuracy. The studies performed at circular and polygonal holes with rounded tops can be used in strength calculations for dynamically loaded plates. The influence of Poisson's ratio on the concentration of dynamic stresses is analyzed.

Keywords: *concentration of dynamic stresses, method of boundary integral equations, Laplace transforms, inversion formulas*

Introduction. A plane dynamic problem of elasticity for a plane with holes is considered. The problem is solved using the Laplace integral time transform and the boundary integral equations method (BIEM). The inverse Laplace transform is performed numerically using Prudnikov's formula, which is adapted to the dynamic problems of elasticity. It is shown that using the proposed approach, the calculation of the stress concentration can be performed with a given accuracy. The solution of the obtained integral equations, where kernels are Cauchy kernels and logarithmic functions, is solved numerically without additional regularization. The results of calculations of dynamic coefficients of stress concentration near holes of different shapes are presented.

Literature review. Boundary integral equations method is widely used to solve 2-dimensional dynamic problems of elasticity. Numerical algorithms for solving integral equations, which are based on Stokes singular solutions, are given in the books by C. A. Brebbia, S. Walker [1] and F. J. Sayas [2]. Such algorithms are quite complex, because the discretization of equations is carried out simultaneously by spatial coordinates and time.

Therefore, other methods for solving dynamic problems of the elasticity theory are widely used in the literature. In particular, in [3] and Article [4] in the equations, the time derivatives are preliminarily replaced by finite differences with subsequent application to the obtained BIEM equations. Let us note that when using these methods, the solutions are found step by step over time, and therefore errors can accumulate.

Simpler algorithms can be built based on the Laplace transform with the additional use of the integral equations method [5] or analytical methods [6]. The main difficulties are to find the originals based on the found images of Laplace. A number of formulas for numerical inversion are used in the literature. However, in their numerical implementation there is a problem of control over the accuracy of calculations, because the problem of inversion in the general case is incorrect. In the works by R. M. Kushnir [7] and T. Ya. Solyar [8] a modified exact Prudnikov formula is used [8, 9] for numerical inversion in problems of nonstationary thermal conductivity, which connects the values of the original of the function and its Laplace transformer. In the proposed work, this formula is modified with respect to the dynamic problems of the theory of elasticity in such a way that when applied, the residual term can be made as arbitrarily small. The convergence of the series that are included in this formula has also been improved. It is shown that with such approach it is possible to control the accuracy of stress concentration calculations near holes of arbitrary shape.

Other methods for solving dynamic problems of the elasticity theory are used in: [10] (based on the integral Fourier transform over time); [11] (the method of Fourier series by spatial coordinates is used); [12] (Galorkin method is used); [13] (the finite element method is used).

The problem statement. Let the plane be weakened by the holes whose boundaries are the contours L_j, $j = 1, ..., J$, to the plate boundary the stresses (p_1, p_2) are applied, where $p_{1,2} = p_{1,2}(M, \tau$, τ – is time; $M(x, y)$ is a point on the boundary. Assume that at initial moment the displacements and stresses are absent. To solve the problem we use the Laplace integral time transform and the BIEM.

Integral representation for the Laplace transform. For the Laplace transforms from displacements $u_{1,2}$ the integral Somiliano representation [1, 5] will be valid

$$\tilde{u}_j(M)=\int_L(\tilde{p}_k U_{jk}(M,M_0)-\tilde{u}_k T_{jk}(M,M_0))ds_0, \qquad (1)$$

where $M(x, y)$ is an arbitrary point of the plane; $M_0(x_0, y_0)$ is a point on the plane boundary, by which we integrate, are Laplace transforms from the stress vector $(\tilde{p}_1, \tilde{p}_2)$, (p_1, p_2), $L = L_1 + ... + L_J$,

$$U_{jk}=\frac{1}{2G\pi}\left(\delta_{jk}f_0-r_j r_k f_2\right);$$

$$T_{jk}=-\frac{1}{2\pi}\left[r_j n_k F_1+\left(\delta_{jk}\frac{\partial r}{\partial n_0}+r_k n_j\right)F_2+r_k r_j\frac{\partial r}{\partial n_0}F_3\right],$$

