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ANALYTICAL STUDIES ON CONSTRAINED PARTICLE SETTLING VELOCITY IN A WATER SUSPENSION OF FLY ASH FROM THERMAL POWER PLANTS

Purpose. To establish analytical dependences for calculating the characteristics of the ash suspension and the velocity of constrained settling of coal and quartz depending on the particle size and density of the medium, which is necessary for calculating the design and determining the operating modes of hydraulic devices for extracting coal from water mineral suspension of fly ash from thermal power plants.

Methodology. The research was carried out on the basis of a cellular suspension model and classical concepts of constrained particle motion in laminar and turbulent flow. For analytical evaluation of the characteristics of the suspension, the defining correlations and the Wend formula for viscosity were used. The Ergun equation and correlation analysis methods were used to calculate and analyze the speed of constrained movement of ash suspension particles.

Findings. Approximating nonlinear functions are obtained for determining the speed of constrained movement of coal and quartz particle size up to 4 mm in an ash suspension with a density of 1.3–1.8 g/cm^3. It is shown that, for both settling and ascending of coal, there is a direct relationship between the velocity and particle size, in both cases it is nonlinear. For any coal size, the speed of ascent depending on the suspension density is of extreme nature; the rational density range is 1.55–1.8 g/cm^3 with a maximum of 1.65 g/cm^3. The established dependencies allow us to determine the size of coal and quartz particles, taking into account the counter-flow of the liquid phase, as well as the boundary size.

Originality. For the regime of weak-turbulence flows there were established dependences and approximation equations of the constrained movement speed of coal and quartz particles – the main components of the water suspension of fly ash from the Novo-Kramatorska TPP, depending on the size and density of the ash suspensions with changes in the characteristics of the medium depending on density. The frames of the Stokes description of processes are established. It is shown that the movement of the liquid phase in a counter-flow with precipitating particles is effective for surfacing of thin coal classes.

Practical value. The described approach can be used for analytical evaluation of the characteristics and velocity of constrained movement in various water suspensions of discrete solid particles in weak-turbulence flows. The advantage is a wider coverage of hydraulic equipment operating modes. The results obtained are necessary for designing and determining the technological modes of operation of various hydraulic devices in the technology of complex processing of fly ash from TPPs.

Keywords: *mineral suspension, fly ash, density, velocity, settling, ascend*

Introduction. The problem of utilization and processing of fly ash from coal-fired thermal power plants (TTPs) has long attracted the attention of scientists, since the relatively high content of carbon, as well as iron, aluminum, and titanium oxides allows us to consider this raw material promising for complex processing [1]. Ash dumps occupy large land areas, and the cost of their construction, reconstruction, and maintenance increases annually [2]. Without the utilization of industrial waste from TPPs, environmental problems are sharply increasing, which leads, in particular, to an increase in tax costs [3].

A review of scientific information shows that the main trend in creating and improving the technology for processing fly ash is the use of relatively cheap gravity enrichment [4–6]. Development of equipment for hydraulic separation of fine-disperse mixtures of fly ash with water is an urgent task. Such work is performed in IGTM NAS of Ukraine.

The design of new hydraulic devices and the determination of rational modes for operating equipment are based on the calculations of constrained particle settling velocity. However, the problem of determining the speed of constrained movement of grains in the mineral water suspension of fly ash from TPPs has not been studied sufficiently.

Analyzing the process of fly ash particle settling in a water suspension, we will consider only coal and quartz particles. Firstly, this is because the feedstock has a high content of carbon and quartz (Tables 1, 2).

Secondly, it has been repeatedly noted in the literature that TPP ash can serve as a raw material for the production of aluminum, iron, and rare metals [2, 4, 7]. For example, in [6], a complex technology for the enrichment of ash and slag to produce aluminum and vanadium is proposed. From Table 2 it can be seen that the fly ash has a high content of Al$_2$O$_3$. These data show that it is possible to obtain a concentrate with a conditioned content of aluminum oxide of 25–28 %. However, in order for such a concentrate to be suitable for the production

Table 1

Analysis of fly ash from the Novo-Kramatorska TPP

Size, mm	Class output, %	Content of *C*, %
+0.25, max 4–5	26.0	11.33
−0.25 + 0.1	10.8	34.43
−0.1 + 0.05	18.17	31.96
−0.05	45.03	14.53
Total	100	19.01

Table 2

Components of fly ash (to determine the weighted average density of solid particles)

Components	Output, %	Density, g/cm^3	Ratio of total density, g/cm^3
C	19.01	1.5	0.284
SiO_2	40.45	2.65	1.072
Al_2O_3	19.45	3.7	0.720
Fe_2O_3	13.37	5.0	0.669
CaO	2.59	3.34	0.087
MgO	1.22	3.58	0.044
K_2O	2.51	2.32	0.058
Na_2O	0.89	2.27	0.020
TiO_2	0.57	4.23	0.024
Total	100.0	–	2.98

of aluminum, it must have the lowest possible silica SiO_2 content. For the optimal removal of silica, it is necessary to study the issue of constrained settling of quartz.

