V. Strutynsky[1], Dr. Sc. (Tech.), Prof.,
orcid.org/0000-0001-5533-9915,
**A. Hurzhii[2], Academician of the National Academy
of Sciences of Ukraine, Dr. Sc. (Tech.), Prof.,**
orcid.org/0000-0001-6729-6254,
L. Kozlov[3], Dr. Sc. (Tech.), Assoc. Prof.,
orcid.org/0000-0001-9652-1270

1 − National Technical University of Ukraine "Igor Sikorsky
Kyiv Polytechnic Institute", Kyiv, Ukraine, e-mail: kvm_
mmi@ukr.net
2 − National Academy of Pedagogical Sciences of Ukraine,
Kyiv, Ukraine
3 − Vinnytsia National Technical University, Vinnytsia,
Ukraine, e-mail: osna2030@gmail.com

DETERMINATION OF STATIC EQUILIBRIUM CONDITIONS
OF A MOBILE TERRESTRIAL ROBOTIC COMPLEX

Purpose. Determination of the location of a robotic complex and an object being manipulated.

Methodology. The applied theoretical methods are based on the solution of the problems of mechanical system statics, Monte-Carlo methods and analytical geometry with approbation of the obtained results on a robotic complex prototype.

Findings. The conditions of the static equilibrium of a mobile terrestrial robotic complex with a lever-type manipulator are established. They are based on the analytical correlation as the sum of moments of gravitational forces acting on the joints of the complex relative to the axes passing through the extreme supporing points of the chassis.

The location of the mass center of the robotic complex with the lever-type manipulator is determined. The area is created by the arc of a circle whose radius depends on the mass of the object captured by the manipulator. The influence of static chassis deformations on the permissible complex area location of the general mass center is defined. It is found how the complex location influences the inclined surface of the tolerable area of the mass center location in terms of the persistency. The efficient conditions of the large mass manipulated object location are set.

Originality. Necessary and sufficient conditions for the terrestrial robotic complex to remain in a static equilibrium when working with objects of significant mass are determined. The area of the general mass center of the complex with all possible positions of the manipulator and different mass of the object being manipulated on the basis of the Monte Carlo methods adapted to the solution of the static problem of the terrestrial robotic complex with a lever- type manipulator is established. In this case, the bearing system deformability of the chassis and the features of the position of the complex being located on an inclined plane are considered.

Practical value. The established conditions of the robotic complex static stability are the basis of the choice of its geometric parameters for providing maximum working capacity in different operating conditions.

Keywords: *mobile robot, lever-type manipulator, static equilibrium, kinematic scheme, mass center location, chassis deformation, inclined plane, efficient working conditions of the manipulator*

Introduction. Terrestrial robotic complexes equipped with manipulators are used to work with hazardous objects [1]. The development of terrestrial robotic systems and the study of their characteristics is an urgent scientific and technical problem for the national security of Ukraine [2].

There are a large number of findings in the field at present [3]. Information on specially designed terrestrial robotic complexes is provided in recent studies and publications [4]. Their kinematic characteristics [5] and design features of drives are investigated [6]. A number of publications are devoted to the study of dynamic characteristics of terrestrial robotic systems [7]. It is noted that dynamic processes significantly affect the accuracy of the robotic complex manipulator position [8]. Special systems and dampers of oscillations are used to compensate for the negative effects of manipulator vibration [9]. The research results of the driving part of robotic complexes are presented [10]. The issues of the output accuracy of the operational body into the position are considered [11]. Several issues of development of the mobile robotic systems designed to work with hazardous objects are described in literary sources [12]. It is noted that the special feature of dealing with dangerous objects is the need for remote control of the complex [13]. At the same time, the information about the object is insufficient, and the accuracy measurements of their parameters are insignificant [14]. Data about the mass of a dangerous object is connected with the problem of ensuring the terrestrial robotic complex static stability [15]. Separate publications are devoted to providing the static and dynamic repeatability of robotic complexes [16]. Special devices and stability systems are used [17]. These characteristics depend on the manipulator design and the working part location within the space [18].