where δ_{jk} is Kronecker's delta; (n_1, n_2) are the direction cosines of outward normal to the boundary contour in $M_0(x_0, y_0)$

$$r=\sqrt{(x-x_0)^2+(y-y_0)^2};\quad r_1=\frac{x_0-x}{r};\quad r_2=\frac{y_0-y}{r};$$

$$\frac{\partial r}{\partial n_0}=\frac{(x_0-x)n_1+(y_0-y)n_2}{r}.$$

Here and further by the parameter, which is repeated, we will sum up

$$f_0 = K_0(q_2r) + \varphi(q_2r);$$

$$f_2 = K_0(q_2r) - \alpha K_0(q_1r) + 2\varphi(q_2r);$$

$$\varphi(z)=\frac{K_1(z)-\beta K_1(\beta z)}{z};$$

$$F_1=\frac{2}{r}f_2+\gamma_0 q_1 K_1(q_1 r);\quad F_2=\frac{2}{r}f_2+q_2K_1(q_2r);$$

$$F_3=-\frac{8}{r}f_2-2q_2[K_1(q_2r)-\beta^3K_1(q_1r)],$$

where $q_{1,2} = s/C_{1/2}$; s is a parameter of the Laplace time transform; $C_1=\sqrt{\dfrac{\lambda+2G}{\rho}}$, $C_2=\sqrt{\dfrac{G}{\rho}}$; $K_0(z)$, $K_1(z)$ are the Macdonald functions; ρ is density; λ, G are Lame constants,

$$\beta=\frac{C_2}{C_1};\quad \alpha=\beta^2=\frac{G}{\lambda+2G}=\frac{1-2\nu}{2-2\nu};\quad \gamma_0=\frac{\lambda}{\lambda+2G}=\frac{\nu}{1-\nu},$$

where ν is Poisson's ratio.

The above relations are also valid for the case of the plane stress state (PSS), if the values G, λ, ν are replaced by G, λ, ν, where $\lambda_* = 2G\nu/(1-\nu)$; $\nu_* = \nu/(1+\nu)$.

For small values of z we have

$$\varphi(z)=-\frac{1-\alpha}{2}\left(\frac{1}{2}-\gamma-\ln\frac{z}{2}\right)-\frac{\alpha}{4}\ln\alpha;\quad f_2=\frac{\alpha-1}{2},$$

where g = 0.57721566400153.

Then to find the Laplace transforms from displacements we will start from the potential representation of the solution of the formulated problem and as such on the basis of (1) we take

$$\tilde{u}_j(M)=\int_L P_k U_{jk}(M,M_0)ds_0,\quad j=1,2, \qquad (2)$$

where $P_k = P_k(M_0)$ are unknown functions, $k = 1, 2$.

We determine the stress vector (Z_1, Z_2), corresponding to displacements (2), at point M on the plane, the normal to which has direction cosine N_1, N_2.

Then

$$Z_j(M)=\int_L P_k(M_0)Q_{jk}(M,M_0)ds_0,\quad j=1,2. \qquad (3)$$

The values $Q_{jk}(M, M_0)$ $(k, j = 1, 2)$ are obtained on the basis of the value $T_{kj}(M_0, M)$ and have the following form

$$Q_{jk}=-\frac{1}{2\pi}\left[R_kN_jF_1+\left(\delta_{jk}\frac{\partial r}{\partial N}+R_jN_k\right)F_2+R_kR_j\frac{\partial r}{\partial N}F_3\right].$$

Where

$$R_1=\frac{x-x_0}{r};\quad R_2=\frac{y-y_0}{r};\quad \frac{\partial r}{\partial N}=R_1N_1+R_2N_2.$$

The expression for the sum of stresses is presented as

$$\sigma_x+\sigma_y=-\frac{q_1}{2\pi(1-\nu)}\int_L\left(P_1R_1+P_2R_2\right)K_1(q_1r)ds. \qquad (4)$$