Unsolved aspects of the problem. When separated in hydraulic devices, the water suspension of fly ash can have a density from 1.3 up to 1.8 g/cm^3. In most cases, the settling of particles in the working area of hydraulic devices is constrained and it is not appropriate to use the laminar flow model and Stokes' law. The flow should be considered weak-turbulent and both laminar and turbulent speed components should be taken into account. The scientific literature has not found solutions to the problem of constrained settling of fly ash particles taking into account the turbulent velocity component. For this task, first, we need to determine the most appropriate equation for calculating the speed. Secondly, since any known equation for speed includes the characteristics of the suspension (viscosity, porosity, and so on), it is necessary to determine the formulas for calculating the characteristics, taking into account the fact that they change with the change in the operating mode of the hydraulic apparatus. Based on this, it will be possible to analyze the processes of constrained settling and ascent of ash suspension particles and obtain correlations for estimating the speed of constrained particle motion that are suitable for design and engineering calculations.

Literature review. Many dependencies have been proposed for calculating the speed of constrained settle (sedimentation), but almost all of them are applicable only for relatively narrow ranges of flow modes that need to be determined beforehand.

The most developed model is the laminar, Stokes flow model for the flow around particles without interrupting the flow. It was used to calculate the speed of settling in early works: R. T. Hancock (1937), Gaudin (1946), D. M. Mints (1955) and others. When studying hydrodynamic processes in coal pulps, this model was developed in the works by A. D. Polulyakh [5], and was reflected in the dissertations by E. I. Nazimko (1997), V. D. Melnichuk (1997), A. P. Levandovich (1996), and others. However, when coal pulp is enriched, the laminar mode is rarely implemented.

For the laminar-turbulent transition zone, complex models are used, for example, the model of a non-equilibrium phase transition whose mechanism is a diffuse bundle [8]. However, this is a narrow zone of the current transition regime, and it is more of academic interest.

In [9], the choice of equations was carried out that most likely describe the processes of settling and floating of solid particles in a turbulent flow. However, this mode is rarely used for particle separation. More often, hydraulic devices implement a weakly-turbulent mode, since the movement of particles occurs in a constrained medium. To describe such a regime, along with the experimental interpolation formulas given, for example, in the monograph by J. Happel and G. Brenner (1976), the equation of Ergun (1952) is known, which is the most theoretically justified.

Also known are the formulas of M. E. Aerov (1955), O. M. Todes (1968), A. N. Planovsky (1967), and a number of others that relate to the region of particle motion in a dense granular layer. Such a layer has a regular packing of particles, usually cubic or rhombohedral, where the porosity (lumen) of the medium is a constant. Such formulas are not suitable for dilute suspensions where the dense packing of particles is disturbed. For example, calculations show that with a decrease in the suspension density, the formula of V. D. Goroshko, R. B. Rosenbaum, and O. M. Todes (1958), better known as the Todes-Rosenbaum formula, gives a discrepancy with the Ergun formula of up to 30 %.

Thus, the analysis of literature sources has shown that for weakly-turbulent flows that are implemented in hydraulic devices, it is advisable to use the S. Ergun formula, which covers the widest range of flow modes. It is obtained on the basis of a formal unification of the laws of linear (Darcy) and nonlinear (A. A. Krasnopolsky, Darcy-Weisbach) filtration and in the final expression has the form

$$150\cdot\omega\cdot\frac{(1-\varepsilon)\cdot\nu}{d^2}+\frac{1.75\cdot\omega^2}{d}=g\cdot\varepsilon^3\cdot\frac{\rho_{\grave{o}}-\rho_s}{\rho_s}, \qquad (1)$$

where ω is velocity; *d* is particle size (equivalent diameter); ρ_m is particle density; ρ_s is medium density; ν is kinematic viscosity; ε is porosity (clearness) of the suspension; $g = 981$ cm/s^2.

In the works by V. G. Ainstein (1967), it is noted that in the formula (1), the Kozeni-Karman formula is used to determine the filtration coefficients, as well as the Y. Kozeni (1927) hypothesis for the transition from a cellular model to a capillary model, numerical values are obtained by analyzing many experimental data.

The first summand of the left part of equation (1) reflects the influence of viscosity forces (laminar component), the second – inertia forces (turbulent). The Ergun equation shows that as the diameter decreases, both summands grow, but the first one grows faster, that is the viscosity forces and the laminar component prevail. When the speed decreases, both summands decrease, but the first one decreases more slowly, and the laminar component also prevails. That is, when ω and *d* decrease, and hence the Reynolds numbers Re, the flow goes into a laminar mode. Accordingly, with increasing ω and *d*, it becomes turbulent.

These processes for particles of water suspension of fly ash from TPPs have not been studied previously. The task was to determine the auxiliary values included in the Ergun equation and, based on it, to analyze the hindered settling of fly ash particles in water mineral suspensions of different densities.

Purpose. To establish analytical dependences for calculating the characteristics of the ash suspension and the speed of constrained settling of coal and quartz depending on the particle size and density of the medium, what is necessary for calculating the design and determining the operating modes of hydraulic devices for extracting coal from water mineral suspension of fly ash from thermal power plants.

Methods. To solve the quadratic equation (1) with respect to ω or d, we need to determine the auxiliary values: the porosity ε and the viscosity ν. As shown in [10, 11] associated values involve a number of other characteristics of the slurry. As the density of solid particles ρ_m, we will take the weighted average value of the density of the components included in the fly ash (by the sulfate-free mass). From Table 2 it can be seen that it was $\rho_m = 2.98$ g/cm^3. The density of the medium ρ_s is the volume density determined by weighing the suspension sample in a container of a known volume. Along with the volume density of the ρ_s, the weight density or percentage of solid θ is used in practice. It is the ratio of the weight of the dry residue to the weight of the wet sample.