As a result of the analysis of literary sources, one can conclude that there is an unsolved part of the general problem, namely developing the functional capacity of a robotic complex under uncertain parameters of hazardous objects. One of the causes of disability is the inadequate static stability of the mobile robotic complex, so an important part of the main problem is to provide static stability of the terrestrial robotic complex.

The purpose of the research described in this article is to determine necessary conditions for the static consistency of a mobile terrestrial robotic complex equipped with a lever-type manipulator.

The objectives of the research were to determine general conditions of the robotic complex static stability, to find the areas of the complex mass center with a different mass of a dangerous object, in particular to restrict the area of the mass center distribution according to the conditions of consistency at placement of the complex on inclined surfaces and approbation of the research results by means of designing and manufacturing experimental prototype of a mobile terrestrial robotic complex.

The theoretical methods mainly used in the research process are based on the solution of the problems of permanent mechanical systems, Monte Carlo methods and analytic geometry. The theoretical statements are confirmed through testing a designed and manufactured prototype of a mobile terrestrial robotic complex.

Results. A typical mobile terrestrial robotic complex includes a caterpillar platform 1 which has a lever-type manipulator (Fig. 1).

The manipulator has a lower lever *2* and an upper lever *3* on the end with a hinge where the bar *4* is installed. A rotary gripper *5* is mounted on the short end of the bar. A camcor-
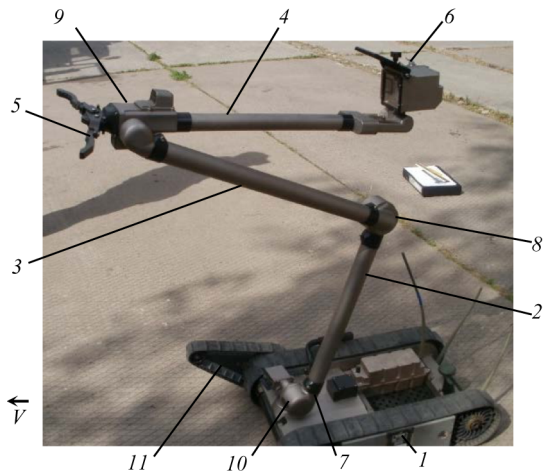
Fig. 1. General view of the terrestrial robotic complex with a lever-type manipulator

der *6* with rotary actuators is installed on the long end of the bar [19].

The levers and the bar form a flat hinge-lever mechanism in which the planes of displacement of the levers and the bars are parallel. Three single-hinged joints *7, 8, 9* are applied when the levers are turned relative to their axels. The lever system is surrounded with the rotary support *10*.

Depending on the position of the manipulator, the settlement of the separate masses of the complex and the location of the mass center is substantially transformed. To provide the permanent consistency of the complex, it has additional caterpillar movers (flippers) *11* which, if necessary, alternate their transverse angular position.

The static equilibrium of the complex depends on the configuration of the manipulator, the gravitational loading G_N of the object being manipulated, which is in the gripper of the manipulator and on the slope of the roadway on which the chassis is located. The conditions of the static equilibrium are determined by the action of the gravitational forces $G_1, ..., G_4, G_N$, and the placement of the general mass center with the corresponding gravitational force G_0 relative to the extreme supporting points of the complex on the ground. These objects are the areas of the caterpillar under the extreme supporing rollers P_1, P_2, Q_1, Q_2 (Fig. 2).

The robotic complex is in a static equilibrium if the projection of the mass center on a horizontal plane lies within the rectangle $P_1P_2Q_1Q_2$.

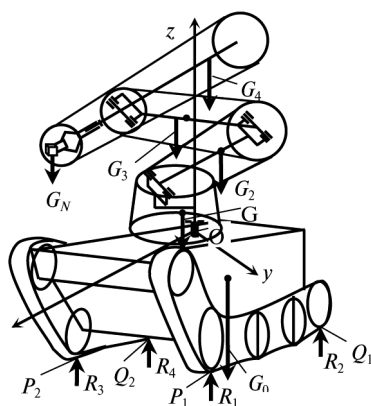Alteration of the manipulator configuration leads to a change in the mass center location of the complex. When extending the console part of the manipulator in a horizontal plane, the projection of the mass center and the corresponding gravitational force G_0 extend beyond the limites of the rectangle P_1, P_2, Q_1, Q_2 (Fig. 3).

