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MATHEMATICAL MODEL OF OSCILLATIONS OF A DRILL TOOL WITH A DRILL BIT OF CUTTING-SCRAPING TYPE

Purpose. Finding the cause and effect relationships between the parameters of non-stationary longitudinal and torsional vibrations of a drilling tool and developing a mathematical model to study their properties.

Methodology. The solution to this problem is based on one of the basic principles of analytical mechanics – the Lagrange principle. The equation of motion of the studied rod mechanical system with four degrees of freedom is made using the Lagrange equation of the second kind, which is one of the next stages of its dynamic analysis.

Findings. In the process of solving this problem on the basis of industrial data, the analysis of axial load functions, angular rotation speed and torque of the drilling tool for their non-stationarity was carried out. The frequency of change in these non-stationary drilling mode functions is set by rapid Fourier transform.

Originality. For the first time, analytical dependences were obtained to determine the forces of cutting and friction on the bit from the second derivatives of the independent generalized coordinates of the mechanical system over time, as well as the relationship between the generalized accelerations of the translational and rotational motions of the bodies of the mechanical system.

Practical value. The practical value of the work is the obtained equations of motion of the system, taking into account the received frequency compositions of the mode parameters of drilling until their subsequent numerical solution. The results of this task will further facilitate the choice of optimal modes of dynamic loading of the drilling tool in order to improve its energy efficiency and reliability. Analysis of the frequency dependences of the change in non-stationary functions of the drilling mode parameters will allow evaluating the level of wear of the rock destruction tool, as well as to predict the durability of the drill string elements.

Keywords: drill string, bit, oscillation, amplitude, frequency

Introduction. The current state of development of the oil and gas industry is updating the development of new approaches for the intensification of drilling of oil and gas wells with the use of drilling and cutting tools, including Polycrystalline Diamond Compact (PDC) bits. This is due to the following important factors:

- a significant expansion of the use of cutting tools;
- the lack of accurate and objective methods for assessing the effectiveness of the destruction of rocks with bits of this type;
- absence of scientifically substantiated principles of the process of dynamic loading of a drilling tool with bits of cutting-chipping action, operating in the mode of non-stationary oscillations;
- the absence of a generalized criterion for evaluating the effectiveness of the dynamic process of destruction of rocks by bits of cutting-chipping action.

These factors determine the formulation of a set of theoretical and experimental studies of the dynamics and kinematics of a drilling tool with bits of cutting and chipping action. Studies of non-stationary oscillation processes of such a drilling tool are of great interest in terms of ensuring its reliability, as well as reducing energy costs in deepening the well.

Literature review. Currently, the proportion of deep wells drilled using PDC bits is increasing. At the same time, the problem of the negative impact on the drilling tool of vibrations arising in the process of destruction of rocks is unsolved. The researchers distinguish three main types of oscillations of the drill string: axial, rotational and transverse. Due to the different nature of the origin, each type has its own set of characteristics.

Because of this, it is not possible to objectively determine the nature of the pothole process and to take measures to avoid accidents unless it is a complex manifestation of several types of oscillations. In these circumstances, it is difficult to predict what measures will lead to a positive outcome.

To clarify the parameters of the load and stress-strain state of the drilling tool during non-stationary oscillations, it is necessary to have a clear methodology for evaluating the characteristic features of the movement of its elements. Therefore, the study of the parameters of non-stationary oscillations during rotation is an urgent problem of the theory of dynamic stability of elastic systems. The solution to this problem is necessary for the correct analysis of the drilling tool in rotary and combined drilling methods.

In this case, the drilling tool is a fairly complex mechanical system with distributed parameters. Depending on the magnitudes of the external and internal loads, as well as the conditions of contact interaction with the face, phenomena of a non-stationary nature, which are characteristic of classical elastic rods, occur in the column or on its individual parts. In addition, the drilling tool is exposed to friction and contact forces, inertia forces, and others. These phenomena have a negative effect on the elements of the drill string and the bit and generally lead to energy losses [1, 2] and a decrease in the technical and economic indices of drilling [3, 4].

It should be noted that in the study of the processes of rotation of the bent parts of the drill string there are some difficulties due to the lack of accurate expressions for the evaluation of the moment of inertia of the bent pipe on the parameters of its deformation. It is difficult to assess the dynamic stability and compliance of the drill string under conditions of unsteady vibration loading [5, 6].