In relation (3) let us direct point M to the plate boundary and assume that N_1, N_2 are the direction cosines of the outward normal to the boundary at this point. Then, having used the Sohotski-Plemelj formula, we obtain the integral equations to determine the unknown functions.

$$\frac{1}{2}P_j(M)+\int_L P_k(M_0)Q_{jk}(M,M_0)ds_0=\tilde{p}_j(M),\quad j=1,2,\quad M\in L. \qquad (5)$$

Similarly, we can find the formula for the transform from the sum of stresses

$$\tilde{\sigma}_x+\tilde{\sigma}_y=\frac{1}{2(1-\nu)}\left(P_1\frac{\partial x}{\partial n}+P_2\frac{\partial y}{\partial n}\right)-$$

$$-\frac{q_1}{2\pi(1-\nu)}\int_L\left(P_1R_1+P_2R_2\right)K_1(q_1r)ds.$$

Numerical algorithm of solution of the system of equations. Integral equations (5) are singular. Besides, the kernels of these equations have logarithmic singularities. Present the direct numerical algorithm of solution of integral equations (5) without their preliminary regularization [14, 15]. Then for simplification we assume that L is one contour. Write down the contour equation in a parametric form $x = \alpha(\eta)$, $y = \beta(\eta)$, $A \le \eta \le B$.

Having put $x_0 = \alpha(\xi)$; $y_0 = \beta(\xi)$; $A \le \xi \le B$ in equation (5), we obtain

$$\frac{1}{2}f_j(\eta)+\int_A^B f_k(\xi)F_{jk}(\eta,\xi)d\xi=g_j(\eta),\quad j=1,2,\quad A\le\eta\le B, \qquad (6)$$

where

$$F_{jk}(\eta,\xi)=s'(\xi)Q_{jk}(M,M_0)\Big|_{x=\alpha(\eta),\,y=\beta(\eta);\,x_0=\alpha(\xi),\,y_0=\beta(\xi)};$$

$$f_j(\xi)=P_j(M_0)\Big|_{x_0=\alpha(\xi),\,y_0=\beta(\xi)};$$

$$g_j(\eta) = \tilde{p}_j(M)\Big|_{x=\alpha(\eta),\, y=\beta(\eta)};$$

$$s'(\xi) = \sqrt{\alpha'^2(\xi) + \beta'^2(\xi)}.$$

Let us set the values for parameter $\eta = \eta_0$, $\eta = \eta_2, \ldots, \eta = = \eta_{2N}$, such that $\eta_0 = A$, $\eta_{2N} = B$; $\eta_{j+1} > \eta_j$. Assume that $\eta_{2j+1} = = (\eta_{2j+2} + \eta_{2j})/2, j = 0, \ldots, N-1$.

Let us introduce the nodal points $x_k = \alpha(\eta_k)$; $y_k = \beta(\eta_k)$; $k = = 0, \ldots, 2N$ on the contour L. To solve equation (6) we use the collocation method, demanding that this equation be satisfied at points $C_k(x_k, y_k)$, $k = 0, \ldots, 2N$. Than we obtain a system of equations

$$\frac{1}{2} f_j(\eta_m) + \int_A^B f_k(\xi) F_{jk}(\eta_m, \xi) d\xi = g_j(\eta_m), \quad j = 1, 2, \quad m = 0, \ldots, 2N. \tag{7}$$

Note that the integrand functions $F_{jk}(\eta_m, \xi)$ in equation (6) have in the vicinity $\eta = \eta_m$ the Cauchy kernel and logarithmic singularity. That is, in the integral equations the integrals of the form are obtained

$$I = \int_A^B F(\xi, c) f(\xi) dt,$$

where $f(\xi)$ is a function discontinuous on the interval $[A, B]$; $F(\xi, c)$ is a function which can have singularity on the interval of integration in the vicinity of point $\xi = c = \eta_m$. To construct a quadrature formula the integrand function $f(\xi)$ for $\eta_{2j-2} < \xi \leq \leq \eta_{2j}$, $j = 1, \ldots, N$ is presented by the Lagrange interpolation polynomial