The robotic complex loses consistency and turns over when the mass center projection of the complex on a horizontal plane passes through one of the axes y_1 or y_2 or x_1 or x_2.

To analyze the static stability condition of the terrestrial robotic complex for analytic determination we have to consider the issue of the complex statics under the impact of gravitational loads. First, let us assume that the complex is placed on an even road.

To solve the static problem let us assume that the places of application of gravitational loads are located in mass centers of manipulator units and chassis. In this case, the sum of the moments of the gravitational forces and of the work load corresponding to ones of the manipulated object G_N create the moments of the forces M_x, M_y acting relative to the longitudinal x and transversal y in the axes

$$M_x = \sum_{i=0}^{N} G_i \cdot y_i + G_N \cdot L_x; \quad M_y = \sum_{i=0}^{N} G_i \cdot x_i + G_N \cdot L_y, \quad (1)$$

where y_i and x_i are arms of gravitational forces G_i relative to axes x and y; L_x, $L_y - x$ and y; N is the total number of dedicated junction points of the robotic complex for which the gravitational forces act; G_N is gravitational force acting on the object of manipulation.

The arms of the gravitational forces of x_i and y_i are changed as a result of upgrading the manipulator configuration. The moments of forces will be substituted relative to the given axes. The static stability loss of the complex occurs when one of the conditions is fulfilled

$$M_{x_1} = 0; \quad M_{x_2} = 0; \quad M_{y_1} = 0; \quad M_{y_2} = 0, \quad (2)$$

where the moments are determined relative to the axes x_1, x_2, y_1, y_2.

Revised settlement of the mass center of the the robotic complex leads to a shift in the equivalent of the supporting responces of the chassis which are defined by means of four forces acting on the extreme rollers of the caterpillar R_1, R_2, R_3, R_4 (Fig. 2).

To find the equivalent chassis supporing responces, we have a static equation

$$\sum_{j=1}^{4} R_j \cdot y_j = M_x; \quad \sum_{j=1}^{4} R_j \cdot x_j = M_y; \quad \sum_{j=1}^{4} R_j = \sum_{i=0}^{N} G_i + G_N. \quad (3)$$

The obtained system of equations of statics (1, 3) is unclosed. Three equations of equilibrium include four unknowns R_1, R_2, R_3, R_4. An additional condition must be introduced for the closure of this system. The condition of the complex symmetry as for a longitudinal or transverse plane is practically important.

Fig. 2. The configuration of the manipulator corresponding to the position of the static equilibrium of the mobile terrestrial robotic complex
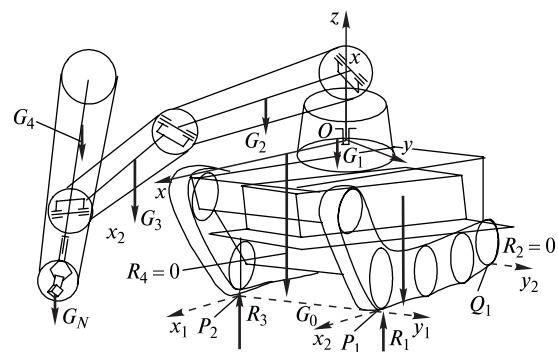

Fig. 3. The position of the manipulator corresponding to the limit of static stability loss

This condition is conducted for a robotic complex manipulator with its levers moving in close parallel planes. Let us consider the kinematic scheme of the manipulator installed on the chassis (Fig. 4).

The kinematic scheme corresponds to the terrestrial robotic complex (Fig. 1). The joints A, B and C of this manipulator have parallel axes and the levers AB, BC and the CDE bar move in close parallel planes (Fig. 4). Manipulators are installed on a rotary support with a possibility of revolving in direction of the z axis at the angle q_1. In this case, the levers move in planes which are adjacent to the radial axis z plane and their position is stipulated with the controlled angular coordinates q_2, q_3, q_4. The turn of the gripper is determined by the angle q_5.