Features of approaches to the dynamic analysis of the drill string depend on the shape of its elastic equilibrium with longitudinal and torsional vibrations. In [7], an analogy between the phenomena of loss of stability in the compression of long rods and the achievement of critical rotational speeds of flexible shafts is revealed, as well as approaches to the practices of avoiding these dangerous states. According to the results of the research, methods for creating non-critical rotors, non-resonant structures and rods that do not lose compression resistance are proposed. The authors of [8, 9] consider the reduction of vibration of the drilling tool as a key to maintaining the dynamic stability of the drill string as a whole.

The modeling of contact interaction in shell-core systems under nonmonotonic loading was carried out in [10, 11] to determine the strength, stiffness and damping ability of these systems, and in [12] to study the process of rock fracture using a pulse washing mode.

The contact interaction of drill string elements with the borehole wall is a key factor that determines the energy intensity of the drilling process as they are rotated. In this direction, the questions of mathematical modeling of the statics and dynamics of the core systems with respect to the problems of elimination of drill string grips remain [13, 14], as well as the processes of drilling fluid circulation [15]. The refinement and analysis of their mathematical models, taking into account the work of the cutting-chisel type with a resilient or inelastic medium, are necessary to ensure its long-term and safe operation as a long-stemmed mechanical system.

Despite the variety of mechanical and mathematical solutions, all known models do not take into account changes in the dynamic characteristics of the drill string due to the damping of pipes. In fact, taking into account these characteristics is possible by establishing and investigating analytical dependencies to determine the laws of motion of sections of the drill string with non-stationary oscillations.

Unsolved aspects of the problem. The models of drilling tool dynamics [1–3, 13, 14] are considered to be a kind of a single complete mathematical model, taking into account a number of certain assumptions. Therefore, analyzing the operation of the drilling tool in the well, the causes and occurrence of its non-stationary random oscillations can be formulated on the basis of the following considerations.

First, non-stationary perturbations on the bit can occur as a result of the destruction of rocks, in which alternately layers of different hardness occur. Their alternate drilling is accompanied by non-stationary vibrations of the tool, which significantly affect the change in axial load on the bit, torque and speed. Examples of non-stationary perturbation are the following phenomena: "unloading" of the bit when it enters the soft layer, "loading" – when it goes from soft to hard layer, as well as the phenomenon of sharp increase in the amplitude of torsional vibrations at the moment of slipping.

Secondly, as a result of the destruction with the bit by the cutting-chipping type, the thickness of the rock of the same hardness on the face produces workings with characteristic grooves, the number of which is basically a multiple of the number of cutting elements of the bit. However, as confirmed by the results of industrial research, in the process of breaking the rocks of different hardness for a long period of time, an atypical well is formed, chaotic in sequence and the size of the hollows and peaks of the profile. Moving the cutting elements along such a hollow can in the future be the root cause of long chaotic oscillations of both the bit and the lower part of the drill string.

Therefore, some key issues relating to the individual cases of non-stationary random oscillations of the drilling tool are unexplained, and, at the same time, both analytical and numerical methods for analyzing these oscillations in general are insufficiently developed.

Purpose. The purpose of the article is to develop a mathematical model for the investigation in bench conditions of the parameters of longitudinal and torsional vibrations of a drilling tool equipped with bits of cutting-chipping type. To achieve this goal it is necessary to solve the following problems:

- to analyze the functions of axial loading, torque and rotational speed of a drilling tool equipped with bits of cutting-chipping type and to establish the presence of their non-stationarity within specific time intervals;
- establish an analytical relationship between the parameters of longitudinal and torsional oscillations;
- estimate the frequency composition of oscillatory processes.

Results. At present, the results of studies [2, 3, 16] established that the condition of the emergence of longitudinal self-oscillations caused by torsional ones is the eigenfrequency of any harmonics of torsional and longitudinal oscillations. In this regard, it is possible to model the process of longitudinal and torsional vibrations of a drilling tool, as a mechanical system with concentrated parameters. A system of this type was successfully used in [1] to study nonlinear longitudinal and torsional vibrations of PDC bits with uneven distribution of cutting elements and formation of holes in the destruction of anisotropic rocks in bench conditions.