$$f(\xi) \approx f_{2j-2} S_1(\xi - \eta_{2j-1}, h_j) + + f_{2j-1} S_2(\xi - \eta_{2j-1}, h_j) + f_{2j} S_3(\xi - \eta_{2j-1}, h_j),$$

where $2h_j = \eta_{2j} - \eta_{2j-2}$; $f_k = f(\eta_k)$;

$$S_1(\xi, h) = \frac{\xi(\xi - h)}{2h^2}; \quad S_2(\xi, h) = \frac{h^2 - \xi^2}{h^2}; \quad S_3(\xi, h) = \frac{\xi(\xi + h)}{2h^2}.$$

Then we obtain a quadrature formula of the form

$$I \approx \sum_{j=0}^{2N} A_j f_j, \tag{8}$$

where $A_0 = I_1^{(1)}$; $A_{2N} = I_3^{(N)}$; $A_{2k-1} = I_2^{(k)}$; $k = 1, \ldots, N$; $A_{2n} = = I_3^{(n)} + I_1^{(n+1)}$; $n = 1, \ldots, N-1$; $I_k^{(n)} = \int_{\eta_{2n-2}}^{\eta_{2n}} F(\xi, c) S_k(\xi - \eta_{2n-1}, h_n) d\xi$.

Note that those coefficients have the following form

$$A_{2j-1} = h_j \int_{-1}^{1} F(t_{2j-1} + h_j \xi, \eta_m)\left(1 - \xi^2\right) d\xi, \quad j = 1, \ldots, N;$$

$$A_{2j} = \frac{1}{2} \int_0^2 \left[h_j F(\eta_{2j} - h_j \xi, \eta_m) + h_{j+1} F(\eta_{2j} + h_{j+1} \xi, \eta_m) \right] \times \times (1 - \xi)(2 - \xi) d\xi, \quad j = 1, \ldots, N-1.$$

Using the last formula we can also find the coefficients A_j for $j = 0$ and $j = N$, when assuming that function $F(\xi) = 0$ for $\xi < A$ and $\xi > B$.

Here the integrand function A_i in integrals has a singularity in the vicinity of point $\xi = 0$ for $m = i$. We can verify that for $m = 2i$ in the integrand function the Cauchy kernel is absent and only logarithmic singularities can exist. The first integral, which for $m = 2j - 1$ is particular, is written as

$$A_{2j-1} = h_j \int_0^1 \left[F(\eta_{2j-1} + h_j \xi, \eta_{2j-1}) + F(\eta_{2j-1} - h_j \xi, \eta_{2j-1}) \right] \times \times (1 - \xi^2) d\xi, \quad j = 1, \ldots, N.$$

We can also verify that the integrand functions can have only logarithmic singularity in the vicinity of point $\xi = 0$. To calculate them we consider such integrals

$$J = \int_0^a g(\xi) d\xi = a \int_0^1 g(au) du, \quad a = \text{const}.$$

Having replaced the variable $u = z^q$, $q > 1$, we obtain

$$J = aq \int_0^1 g_*(z) dz; \quad g_*(z) = z^{q-1} g(az^q).$$

Here the integrand function $g_*(z)$, if we choose $q \geq 3$, is discontinuous and limited together with the derivative. To calculate this integral it is advisable to use Gauss quadrature formula with nodal points, which do not include point $z = 0$ (at this point the initial points have a singularity).

Having applied the quadrature formula (8) to the integrals in equation (6), we obtain the systems of algebraic equations for determination of the unknown functions at nodal points on the body boundary

$$\frac{1}{2} f_{j,m} + \sum_{n=0}^{2N} A_{jk,mn} f_{k,n} = g_j(\eta_m), \quad j, k = 1, 2; \quad m = 0, \ldots, 2N. \tag{9}$$

Here the coefficients $A_{jk,mn} = A_n\Big|_{F(\xi,c) = F_{jk}(c,\xi)}$ for $c = \eta_m$.