In [11], it is proposed to determine the characteristics θ (%), ν (cm^2/s), ε (item) as functions of only one argument – the suspension density ρ_s; the following equations are also given

$$\theta=\frac{V_m\cdot\rho_m}{V_m\cdot\rho_m+V_l\cdot\rho_l}\cdot100;\quad \rho_s=\frac{1}{1-\frac{\theta}{100}\left(1-\frac{1}{\rho_m}\right)};$$
$$\varepsilon=1-\frac{\rho_s}{\rho_m}\cdot\frac{\theta}{100};\quad \nu=\nu_0\exp\frac{2.5\cdot\beta+0.67\cdot\beta^2}{1-0.609\cdot\beta}, \tag{2}$$

where V_m, V_l and ρ_m, ρ_l are volume and density of solid and liquid phases, respectively; ρ_s is the density of suspension; ρ_m is the density of solid particles; $\beta = 1 - \varepsilon$; $\nu_0 = 0.01$ cm^2/s – kinematic viscosity of water at 20°.

In contrast to [11], we will slightly change the method for calculating the characteristics of the suspension, and also provide a more convenient formula for calculating θ. To calculate the kinematic viscosity, as in [11], we use the Wend formula (1948).

The method for determining auxiliary values is that for a given density of solid particles ρ_m, we will set and variate the value of the suspension density ρ_c. For each value of ρ_s, first, we determine θ, then ε and $\beta = 1 - \varepsilon$, and then v using the following formulas

$$\theta=\frac{100\cdot\rho_{\grave{o}}\cdot(\rho_s-1)}{\rho_{\grave{o}}\cdot(\rho\quad-1)}\cdot100;\quad \varepsilon=1-\frac{\rho_{\tilde{n}}}{\underset{\grave{o}}{}\ \rho}\cdot\frac{\theta}{100};$$
$$\nu=\nu_0\exp\frac{2.5\cdot\beta+0.67\cdot\beta^2}{1-0.609\cdot\beta}. \tag{3}$$

Let us define the features of using the Ergun formula (1). Denote respectively for coal and quartz

$$\Delta_c=\frac{\rho_c-\rho_s}{\rho_s}\quad\text{and}\quad\Delta_q=\frac{\rho_q-\rho_s}{\rho_s}.$$

Then the velocity of constrained settling of quartz particles is

$$\omega_q^2\cdot d_q+\omega_q\cdot85.71\cdot(1-\varepsilon)\cdot\nu-560.57\cdot d_q^2\cdot\varepsilon^3\cdot\Delta_q=0. \tag{4}$$

In equation (4), the value Δ_q is always positive, since the density of the suspension ρ_s is less than the density of quartz particles $\rho_q = 2.65$ g/cm^3. So, for the most dense rhombohedral packing of particles, when the porosity is minimal ε = 0.259, for an ash suspension (at $\rho_m = 2.98$ g/cm^3), we get $\rho_s = \rho_m(1 - \varepsilon)$ and $\rho_s = 2.47$ g/cm^3. This value is less than the density of quartz. For a cellular model, a higher density of ash suspension is not feasible.

The speed of constrained movement of coal particles is determined by the equation

$$\omega_c^2 d_c+\omega_c\cdot85.71\cdot(1-\varepsilon)\cdot\nu-560.57\cdot d_c^2\cdot\varepsilon^3\left|\Delta_c\right|=0. \tag{5}$$

A special feature of the formula (5) is that the value of the *Δ*$_c$ can be either positive or negative, that is, both settling and lifting of coal are realized. The density of coal is 1.5 g/cm^3. If the suspension density is less than 1.5 g/cm^3, then $\Delta_c > 0$, coal is settling. This situation is similar to the deposition of quartz according to equation (4).

But if the density of the suspension is higher than the density of coal, $\rho_s > \rho_c$, then the value of Δ_c is negative. This is accompanied by the ascent of coal in practice. However, for $\Delta_c < 0$, the quadratic equation (5) has two roots and both of them are negative, which does not reflect the physics of the process.

Given this, in formula (5) we will use the modulus of the Δ_c value, and calculate the velocity as follows. We calculate the Δ_c and fix (remember) the sign at the Δ_c. Next, take Δ_c module and calculate the velocity ω by the formula (5). If the sign in the module body was negative, then the calculation result will be the ascent speed, if positive, then the settling (sedimentation) velocity.

Note that equations (4, 5) describe the process of gravitational settling of particles, which is observed during simple sedimentation of the suspension. In hydraulic devices, it is very common to artificially create an additional movement of the liquid phase in a direct or counter-flow with precipitating particles. Consideration of the speed of such an additional flow will be considered below after the analysis of gravitational settling. Calculations were performed in the Mc. Excel program, which is freely available.

Results. The constrained settling of quartz. The task was to construct the dependences of $\omega_q = f(d_q)$ and $d_q = f(\omega_q)$ for different densities of the ash suspension ρ_s using formulas (3, 4) and obtain their approximating functions. Dimension of values: d_q – mm, ω_q – mm/s.