Theoretical justifications as for expediency require the development of new conceptual models of longitudinal and torsional vibrations of a drilling tool armed with these bits. Therefore, for the purpose of further analytical studies, we consider a drilling tool consisting of a drill pipe and a bit that perform two relative movements. To develop a model of oscillatory processes, we make some assumptions: the chisel and the drill pipe are fed as concentrated masses m_1, m_2 with moments of inertia J_1 and J_2 respectively; the mass and inertia of the spindle of the rotator is neglected.

Axial x_1, x_2, x_3 and angular φ_1, φ_2, φ_3 coordinates determine the position of the bodies of the system at any given time. The threaded connections of the bodies of the mechanical system are modeled by elastic-damping elements with rigidities k_1, k_2 and damping factors α_1 and α_2. The connection of the bit with the drill pipe is structurally made in the form of a Morse cone, which is modeled by rigidity c_1. Deformation of bodies, masses of springs and dampers, as well as changes in the energy of the oscillatory system and the action of the ejection force of the washing fluid are neglected. The power actuator is not modeled and is considered to provide constant angular and axial speed of the drilling tool. Therefore, the boundary conditions at the top of the drill tool will be the angular and axial displacements of the drill bit rotator spindle. Such assumptions are convenient for a greater ease of investigation of the oscillations of a given mechanical system.

In the process of drilling, an axial force acts on the mechanical system $\vec{P}$ and the reactive moment M. These forces have two components – cutting and friction. Based on [2, 3], they can be written as follows

$$P_c = zr\zeta\varepsilon h; \tag{1}$$

$$M_c = \frac{1}{2} zr^2 \varepsilon h; \tag{2}$$

$$P_{fr} = \frac{1}{2} zrl\sigma\left[1 + \operatorname{sign}(\dot{x}_1)\right]; \tag{3}$$

$$M_{fr} = \frac{1}{4} zr^2 \xi\mu l\sigma\left[1 + \operatorname{sign}(\dot{x}_1)\right], \tag{4}$$

where z – number of chisel cutters; r – bit radius; ε – the internal specific energy of cutting the rock; ζ – parameter that determines the position of the cutter; l – the length of the active (worn) part of the cutter; μ – coefficient of friction; ξ – parameter that characterizes the intensity of the cutters; h – the value of the recess with the cutter of the face.

In the presence of four relative motions, we impose an additional geometric restriction on the displacements and angles of rotation of the bit and the pipe

$$x_2 - x_1 = d(\varphi_2 - \varphi_1), \tag{5}$$

where d – a parameter that takes into account the structural features of the Morse cone. The deepening of the chisel (Fig. 1) is determined by the height of the rock before the cutter

$$h(t) = x_1(t) - x_1(t - t_n). \tag{6}$$

The considered mechanical oscillation system is idealized to conservative, and its motion can be described by differential Lagrange equations of the second kind

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial \Pi}{\partial q} = Q_j + P\lambda, \tag{7}$$

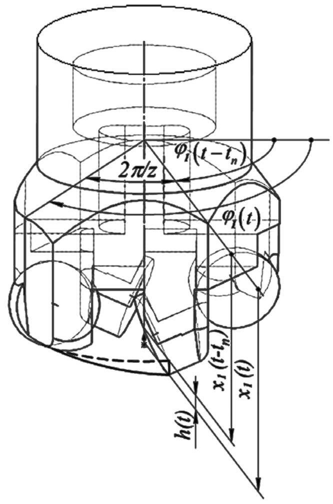


Fig. 1. Calculation chart of the bit

where T – kinetic energy of the mechanical system; $q = [x_1, x_2, \varphi_1, \varphi_2]$ – generalized coordinates; Q_j – generalized power factors of non-conservative origin; $P\lambda$ – generalized forces (bonding reactions); λ – the Lagrange multiplier. We present the kinetic energy of the mechanical system as the sum of their kinetic energies of its bodies, which perform reciprocating and rotational motions

$$T = \frac{m_1\dot{x}_1^2}{2} + \frac{m_2\dot{x}_2{}^2}{2} + \frac{J_1\dot{\varphi}_1^2}{2}\frac{J_2\dot{\varphi}_2^2}{2}. \qquad (8)$$

We present the potential energy of the mechanical system as the sum of the potential energies of the springs

$$\Pi = \frac{k_1\left(x_2 - x_1\right)^2}{2} + \frac{k_2\left(x_2 - x_3\right)^2}{2} + \frac{c_1\left(\varphi_2 - \varphi_3\right)^2}{2}. \qquad (9)$$