Formula of inversion of the Laplace transform. Consider the problem of finding the function $f(t)$ based on its integral Laplace transform

$$F(s) = \int_0^\infty f(t) \exp(-st) dt.$$

We will start from the exact Prudnikov formula, which connects the values of originals and their presentations [7–9]

$$f(t) = \frac{1}{l} \exp(ct/l) \sum_{n=-\infty}^{\infty} F(s_n) \exp(2\pi nit/l) - R_1, \tag{10}$$

where $0 < t < l$, $s_n = (c + 2\pi ni)/l$; c and l are constants, choosing which we can improve the convergence of solution, where $\mathrm{Re}(c) > 0$,

$$R_1 = \sum_{n=1}^{\infty} \exp(-nc) f(t + nl). \tag{11}$$

We assume that the known values of original and its derivative for $t = 0$are $f(0)$, $f'(0)$. Let us also consider the case when the value of original for large values of the variable t, denoted by $f_\infty = \text{const}$ is known. Then formula (10) can be written in the form [7]

$$f(t) = \frac{1}{l} \exp\left(c \frac{t}{l}\right) \sum_{n=-\infty}^{\infty} F_n \exp\left(2\pi ni \frac{t}{l}\right) + +(1 + \gamma)\left[f(0) + f'(0)\left(t + \gamma l\right)\right] - \gamma f_\infty - R_2, \tag{12}$$

where

$$F_n = F(s_n) - \frac{f(0)}{s_n} - \frac{f'(0)}{s_n^2};$$

$$R_2 = \sum_{n=1}^{\infty} \exp(-nc)\left[f(t + nl) - f_\infty \right]; \quad \gamma = \frac{1}{\exp(c) - 1}.$$

Formula (12) is also exact. The series in (12) is fast convergent since for assumed assumptions the coefficients in it are of the order $F_n = O(n^{-3})$ for θ. When we choose parameter l so that for $t > l$ $f(t) \approx f_\infty$ and assume $c > 3$, then the value R_2 can be small and it can be neglected. In particular l, if we choose parameter $|f(t) - f_\infty| < \varepsilon$ such that $t > l$, then the residual term will be of the for

$$\left| R_2 \right| \leq \sum_{n=1}^{\infty} \exp(-nc) \left| f(t + nl) - f_\infty \right| \leq \varepsilon \frac{\exp(-c)}{1 - \exp(-c)}.$$

When $c = 3$ for residual term, the estimate $|R_2| \leq 0.0524\varepsilon$ is valid.

Consider now the problem on determination of the hoop stresses which are the basis for strength calculations on the hole boundary by means of (12). The value of the Laplace transform from these stresses will be found by the BIE method, having put above the transform parameter $s = s_n$, $n = 0, 1, \ldots$

Find the value of hoop stresses at initial moment. Then consider the case when at the initial moment the displacements and velocities in the plane are zero. To find the hoop stresses we use Hook's law, which in the case of plane deformation is of the form [16]

$$\varepsilon_\theta = \frac{1}{E_1}(\sigma_\theta - \nu_1\sigma_n), \qquad (13)$$

where σ_θ, σ_n are circular and normal stresses on the boundary $E_1 = \frac{E}{1-\nu^2}$, $\nu_1 = \frac{1}{1-\nu}$; ε_θ – hoop deformations on the hole boundary which are determined by the formula $\varepsilon_\theta = \frac{\partial u_\theta}{\partial s} + ku_n$; u_θ, u_n are displacements in the hoop and nodal directions on the boundary; k is the curvature.

Then from (13) at the initial moment on the boundary of the hole the hoop stresses will be $\sigma_\theta = \nu_1\sigma_n$, here the normal stresses σ_n are known. Find the time derivative for $\tau = 0$

$$\frac{d\sigma_\theta}{d\tau} = \nu_1\frac{d\sigma_n}{d\tau}.$$

That is the hoop stresses at the initial moment on the boundary are not zero as it is assumed in some publications [4, 17].