When reversing the configuration of the manipulator with the load, the distribution of mass changes and accordingly the line of gravitational load changes. To establish the quantitative characteristics of the changes in the action of gravitational forces, a planar diagram of the manipulator is used in which all the levers move in the same plane which is adopted as a coincident one with the axis of the symmetry of the chassis (Fig. 5).

The track gear has a small mass that is comparable to the weight of the manipulator and the load. The mass of the running gear is centered on two points that correspond to the centers of the caterpillar driving drums. The proportioning of masses along the length of the platform is nearly uniform. Therefore, the masses M_1 and M_2 are close. The weight of the manipulator drive is included into the M_1 mass.
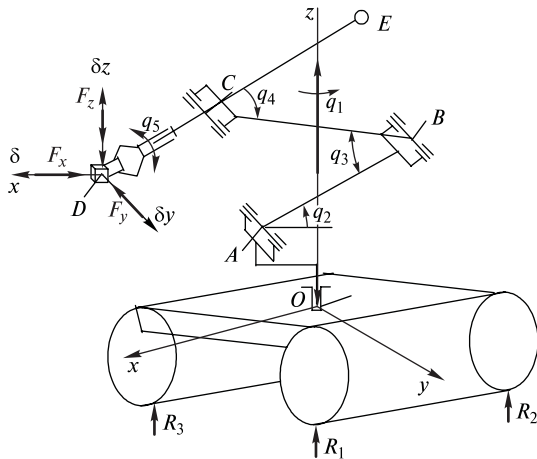

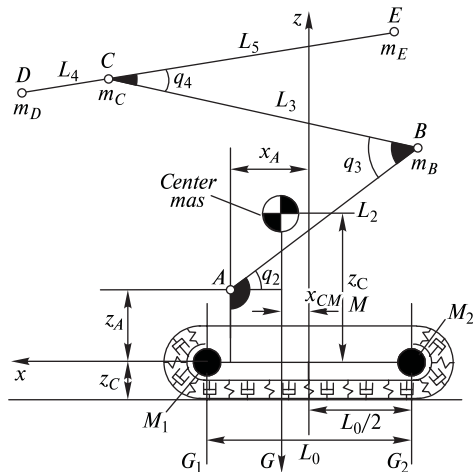*Fig. 4. Kinematic diagram of the lever-type manipulator installed on the chassis*

Fig. 5. The planar scheme of the robotic complex manipulator is adopted to determine its mass center

We introduce a coordinate system whose axes pass through two concentrated masses of the $M_1 M_2$ platform, and the z axis through the mass center of the platform, located in the middle between the masses M_1 and M_2. To calculate the mass center location, it is assumed that the mass of the manipulator is concentrated at 5 points. The mass of the drive located at point A with the first part of the weight of the lever $AB = L_2$ is reffered to the mass of the platform. The mass of the drive placed at point B, the second part of the lever L_2 and the part of the lever $BC = L_3$ is focused at point B and is m_B. The mass of the load being held by the gripper together with the mass of the gripper, the weight of the drive clamp and the drive gripper, and a part of the mass of the bar $DC = L_4$ represents a concentrated mass placed at point D that it is m_D. The mass of the part of the lever BC, the part of the bar DC and the part of the bar CE with rotary drives is at point C and is m_C. The mass at point E includes the mass of the camera, the actuators of cameras and the weight of the bar $CE = L_5$. Mass volume at this point is m_E.

Let us designate the coordinates of the point masses m_B, m_C, m_D, m_E in coordinate system xz.