At $q = [x_1, x_2, \varphi_1, \varphi_2]$ generalized power factors of a non-conservative origin acting on a mechanical system will look like this

$$Q_{n.c.} = \begin{bmatrix} -P - \alpha_1\left(\dot{x}_1 - \dot{x}_2\right) \\ \alpha_1\left(\dot{x}_1 - \dot{x}_2\right) - \alpha_2\dot{x}_2 \\ -M \\ 0 \end{bmatrix}. \qquad (10)$$

To determine $P\lambda$ equate expression (5) to zero and differentiate it by generalized coordinates

$$\begin{aligned} h(q) &= \left(x_1 - x_2\right) + d\left(\varphi_2 - \varphi_1\right) = 0; \\ P &= \frac{\partial h}{\partial q} = \left[1, -1, -d, d\right]; \qquad (11) \\ \text{Đ}\lambda &= \left[\lambda, -\lambda, -d\lambda, d\lambda\right]. \end{aligned}$$

Express the derivatives for kinetic and potential energy (8, 9)

$$\begin{cases} \dfrac{\partial T}{\partial q} = 0 \\ \dfrac{\partial T}{\partial \dot{q}} = \left[m_1\dot{x}_1, m_2\dot{x}_2, J_1\dot{\varphi}_1, J_2\dot{\varphi}_2\right] \\ \dfrac{d}{dt}\dfrac{\partial T}{\partial \dot{q}} = \left[m_1\ddot{x}_1, m_2\ddot{x}_2, J_1\ddot{\varphi}_1, J_2\ddot{\varphi}_2\right] \\ \dfrac{\partial \Pi}{\partial q} = \begin{bmatrix} -k_1\left(x_2 - x_1\right) \\ k_1\left(x_2 - x_1\right) + k_2\left(x_2 - x_3\right) \\ c_1\left(\varphi_2 - \varphi_3\right) \\ 0 \end{bmatrix} \end{cases}. \qquad (12)$$

After substituting (10–12) into (7) and a series of transformations, we obtain a system of equations of motion

$$\begin{cases} m_1\ddot{x}_1 - \alpha_1\left(\dot{x}_2 - \dot{x}_1\right) - k_1\left(x_2 - x_1\right) = -P_p - P_{mp} + \lambda \\ m_2\ddot{x}_2 + \alpha_2\dot{x}_2 + \alpha_1\left(\dot{x}_2 - \dot{x}_1\right) + k_2\left(x_2 - x_3\right) + k_1\left(x_2 - x_1\right) = -\lambda \\ J_1\ddot{\varphi}_1 = -M_p - M_{mp} - d\lambda \\ J_2\ddot{\varphi}_2 + c_1\left(\varphi_2 - \varphi_3\right) = d\lambda \end{cases}. \qquad (13)$$

On the basis of (5) we write the expression for the angle of rotation and angular acceleration of the pipe

$$\begin{cases} \varphi_2 = \dfrac{x_2 - x_1}{d} + \varphi_1 \\ \ddot{\varphi}_2 = \dfrac{\ddot{x}_2 - \ddot{x}_1}{d} + \ddot{\varphi}_1 \end{cases}. \qquad (14)$$

Substituting (14) into the fourth equation of system (13), we obtain

$$J_2\left(\frac{\ddot{x}_2 - \ddot{x}_1}{d} + \ddot{\varphi}_1\right) + c_1\left(\frac{\ddot{x}_2 - \ddot{x}_1}{d} + \ddot{\varphi}_1 - \varphi_3\right) = d\lambda,$$

where is the Lagrange multiplier

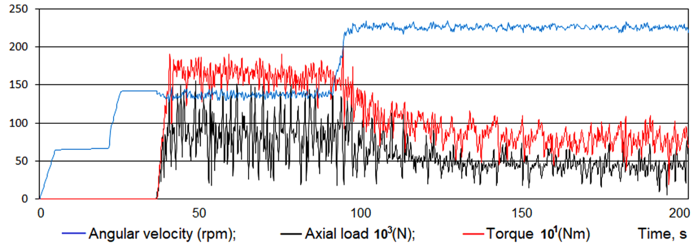
$$\lambda = \frac{J_2}{d}\left(\frac{\ddot{x}_2 - \ddot{x}_1}{d} + \ddot{\varphi}_1\right) + \frac{c_1}{d}\left(\frac{\ddot{x}_2 - \ddot{x}_1}{d} + \ddot{\varphi}_1 - \varphi_3\right). \qquad (15)$$

Analytical dependencies (13–15) make a mathematical model for the study of the parameters of longitudinal and torsional vibrations of a two-mass drilling tool with four degrees of freedom, armed with a cutting-type bit. By applying these dependencies, known non-stationary oscillation processes can be analytically investigated and simulated at empty slots, as is known in [15].