We assume that the load applied to the holes boundary is the following: $p_1 \to p_1^\infty$, $p_2 \to p_2^\infty$ for $t \to \infty$. Then the asymptotic values for hoop stresses σ_θ^∞, which are necessary for the application of the formula (12), are obtained by solution of the static problem for the plane with holes and boundary stress (p_1^∞, p_2^∞).

Results of calculations. Consider the plane with a circular hole of radius *a,* to whose boundary the stress $\sigma_r = -p(1 + \cos 2\theta)H(\tau)$, $\tau_{r\theta} = 0$ is applied, where θ is an angular coordinate, $H(t)$ is the Heaviside function. The calculated hoop stresses on the hole boundary, which are referred to parameter p depending on the dimensionless time coordinate $t = C_2\tau/a$ for $\theta = 0$ the values of Poisson's coefficients $\nu = 0.1$, 0.3, 0.49 are given in Fig. 1 (values ν are given at the curves). When solving the problem at 40–80 nodal points on the hole boundary were chosen. When using inversion formula (12) it was set that $c = 3$, $l = 15$ and 60–120 forms of the series were considered.

The plots in Figs. 1, 2 are originally different from the results of calculations obtained for this case in [4], using the method of finite differences by the time coordinate. Therefore, the considered problem was also solved analytically. For this purpose the case is considered when on the boundary of a circular hole the Laplace transforms from the applied stresses read

$$\tilde{\sigma}_r = \sum_{n=0}^{\infty} p_n \cos n\theta; \quad \tilde{\tau}_{r\theta} = \sum_{n=1}^{\infty} t_n \sin n\theta.$$

The analytical solution of the problem is constructed in [1, 4]. The transforms from the hoop stresses on the hole boundary are determined by the formulas

$$\tilde{\sigma}_\theta = \sum_{n=0}^{\infty}\left(A_n f_{\theta n} + B_n g_{\theta n}\right)\cos n\theta,$$

where A_n, B_n – are the constants determined from the system of equations

$$\begin{cases} A_n f_{rn} + B_n g_{rn} = p_n \\ A_n f_{tn} + B_n g_{tn} = 2t_n \end{cases}.$$

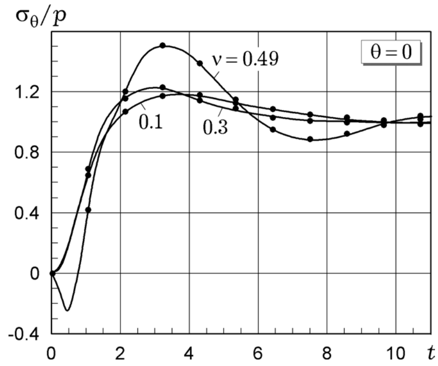

Fig. 1. Relative hoop stresses for $\theta = 0$

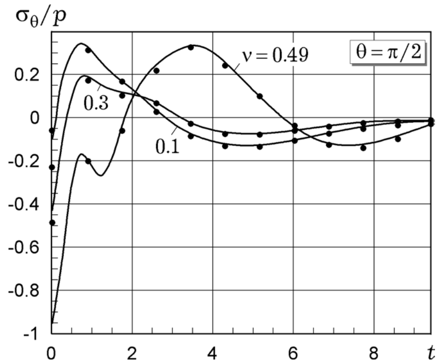

Fig. 2. Relative hoop stresses for $\theta = \pi/2$ (*the dots represent are data, calculated by analytical solution*)

Here

$$f_{rn} = s_1^2\left(C + \partial^2\right)K_n(s_1 r); \quad g_{rn} = -\frac{n}{r}\left(\frac{1}{r} - s_2\partial\right)K_n(s_2 r);$$

$$f_{tn} = \frac{2n}{r}\left(\frac{1}{r} - s_1\partial\right)K_n(s_1 r); \quad g_{tn} = \left(-\frac{n^2}{r^2} + \frac{s_2}{r}\partial - s_2^2\partial^2\right)K_n(s_2 r);$$

$$f_{\theta n} = \left(-\frac{n^2}{r^2} + \frac{s_1}{r}\partial + Cs_1^2\right)K_n(s_1 r); \quad g_{\theta n} = \frac{n}{r}\left(\frac{1}{r} - s_2\partial\right)K_n(s_2 r),$$

where $r = a$; $s_j = s/C_j$; $j = 1, 2$; $\partial K_n(z) = K_n'(z)$; $\partial^2 K_n(z) = K_n''(z)$; $C = \nu/(1-2\nu)$.