It is found that the dependences $\omega_q = f(d_q)$ are poorly approximated by even high-degree polynomials. Therefore, each

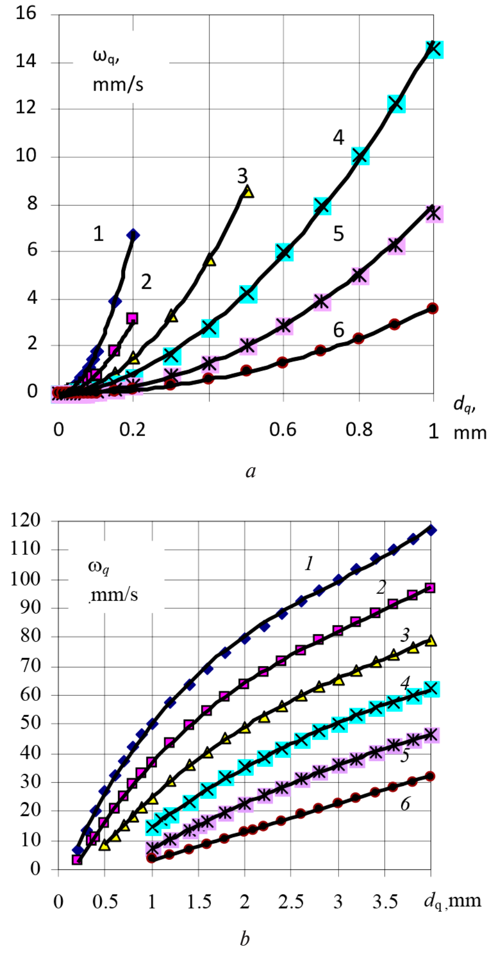


Fig. 1. Constrained settling speed of fine (*a*) *and coarse* (*b*) *quartz at different suspension densities:*
ρ_s, *g/cm*^{3}: *1 – 1.3; 2 – 1.4; 3 – 1.5; 4 – 1.6; 5 – 1.7; 6 – 1.8*

of them was divided into two parts – separately for small and large classes, up to 4 mm inclusive (Fig. 1).

The range of splitting of each dependence $\omega_q = f(d_q)$ by size was refined in the process of checking the accuracy of the approximating equations (Table 3).

According to Table 3, it can be seen that for small classes, the equations have the form of Stokes law: $\omega \sim k \cdot d^2$. However, the k coefficients for constrained and free settling differ significantly. According to Stokes law, the free settling speed is defined as $\omega = 54.481 \cdot \nu^{-1} \cdot ((\rho_q - \rho_s)/\rho_s) \cdot d^2$, where d is cm, ω is cm/s. Calculations show that for fine quartz in a medium with the same characteristics ν and ε at $\rho_s = 1.3$; 1.4; 1.5; 1.6; 1.7 g/cm^3 the velocity of constrained settling is less than that of free settling by 20.8; 33.2; 50.7; 75.6; 110.8 times, respectively. The ratio of these velocities increases with increasing density of the medium according to the law of the quadratic parabola.

It follows that the calculation of hydraulic processes taking into account only the laminar regime and the Stokes law is limited. The more dilute the suspension, the smaller the particle size, the movement of which obeys the Stokes law and the smaller the range of size that it covers. For example, at medium density of 1.3, 1.4 g/cm^3, for quartz particles this range is only 0–0.2 mm (Table 3). For larger particles – the accuracy of the speed approximation by the Stokes law is significantly reduced, and the turbulent component must be taken into account. In this case, outside the range of 0–0.2 mm, these equations have the form of third-order polynomials.

Consider the dependencies $d_q = f(\omega_q)$. Table 3 shows that for small classes they have the form $d_q \sim f(\omega_q)^{0.5}$. For large classes the equations are obtained by constructing the inverse dependencies shown in Fig. 1, *b*. Approximating functions of the dependencies $d_q = f(\omega_q)$ for small and large classes are shown in Table 4.

The equations in Tables 3 and 4 have a high accuracy of approximation, which is estimated by the square of the correlation coefficient R^2. In comparison with the solution of quadratic equations of the form (4), they are convenient to use when solving problems of designing equipment for hydraulic separation of ash suspensions.

Constrained ascent and settling of coal. When calculating, we take the density of the coal particle 1.5 g/cm^3. Using equations (5, 3), the dependences of the velocity of constrained ascent (Fig. 2, *a*) and settling (Fig. 2, *b*) of coal with a size of 1.5; 1.0 and 0.5 mm on the density of the medium are obtained.

Fig. 2, *a* shows that the larger the coal particle is, the faster it floats to the surface, and when the medium density is less than the density of coal (Fig. 2, *b*), the faster it is settling. It is characteristically that for any size, the dependence of the velocity of coal ascent on the density of the medium is of extreme nature. The optimal range of suspension density for coal ascent (Fig. 2, *a*) is 1.55–1.8 g/cm^3 with a maximum of 1.65 g/cm^3.