Point B position

$$x_B = x_A - L_2 \cos q_2; \quad z_B = z_A + L_2 \sin q_2.$$

Point C position

$$x_C = x_B + L_3 \cos(q_3 - q_2); \quad z_C = z_B + L_3 \sin(q_3 - q_2).$$

Point D position

$$x_D = x_C + L_4 \cos(q_3 - q_3 + q_2); \quad z_D = z_C - L_4 \sin(q_4 - q_3 + q_2).$$

Point E position

$$x_E = x_C - L_5 \cos(q_4 - q_3 + q_2); \quad z_E = z_C + L_5 \sin(q_4 - q_3 + q_2).$$

Let us fix the coordinates of the mass center of the "chassis-manipulator" system

$$x_{CM} = \frac{m_B x_B + m_C x_C + m_D x_D + m_E x_E}{M_1 + M_2 + m_B + m_C + m_D + m_E}; \quad (4)$$

$$z_{CM} = \frac{m_B z_B + m_C z_C + m_D z_D + m_E z_E}{M_1 + M_2 + m_B + m_C + m_D + m_E}. \quad (5)$$

The coordinates of the mass center of the complex depend on the controlled coordinates of the manipulator q_2, q_3, q_4. These coordinates can acquire uncontrolled values in the ranges $[0 \ldots \pi]$. At the same time, the mass center of the mobile complex is changing and is located on a certain area of space. Calculations based on the Monte Carlo methods are performed to find the location of the mass center [20]. Random values of the controlled coordinates q_2, q_3, q_4 are set in formulas (4, 5). They are defined as random numbers with uniform allotment laws in the revised ranges and are calculated according to the procedure: $q_{kj} = rnd(\pi)$, $k = 2$, 3, 4, j = 0,...,1000. Found coordinates of the mass center position illustrated in the graph as points (designated by the sign x) for each jth set of controlled coordinates from (4, 5).

Calculations are made for different values of the mass of the object of manipulation.

The results show that the location of the mass center of the robotic complex significantly increases with increasing mass of the object of manipulation (Fig. 6).

Areas of the mass center location are approximated by analytic dependencies for the analytical description of the mass center location of the complex. It was established that the mass center location of the complex with the maximum mass of the manipulated object approximates the circle with a radius r_m whose center is at point A. The contour shaped as a circle is close to the mass center location of the complex except for the peculiar area PQ which corresponds to the area between supporting points of caterpillars on the support surface.
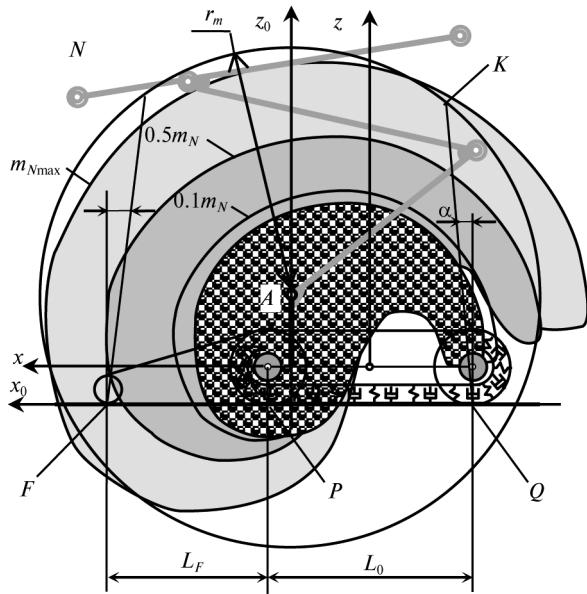
Fig. 6. Radial cross-sections of the mass centers areas of the terrestrial robotic complex with different masses of the manipulated object

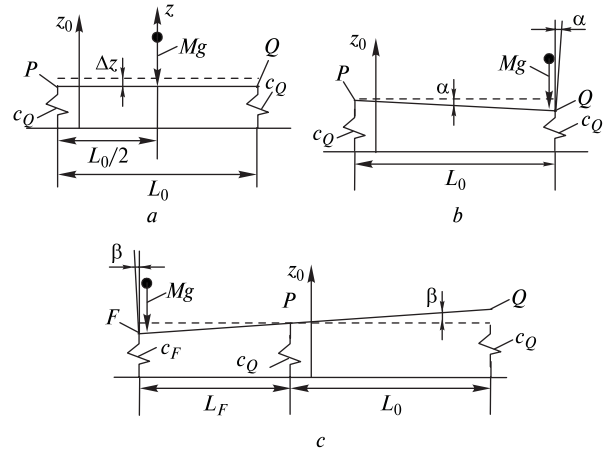
Fig. 7. The chassis static deformations of the complex under the action of gravitational forces with different location of the mass center of the complex:

a − plane-parallel displacement of the chassis with the location of the mass center on the symmetry axis of the complex; b − rotation of the chassis with the location of mass center in the area of the supporting point Q; c − rotation of the complex with the location of the mass center in the area at the end of the flipper