The originals of the stresses were found by the given inversion formulas of the Laplace transform. The stresses obtained on the basis of this solution are practically exact since the errors in calculations arise only due to neglecting the residual term R_2, which in this case is small in consequence of substantially selected parameter l. The stresses calculated by analytical solution are given in Figs. 1, 2 by dots. That is, the used above BIE method guarantees the practically exact results.

Consider a plane with an elliptic hole with semi-axes $\sigma_n = -pH(\tau)$ and $p = \text{const}$ to whose boundary the normal stresses are applied.

The multipliers for the dynamic stress concentration factors (SCF) are calculated, that is, the relations of SCF under dynamic and static loads, for an elliptic hole with a ratio of semi-axes 0.5 for different values of Poisson's ratio ν. Curves *1–5* in Fig. 3 correspond to the results of calculations at the boundary of the hole at point *A* (where the radius of curvature is minimal), which are carried out at $\nu = 0.1$, 0.2, 0.3, 0.4, 0.45. Curves *1′–5′* correspond to the annular stresses assigned to p at point *B* (where the radius of curvature is maximum).

A rectangular hole with semi-sides *a*, *b* and rounded angles of a quadrant of radius *R,* to whose boundary normal stresses $\sigma_n = -pH(\tau)$ are applied, is considered. The results of calculations for $a = b$, $R = ma$, $\nu = 0.3$ are given in Fig. 4. Here near the curves the value of *m* is given, which is assumed to be 0.2, 0.5,

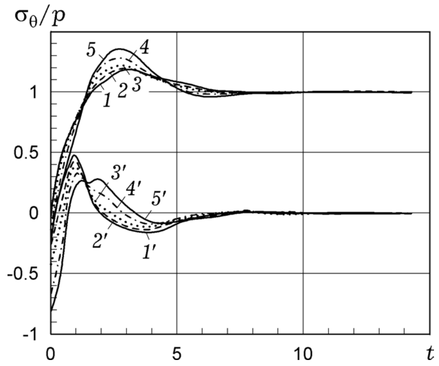

Fig. 3. Relative stresses on the boundary of an elliptic hole

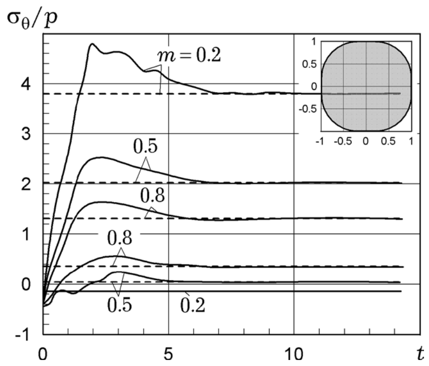

Fig. 4. Relative stresses on the boundary of a square hole with nodal apexes

0.8. The solid lines correspond to the hoop stresses, related to p, at the center of a rounded apex and the dashed ones – in the middle of sides t. In the same figure the straight lines represent the values of relative stresses under static loading that is based on algorithm [17].

Similar results of calculation are obtained for rectangular holes for relation of the sides $a/b = 1, 2, 4$, when the apexes are rounded by quadrants of radius $R = 0.2b$. The calculated relative maximal hoop stresses on the boundary are given in Fig. 5 by solid lines, near which the relations of sides are presented. The dotted lines represent the relative stresses in the middle of the shorter side.

The calculated dynamic stresses at the values of the time coordinate $t > 6$ go to the set mode and coincide with the corresponding values for the static problem. From Figs. 4, 5 it is seen that the maximum dynamic stresses occur in the region of rounding the vertices of the rectangle. In the middle of the sides of the rectangular and square holes stresses are low.