Table 3

Approximating functions of the dependency
of the constrained settling velocity of quartz from the size (Fig.1)

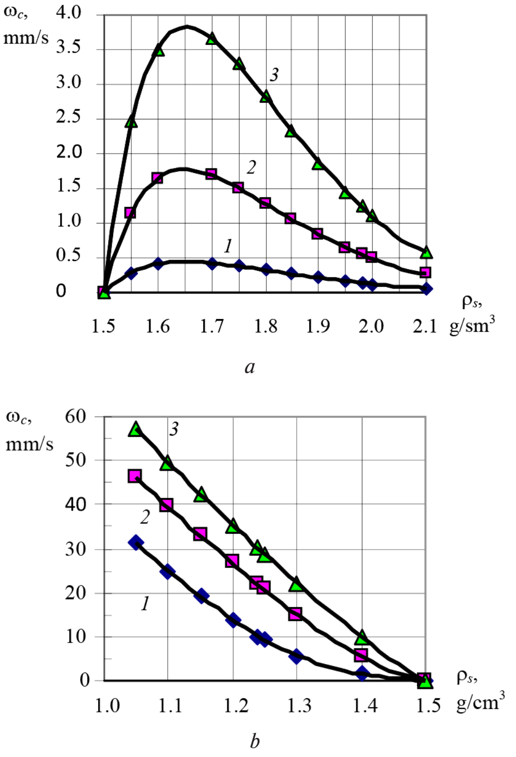
ρ_s, g/cm^3	$d \le 0.2$	$0.2 < d < 4.0$
1.3	$\omega_q = 176 \cdot d^2$	$\omega_q = 1.4573d^3 - 14.058d^2 + 62.26d + 0.0858$, $R^2 = 0.9997$
1.4	$\omega_q = 79.8 \cdot d^2$	$\omega_q = 1.2546d^3 - 12.47d^2 + 56.319d - 8.595$, $R^2 = 0.9998$
	$d \le 0.5$	$0.5 < d < 4.0$
1.5	$\omega_q = 37.1 \cdot d^2$	$\omega_q = 0.7047d^3 - 8.054d^2 + 43.745d - 11.67$, $R^2 = 0.9999$
1.6	$\omega_q = 17.3 \cdot d^2$	$\omega_q = -2.2425d^2 + 26.91d - 9.8744$, $R^2 = 0.9998$
	$d \le 1.0$	$1.0 < d < 4.0$
1.7	$\omega_q = 8 \cdot d^2$	$\omega_q = -1.1717d^2 + 19.023d - 10.666$, $R^2 = 0.9999$
1.8	$\omega_\mathrm{q} = 3.7 \cdot d^2$	$\omega_q = 0.0081d^2 + 9.6935d - 6.6314$, $R^2 = 0.9993$

Table 4

Approximating functions of dependences of quartz particle size on the constrained settling velocity

ρ_s, g/cm^3	$d \le 0.2$	$0.2 < d < 4.0$
1.3	$d_q = (\omega/176)^{0.5}$	$d_q = 0.0003\omega^2 + 0.002\omega + 0.2353$, $R^2 = 0.9997$
1.4	$d_q = (\omega/79.8)^{0.5}$	$d_q = 0.0003\omega^2 + 0.0073\omega + 0.2725$, $R^2 = 0.9993$
	$d \le 0.5$	$0.5 < d < 4.0$
1.5	$d_q = (\omega/37.1)^{0.5}$	$d_q = 0.0004\omega^2 + 0.0118\omega + 0.4212$, $R^2 = 0.9996$
	$d \le 1.0$	$1.0 < d < 4.0$
1.6	$d_\mathrm{q} = (\omega/17.3)^{0.5}$	$d_q = 0.0006\omega^2 + 0.0194\omega + 0.6216$, $R^2 = 0.9999$
1.7	$d_q = (\omega/8)^{0.5}$	$d_q = 0.0005\omega^2 + 0.0475\omega + 0.6505$, $R^2 = 0.9997$
1.8	$d_q = (\omega/3.7)^{0.5}$	$d_q = 0.1027\omega + 0.6871$, $R^2 = 0.9993$

The analysis of the speed of coal particles is performed similarly to the previous one for quartz. Fig. 3 shows the results of calculating the functions $\omega_c = f(d_c)$ and $d_c = f(\omega_c)$ for the coal size of 0.001–4.0 mm using equations (3, 5).



a

b

Fig. 2. The velocity of constrained movement of coal particles depends on the density of the medium:

a – ascent; b – settling; 1 – 0.5 mm; 2 – 1.0 mm; 3 – 1.5 mm

Dependences *1* and *2* of Fig. 3 relate to the sedimentation of coal, $\rho_s < 1.5$ g/cm^3. They show that the settling velocity increases with increasing size. At $\rho_s > 1.5$ g/cm^3, coal ascends (these dependences are not indicated), and the larger the par-

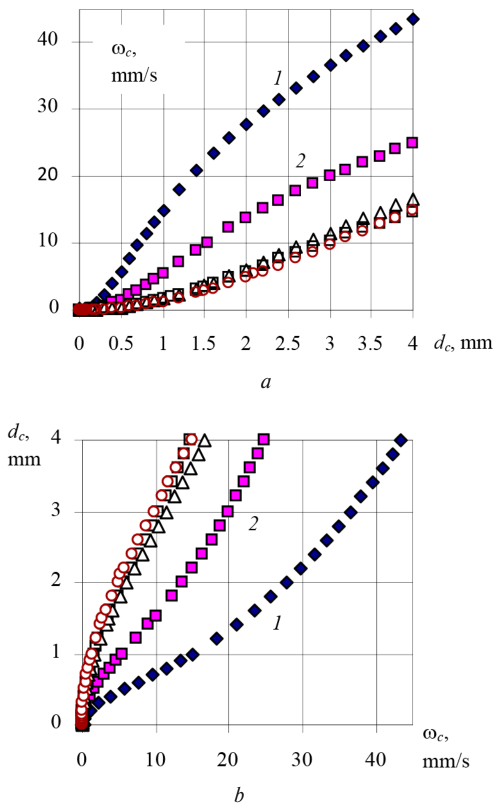


Fig. 3. Dependences of the constrained movement of coal with a size of 0 ÷ 4 *mm at different medium densities:*