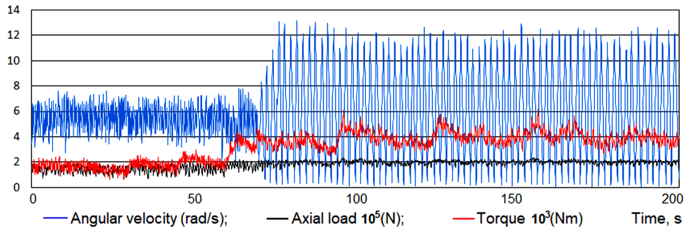
Non-stationary oscillation models can also be obtained by numerically realizing time equations of motion in a MapleSim environment based on the laws of change in the kinematic and force parameters of one of the sections of the instrument. Such regularities can be oscillograms of axial load, torque, angular rotational speed and axial vibration of the drill string, although in practice the first three are most often controlled (Fig. 2).

The frequency composition of oscillatory processes in the frequency domain is determined by spectral analysis based on Fourier transforms. To determine the frequencies of noise, to find possible resonant frequencies of non-stationary processes in transient modes, we use the amplitude-frequency characteristics (frequency response) obtained by the fast Fourier transform (FFT). The standard FFT method uses a uniform time resolution to calculate the frequency response. MathLab software, which automatically searches for stationary sections with the subsequent calculation and visualization of spectrograms, was used to calculate parameters quickly and to build the frequency response of oscillatory processes at predetermined time intervals. Such spectrograms allow us to study the variation of the spectrum of oscillation frequencies from time to time and the speed of rotation at variations of the speed of rotation to identify possible resonances in the system.

Torsional oscillations, manifested in the form of a jumping motion of the drilling tool, caused by sharp accelerations and decelerations during its rotation (Figs. 2, *a, b*). During rotation, the chisel is paused (lasting the order of tenths of a second) at regular intervals, which causes an increase in torque and twist of the entire column. When the twisting moment is exceeded over the moments of resistance to cutting the rock and the moment of friction forces against the well wall, there is a sharp acceleration of the bit – slipping when its angular velocity increases sharply (2–3 times). The duration of such a process may take several minutes and the maximum intensity of oscillation occurs at the bottom of the column. Constructed frequency response of the angular velocity shows that the frequency of its



a



b

Fig. 2. Waveforms of angular velocity, axial loaing and torque of the layout of the bottom of the drill string with bits of cutting-chipping type:

a – the case of loading; b – the case of "slipping"

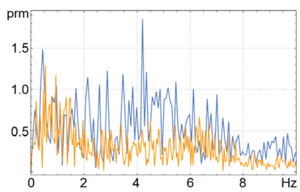
change in the case of "loading" of the bit is 1–5 Hz (Fig. 3, *a*), and in the case of "slippage" – 0.5 Hz (Fig. 3, *d*).

Most often, torsional vibration occurs when drilling PDC bits, and in some cases, the process is accompanied by transverse oscillations of the downhole equipment. According to the frequency response (Figs. 3, *c, f*), the frequency of torque change during both "loading" and "slipping" of the bit is 1–2 Hz. However, oscillations of this type, known in the literature as "stick-slip", can have frequencies below 1 Hz and can have far more serious effects than axial oscillations. The main damage is: breakage of the armament of the chisel, especially in the area that forms the diameter of the well walls; fatigue triggering of threaded connections due to their tightening of the torque and the risk of the equipment turning and falling into the well.