The performed calculations show a significant influence of the mechanical characteristics of the materials on the concentration of dynamic stresses near the holes (for the considered problems at static loads the stresses do not depend on elastic constants). For materials with low Poisson's ratios (cast iron, concrete, carbon steels) SCF significantly increases for materials where Poisson's ratios are close to 0.5. Such materials include, in particular: lead, cold-rolled brass, molybdenum.

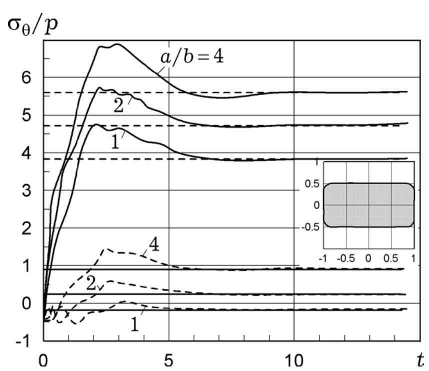

Fig. 5. The relative stresses on the boundary of a rectangular hole

Conclusions. The paper presents an approach to solution of the plane boundary dynamic problems of elasticity which is based on the Laplace transform and BIEM. Singular integral equations, whose kernels contain additionally logarithmic singularities, are solved by a direct method (without their regularization). The proposed formula for the inverse of the Laplace transform makes it possible to calculate the stress concentration under dynamic loads with controlled accuracy. In particular, the above dynamic stresses calculated for large values of time coincide with corresponding static stresses, and the stresses for small values of time agree well with corresponding values determined analytically. The series in inversion formula proved to be fast convergent. A residual term R_2 not considered in dynamic problems of elasticity can be small. From the presented calculations we can see that dynamic SCF increases with the increase in Poisson's ratio. In the holes of rectangular form with rounded angles, the maximal stress concentration appears near the rectangle apex.

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Визначення концентрації напружень біля отворів при динамічних навантаженнях

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Мета. Розробити методику розрахунку напруженого стану пластинчатих елементів конструкцій з отворами при динамічних навантаженнях з контрольованою точністю.

Методика. Дослідження виконано на основі перетворення Лапласа та методу інтегральних рівнянь.

Результати. Запропоновано підхід до визначення динамічних напружень біля отворів у пластинках, що включає: перетворення Лапласа за часовою координатою; числовий метод визначення трансформант переміщень і напружень на основі методу інтегральних рівнянь; знаходження оригіналів на основі адаптованої до динамічних задач теорії пружності формули обернення Прудникова. Задача визначення зображень Лапласа для переміщень зведена до розв'язування сингулярних інтегральних рівнянь. Розв'язування інтегральних рівнянь проведено чисельно на основі підходів, розроблених у методі граничних елементів. Для знаходження оригіналів переміщень і напружень використані адаптовані до динамічних задач формули обернення перетворення Лапласа, що запропоновані Прудніковим. Проведено дослідження динамічних напружень біля отворів різної форми.

Наукова новизна. Розроблено новий підхід до регуляризації формули Пруднікова для обернення перетворення Лапласа стосовно до динамічних задач теорії пружності. Для його реалізації: покращена збіжність рядів Фур'є на основі попередньо встановлених напружень у початковий момент часу; ураховано залишковий член у формулі обернення.

Практична значимість. Розроблена методика розрахунку концентрації напружень біля отворів довільної форми у пластинчатих елементах конструкцій при динамічних навантаженнях. Запропонований підхід дозволяє визначати напруження з контрольованою точністю. Виконані дослідження біля кругових і многокутних отворів із закругленими вершинами можуть бути використані в розрахунках на міцність динамічно навантажених пластин. Проаналізовано вплив коефіцієнта Пуассона на концентрацію динамічних напружень.

Ключові слова: *концентрація динамічних напружень, метод граничних інтегральних рівнянь, перетворення Лапласа, формули обернення*

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