a — $\omega_c = f(d_c)$; *b* — $d_c = f(\omega_c)$; *settling*, ρ_s : *1* — 1.3 *g/cm*^{3}; *2* — 1.4 *g/cm*^{3}; *ascend* (*is not indicated*) $\rho_s = 1.6$; 1.7; 1.8 (*g/cm*^{3})

ticle is, the higher the ascend speed. A special feature is that for dense suspensions of 1.6–1.8 g/cm^3, the difference in ascend velocities is small. For example, for coal with a size of 1.5 mm at ρ_s = 1.6; 1.7; 1.8 g/cm^3, the ascend velocity is 3.5; 3.66; 2.83 mm/s, respectively.

To improve the accuracy of the approximation, each of the dependencies in Fig. 3 was divided into two parts - separately for small and large classes, the corresponding equations are given in Tables 5 and 6.

Table 5

Dependence of the speed of the constrained movement of coal on the size (Fig. 3, *a*)

The speed of coal settling		
ρ_s g/cm^3	$d \le 0.2$	$0.2 < d < 4.0$
1.3	$\omega_c = 26.2 \cdot d^2$	$\omega_c = 0.4965d^3 - 5.2027d^2 + 25.157d - 5.5446$, $R^2 = 0.9999$
	$d \le 0.5$	$0.5 < d < 4.0$
1.4	$\omega_c = 6.4 \cdot d^2$	$\omega_c = -0.0579d^3 - 0.3919d^2 + 9.5218d - 3.4223$, $R^2 = 0.9997$
The velocity of coal ascent		
ρ_s, g/cm^3	$d \le 1.0$	$1.0 < d < 4.0$
1.6	$\omega_c = 1.67 \cdot d^2$	$\omega_c = 0.014d^2 + 4.385d - 2.9651$, $R^2 = 0.9995$
1.7	$\omega_c = 1.69 \cdot d^2$	$\omega_c = 0.154d_c^2 + 4.3959d_c - 3.1573$, $R^2 = 0.9992$
1.8	$\omega_c = 1.29 \cdot d^2$	$\omega_c = 0.387d^2 + 2.7295d - 2.0421$, $R^2 = 0.9993$

Table 6

Dependence of coal size on the speed of constrained movement (Fig. 3. *b*)

The speed of coal settling		
ρ_s, g/cm^3	$d \le 0.2$	$0.2 < d < 4.0$
1.3	$d_c = (\omega_c/26.2)^{0.5}$	$d_c = 0.0016\omega^2 + 0.0137\omega + 0.4139$, $R^2 = 0.9997$
	$d \le 0.5$	$0.5 < d < 4.0$
1.4	$d_c = (\omega_c/6.4)^{0.5}$	$d_c = 0.0025\omega^2 + 0.0801\omega + 0.445$, $R^2 = 0.9992$
The speed of coal ascent		
ρ_s, g/cm^3	$d \le 1.0$	$1.0 < d < 4.0$
1.6	$d_c = (\omega_c/1.67)^{0.5}$	$d_c = 0.2231\omega + 0.696$, $R^2 = 0.9995$
1.7	$d_c = (\omega_c/1.69)^{0.5}$	$d_c = 0.1912\omega + 0.8$, $R^2 = 0.9994$
1.8	$d_c = (\omega_c/1.29)^{0.5}$	$d_c = 0.2085\omega + 0.9447$, $R^2 = 0.9967$

Analysis of the data in Tables 5 and 6 shows that the patterns obtained earlier for quartz are confirmed for coal. Thus, Table 5 shows that for small coal particles, the equation for velocity has the form of Stokes law. However, the proportionality coefficient for d^2 is much less when the movement is constrained than when it is free: for ρ_s = 1.3; 1.4; 1.6; 1.7 g/cm^3, respectively, 20.7; 33.1; 74.6; 110.4 times.

This suggests that, in contrast to Stokes settling, the Ergun velocity calculation takes into account the tightness in the medium and the energy dissipation for particle collisions, which is relevant for weak-turbulent flows.

In the entire range of coal size, both for settling and for ascent, there is a direct relationship between the velocity and size, which is nonlinear in both cases. The more diluted the suspension, the smaller the size range described by Stokes' law. For example, when the medium density is 1.3 g/cm^3 for coal particles, this range is up to 0.2 mm (Table 5).

With an increase in size, in this case above 0.2 mm, the movement pattern differs significantly from the Stokes one. As can be seen from Table 5, it can be described by polynomials of the *2nd* and *3rd* degree. Moreover, if for liquid suspensions 1.3, 1.4 g/cm^3 approximation is performed by third-degree polynomials, then for dense suspensions 1.6–1.8 g/cm^3 by second-degree polynomials.