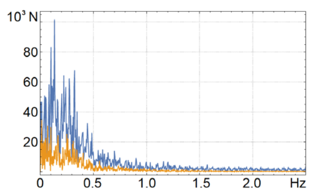
Analysis of the frequency response of longitudinal oscillations indicates that the most commonly given type of oscillation occurs when changing fractured layers with different properties, as well as the passage of solid layers. As the oscillation amplitude increases, the chisel starts to bounce, losing contact with the bottom. As a result, there is additional energy that contributes to the destruction of the rock, but at the same time adversely affects the rock destroying tool itself. This leads to excessive wear and premature destruction of the bit, failure of the seals, depressurization and damage to the supports and, as a consequence, to reducing drilling speed. However, as practice shows, when drilling at shallow depths, the axial oscillations can reach the wellhead, thereby signaling the instability of the drilling process. Based on the amplitude-frequency analysis, it was found that the axial oscillations have a frequency of 1–10 Hz (Figs. 3, *b, e*) and are extinguished by the drill string itself. Therefore, the speed of their attenuation depends directly on the weight and rigidity of the layout.

The desire of drillers to increase the mechanical speed of drilling leads to a distortion of the balance between the stability of the column and the reactive torque from the bottom, which in turn leads to the exit from the zone of stable drilling and the emergence of torsional vibrations. The same thing happens with the wrong mode of operation, when with too high load and low speed of rotation the chisel is deeply immersed in the rock for one revolution.

It should also be noted that the probability of torsional oscillations increases with the depth and zenith angle of the well, as well as with the passage of hard layers. The most characteristic features on the mouth of the "stick-slip" effect are a sharp drilling speed of up to 30–40 %, as well as a periodic change in the speed of rotation and torque by 20–25 %.



a



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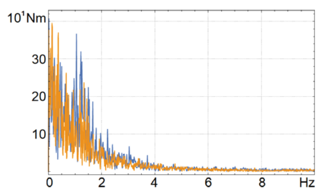

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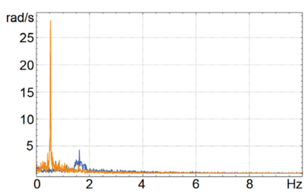

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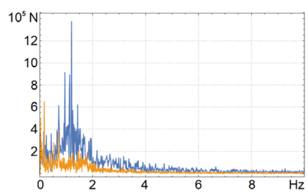
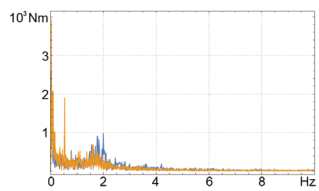


e



f

Fig. 3. Frequency response of the parameters of the bottom of the drilling tool with bits of cutting-chipping type:

a – angular velocity; b – axial load; c – torque ("loading" of the bit in interval: ——— – 40–80 s; ——— – 80–120 s); d – angular velocity; e – axial load; f – torque ("sliding" of the bit in interval: ——— – 0–60 s; ——— – 60–120 s)

In general, we note that the non-stationary oscillations of a drilling tool with small-sized bits of the cutting-chipping type are relaxing and contain areas with fast and slow amplitude and frequency changes. For a long-tube mechanical system under real conditions, such modes of motion pose a serious danger, since they can be accompanied by unscrewing the bit, breaking it with diamond cutters and the general destruction of the column.

Therefore, the selection of rigid and damping parameters of the drilling tool incorporated in the mathematical model is one of the most effective methods for reducing the intensity of non-stationary oscillations. In particular, for the damping of longitudinal oscillations in the bottom of the drill string it is possible to enter additional weighted drill pipes.

However, weighting the column without changing drilling modes can increase the impact impulse and damage the downhole equipment. Therefore, with the advent of axial oscillations of large amplitude, it is necessary to increase the load and reduce the speed of rotation of the bit. In this case, it is preferable to use bits that create low reactive moment. Also, the most effective methods for combating axial oscillations include the use of various damping devices, shock absorbers, vibration dampers, which allow increasing the life of drilling equipment. Quenching of torsional vibrations is also carried out by the drill string above the chisel and the general friction resistance against the well wall. However, unlike the axial direction, the rigidity of the column in the tangential direction is not sufficient for complete damping of oscillations, despite the introduction of additional weighted drill pipes, calibrators and centrifuges into its lower part. Therefore, as in the case of axial vibration, the main way to eliminate torsional vibrations is to change the drilling mode. To do this, limit the axial load and increase the speed of rotation. It is also possible to incorporate high speed rotating motors and damping devices into the bottom of the drill string. Because such dampers create additional elasticity of the column, the purpose of their application is to reduce the tangential shock load on the arm of the bit. Most often, this is justified when drilling at greater depths, when the resource of the destroying tool is more important than the speed of passage.