It follows that taking into account the turbulent component, when calculating the velocity has the form of Stokes law of constrained movement, leads to the fact that with increasing size and (or) density of the medium, the nature of movement changes. Newtonian flow around particles with separation of the flow begins to prevail. In the limit, for Reynolds numbers above 3000, the motion passes into a turbulent region, where it will obey the Newton-Rittinger law with a relatively weak power-law relationship between speed and size $\omega \sim d^{0.5}$.

The established dependences in Figs. 1–3 and the obtained equations in Tables 3–6 describe the process of gravitational settling of particles during simple sedimentation of the suspension. However, this process is not often used in hydraulic devices. Settling is much more widely used when the liquid phase moves simultaneously in a direct- flow or counter- flow with the settling particles.

In counter-flow, the vertical speed component is additionally superimposed with the upstream velocity *V*. It is determined by the operating mode and design of the hydraulic apparatus. The particle size at $\omega = V$ is a boundary value. At this size, the particle is suspended in a suspension, the larger ones settle, the smaller ones ascend.

The results obtained above allow us to determine the boundary particle size from a given upstream velocity and vice versa. Here is an example of such a calculation for the vertical velocity component.

In a suspension with a density of 1.4 g/cm^3, all coal particles will be settled (deposited) because coal density of 1.5 g/cm^3. For example, a 0.1 mm coal particle will be deposited at a speed of 0.064 mm/s.

If in the device creates an upward flow at a velocity of $V =$ $= 10$ mm/s, then the velocity of a particle with a size of 0.1 mm will be $|\omega_c - V| = |0.064 - 10| = 9.94$ mm/s. Since the sign in the module body is negative, this will be the ascent velocity.

At $\omega_c = V$, the boundary size of coal particles is 1.5 mm. As noted above, there is a direct relationship between velocity and size. This means that all coal particles smaller than 1.5 mm, including 0.1 mm, will ascend and larger ones will be settled.

The boundary size of quartz at $\omega_q = V = 10$ mm/s is 0.37 mm. Larger particles will settle, smaller ones will ascend.

Thus, in an ash suspension with a density of 1.4 g/cm^3 at an upward flow velocity of 10 mm/s, coal particles with a size of 1.5 mm or less and quartz particles with a size of 0.37 mm or less will ascend.

Conclusions. Millions of tons of waste are accumulated at the existing and developed ash and slag dumps of the coal-fired TPPs. Its processing will make it possible to obtain a number of valuable products (coal, oxides of iron, aluminum, titanium, and others), extend the operation period of dumps, reduce the cost of their maintenance, and improve the environmental situation of the areas where they are located. The content of unburned small coal fractions in fly ash is higher than in slags. Storage of fly ash is carried out separately from the slag by the wet method along hydraulic lines. Taking this into account, it is advisable to use hydraulic devices for extracting coal from a water ash suspension for primary processing of ash. Calculation of the design and determination of operating modes of such devices are based on determining the speed of constrained movement of particles in fly ash water suspension. For weak-turbulent flows in the working area of hydraulic devices, the question of determining the specified velocity, as well as the properties of the ash suspension, has not been studied sufficiently.

The aim of the work was to establish analytical dependences for calculating the characteristics of the ash suspension and the velocity of constrained settling of coal and quartz depending on the particle size and density of the medium, which is necessary for calculating the design and determining the operating modes of hydraulic devices for extracting coal from water mineral suspension of fly ash from thermal power plants.

When determining the velocity of constrained particle movement, it is necessary to know the characteristics of the suspension, such as viscosity, weight density, porosity or clearness. These characteristics were determined for the weighted average density of mineral particles that make up the water suspension of fly ash from the Novo-Kramatorska TPP. Here are the equations in which the characteristics depend on only one variable parameter – the volume density of the medium, which is easily measured in practice.

As a result of the analysis of literature sources, it is found that it is advisable to use the Ergun equation to calculate the velocity of constrained motion, since it is the most theoretically justified and covers the widest range of modes of flow around particles during settling.

Graphical dependences and correlation equations are obtained for direct and inverse functions of velocity of constrained motion on the size of coal and quartz particles with a size of up to 4 mm and movement in an ash suspension with a density of 1.3–1.8 g/cm^3. It is shown that during quartz settling and coal settling and ascent, there is a direct relationship between the velocity and size, in both cases it is nonlinear. For any size of coal, the velocity of ascent, depending on the density of the suspension, is extreme. The rational range of suspension density for coal ascent is 1.55–1.8 g/cm^3 with a maximum of 1.65 g/cm^3.

It was found that for very small particles of coal and quartz in an ash suspension, constrained settling is similar to Stokes, but with a much lower coefficient of proportionality between the velocity and the square of the equivalent diameter, approximately 20–100 times at a suspension density of 1.3–1.7 g/cm^3. At the same time, the more diluted the suspension is, the smaller the size of the particles whose motion obeys the Stokes law is and the smaller the range of size it covers.

The obtained dependences make it possible to determine the size of coal and quartz, taking into account the counterflow of the liquid phase, as well as the boundary size. Using the example of an ash suspension with a density of 1.4 g/cm^3, it is shown that the creation and control of the upward flow of the liquid phase is an effective tool for extracting small fractions of coal.

The results of determining the velocity of constrained particle motion in an ash suspension are the basis for calculations in the design of hydro-classifiers, hydro-separators, thickeners and other equipment.

Also, the research results are used to determine the rational technological modes of operation of hydraulic devices in the technology of complex processing of fly ash from thermal power plants.

The described approach can also be used to evaluate the characteristics of any other suspensions consisting of a liquid and solid dispersed phase and calculate the velocity of constrained particle motion in such suspensions of different densities.

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Аналітичні дослідження швидкості стисненого осадження частинок у водній суспензії золи виносу ТЕС

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Мета. Встановити аналітичні залежності для розрахунку характеристик зольної суспензії та швидкості туги осадження вугілля та кварцу в залежності від крупності частинок і щільності середовища, що необхідно для розрахунку конструкції й визначення режимів гідравлічних апаратів для добування вугілля з водної мінеральної суспензії золи виносу ТЕС.

Методика. Дослідження виконувалися на підставі чарункової моделі суспензії та класичних уявлень про обмежений рух частинок при ламінарному й турбулентному обтіканні. Для аналітичної оцінки характеристик суспензії використовувалися визначаючі співвідношення та формула Венда для в'язкості. Для розрахунку та аналізу швидкості стиснутого руху частинок зольної суспензії використовувалося рівняння Ергана й методи кореляційного аналізу.

Результати. Отримані апроксимуючі нелінійні функції для визначення швидкості стисненого руху частинок вугілля та кварцу розміром до 4 мм у зольній суспензії з густиною 1,3–1,8 г/см^3. Показано, що, як для осадження, так і для спливання вугілля має місце пряма залежність між швидкістю й розміром частинок, в обох випадках вона нелінійна. Для будь-якої крупності вугілля швидкість спливання в залежності від густини суспензії носить екстремальний характер, раціональний діапазон густини становить 1,55–1,8 г/см^3 при максимумі 1,65 г/см^3. Встановлені залежності дозволяють визначати крупність частинок вугілля та кварцу з урахуванням протитечії рідкої фази, а також граничну крупність.

Наукова новизна. Для режиму слабкотурбулентних течій встановлені залежності та апроксимаційні рівняння швидкості стисненого руху частинок вугілля та кварцу - основних компонентів водної суспензії золи виносу Ново-Краматорської ТЕС, у залежності від крупності й густини зольних суспензій з урахуванням зміни характеристик середовища в залежності від густини. Встановлені межі стоксівського опису процесів. Показано, що рух рідкої фази у протитечії з осаджуваними частинками є ефективним для спливання тонких класів вугілля.

Практична значимість. Викладений підхід можна застосовувати для аналітичної оцінки характеристик і швидкості стисненого руху в різних водних суспензіях дискретних твердих частинок при слабкотурбулентних течіях. Перевагою є більш широке охоплення режимів роботи гідравлічного обладнання. Отримані результати необхідні при проектуванні й визначенні технологічних режимів роботи різних гідравлічних апаратів у технології комплексної переробки золи виносу ТЕС.

Ключові слова: *мінеральна суспензія, зола виносу, густина, швидкість, осадження, спливання*

Аналитические исследования скорости стесненного осаждения частиц в водной суспензии золы уноса ТЭС

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Цель. Установить аналитические зависимости для расчета характеристик зольной суспензии и скорости стесненного осаждения угля и кварца в зависимости от крупности частиц и плотности среды, что необходимо для расчета конструкции и определения режимов работы гидравлических аппаратов для извлечения угля из водной минеральной суспензии золы уноса ТЭС.

Методика. Исследования выполнялись на основании ячеистой модели суспензии и классических представлений о стесненном движении частиц при ламинарном и турбулентном обтекании. Для аналитической оценки характеристик суспензии использовались определяющие соотношения и формула Вэнда для вязкости. Для расчета и анализа скорости стесненного движения частиц зольной суспензии использовалось уравнение Эргана и методы корреляционного анализа.

Результаты. Получены аппроксимирующие нелинейные функции для определения скорости стесненного движения частиц угля и кварца крупностью до 4 мм в зольной суспензии с плотностью 1,3–1,8 г/см^3. Показано, что, как для осаждения, так и для всплытия угля имеет место прямая зависимость между скоростью и крупностью частиц, в обоих случаях она нелинейная. Для любой крупности угля скорость всплытия в зависимости от плотности суспензии носит экстремальный характер, рациональный диапазон плотности 1,55–1,8 г/см^3 при максимуме 1,65 г/см^3. Установленные зависимости позволяют определять крупность частиц угля и кварца с учетом противотока жидкой фазы, а также граничную крупность.

Научная новизна. Для режима слаботурбулентных течений установлены зависимости и аппроксимационные уравнения скорости стесненного движения частиц угля и кварца - основных компонентов водной суспензии золы уноса Ново-Краматорской ТЭС, в зависимости от крупности и плотности зольных суспензий с учетом изменения характеристик среды в зависимости от плотности. Установлены границы стоксовского описания процессов. Показано, что движение жидкой фазы в противотоке с осаждающимися частицами является эффективным для всплытия тонких классов угля.

Практическая значимость. Изложенный подход можно применять для аналитический оценки характеристик и скорости стесненного движения в различных водных суспензиях дискретных твердых частиц при слаботурбулентных потоках. Преимуществом является более широкий охват режимов работы гидравлического оборудования. Полученные результаты необходимы при проектировании и определении технологических режимов работы различных гидравлических аппаратов в технологии комплексной переработки золы уноса ТЕС.

Ключевые слова: *минеральная суспензия, зола уноса, плотность, скорость, осаждение, всплытие*

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