Conclusions. According to the analysis of the literature sources, insufficient study of the non-stationary oscillations of the BC, lack of widespread use of results and mathematical models of previous studies in the practice of drilling due to the complexity of their analytical solution.

The model of oscillatory processes on the basis of a two-mass inertial system with consideration of design features and specificity of destruction of rocks with bits of cutting-chipping action is developed.

On the basis of the frequency response analysis of longitudinal oscillations of the drilling tool, general recommendations were developed to reduce the likelihood of their occurrence and reduce the impact on the tool's performance.

Further research will be directed to confirm the validity of the proposed mathematical model by conducting experimental studies in the laboratory.

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Математична модель коливань бурильного інструмента з долотом ріжучо-сколюючого типу

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Мета. Встановлення причинно-наслідкових зв'язків між параметрами нестаціонарних поздовжніх і крутильних коливань бурильного інструменту та розроблення математичної моделі для дослідження їх властивостей.

Методика. Вирішення поставленої задачі базується на одному з основних принципів аналітичної механіки – принципі Лагранжа. Складання рівнянь руху досліджуваної стержневої механічної системи з чотирма ступенями вільності здійснюється із застосуванням рівняння Лагранжа II роду, що є одним із чергових етапів її динамічного аналізу.

Результати. У ході вирішення поставленої задачі на основі промислових даних проведено аналіз функцій осьового навантаження, кутової швидкості обертання та крутного моменту бурильного інструменту на предмет їх нестаціонарності. Частота зміни зазначених нестаціонарних функцій режимних параметрів буріння встановлена шляхом швидкого перетворення Фур'є.

Наукова новизна. Уперше отримані аналітичні залежності для визначення силових факторів різання й тертя на долоті від других похідних незалежних узагальнених координат механічної системи за часом, а також взаємозв'язку між узагальненими прискореннями поступального та обертального рухів тіл механічної системи.

Практична значимість. Практичною цінністю роботи є рівняння руху системи з урахуванням отриманих частотних складів режимних параметрів буріння до їх подальшого чисельного розв'язку. Результати вирішення поставленої задачі в подальшому сприятимуть вибору оптимальних режимів динамічного навантаження бурильного інструменту з метою підвищення його енергоефективності та надійності роботи. Аналіз залежностей частот зміни нестаціонарних функцій режимних параметрів буріння надасть змогу здійснювати оцінювання рівня зношування породоруйнуючого інструменту, а також прогнозувати довговічність елементів бурильної колони.

Ключові слова: *бурильна колона, долото, коливання, амплітуда, частота*

Математическая модель колебаний бурильного инструмента с долотом режуще-скалывающего типа

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Цель. Установление причинно-следственных связей между параметрами нестационарных продольных и крутильных колебаний бурильного инструмента и разработка математической модели для исследования их свойств.

Методика. Решение поставленной задачи базируется на одном из основных принципов аналитической механики – принципе Лагранжа. Составление уравнений движения исследуемой стержневой механической системы с четырьмя степенями свободы осуществляется с применением уравнения Лагранжа второго рода, что является одним из очередных этапов ее динамического анализа.

Результаты. В ходе решения поставленной задачи на основе промышленных данных проведен анализ функций осевой нагрузки, угловой скорости вращения и крутящего момента бурильного инструмента на предмет их нестационарности. Частота смены указанных нестационарных функций режимных параметров бурения установлена путем численного кратковременного преобразования Фурье.

Научная новизна. Впервые получены аналитические зависимости для определения силовых факторов резания и трения на долоте от вторых производных независимых обобщенных координат механической системы по времени, а также взаимосвязи между обобщенными ускорениями поступательного и вращательного движений тел механической системы.

Практическая значимость. Практической ценностью работы являются уравнения движения системы с учетом полученных частотных составов режимных параметров бурения до их дальнейшего численного решения. Результаты решения поставленной задачи в дальнейшем будут способствовать выбору оптимальных режимов динамической нагрузки бурильного инструмента с целью повышения его энергоэффективности и надежности работы. Анализ зависимостей частот изменения нестационарных функций режимных параметров бурения позволит осуществлять оценку уровня износа породоразрушающего инструмента, а также прогнозировать долговечность элементов бурильной колонны.

Ключевые слова: *бурильная колонна, долото, колебания, амплитуда, частота*

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