The equation of the circular contour describing the mass center arrangement in the coordinate system xz is written as

$$\left(x - x_A\right)^2 + \left(z - z_A\right)^2 = r_m^2,$$

where x_A, z_A are coordinates of point A which refer to the hinge A of the manipulator; r_m is the radius of the circle, which approximates the area of the mass center.

The coordinate system whose axis x_0 coincides with the supporting surface and the z_0 axis passes through point A, is characterized by the equation of the circular contour which will be as follows

$$x_0^2 + \left(z_0 - z_A - z_c\right)^2 = r_m^2, \qquad (6)$$

where z_C is the distance from the supporting surface to the middle line of the chassis corresponding to the x axis.

Using the analytical dependence (6), the terms of static stability of the complex which is on a horizontal supporting surface, are indicated.

The constraint of the complex stability is to find the projection of its mass center on a horizontal plane between extreme points of the complex on the supporting surface.

Let us consider the flat problem of evaluation the static stability of the complex with a manipulator whose plane is the same as the gear symmetry one.

The extreme supporting points for a caterpillar mover are points P and Q with coordinates

$$x_P = -L/2; \quad x_Q = L/2.$$

This condition is graphically interpreted by the mass center arrangement of the complex within the limits between the vertical lines passing through the points P and Q (Fig. 7). Having analyzed the size and location of this band respective to the areas of the mass center we can conclude that when manipulating objects in the front part of the chassis static stability loss can occur even in the absence of an object of manipulation. Thus, the complex will probably be unable to function when lifting the load in the front part of the chassis.

Additional caterpillar movers (flippers) are used to ensure the performance of the complex. Their application shifts the extreme front supporting points of the complex into the area of the point F with coordinates

$$x_F = L_F + L/2 - x_A,$$

where L_F is the distance between supporting point P and point of the interaction F of the caterpillar flipper with the supporting surface.

Here the area of static stability expands and relates to the band between vertical lines passing through points F and Q. Insignificant areas of static instability outside the band are negligible and do not affect the overall operation of the complex.

The abovementioned condition of static stability does not consider the deformability of the caterpillar chassis.

Let us define the influence of the deformation of caterpillars on the static stability of the complex. First, let us consider the chassis without flippers. The deformation of caterpillars under supporting rollers occurs at points P and Q under the effect of gravitational forces. If the mass center of the complex is on the axle z, the platform of the chassis without flippers under the action of gravitational forces is shifted down without violation of its transverse angular position (Fig. 7, *a*).

Displacement of the chassis in the vertical direction with symmetrical load is determined by the dependence

$$\Delta z = Mg/(2c_Q),$$

where c_Q is an equivalent to the rigidity of the caterpillar at points P and Q; g is gravity acceleration; M is the total mass of the complex.

In case of a flat-parallel displacement of the chassis the stability area of the complex is the same and its limits are vertical lines passing through points P and Q (for chassis without flippers).

Shifting the mass center of the complex towards the axis x, efforts perceived by the caterpillar at points P and Q vary and there is a rotation of the chassis at a certain angle. The maximum rotation of the chassis will occur when the mass center is located on a vertical line passing through point P or Q (Fig. 7, *b*). The angle of rotation will be determined under the dependence

$$\alpha = \frac{Mg}{C_Q L_0}. \qquad (7)$$

Therefore, vertical lines penetrating points P and Q and limiting the permissible range of the masses center will be rotated to the angle α towards the central part of the complex

because of the chassis deformability. The reversed line will determine the segment QK (Fig. 7) which defines the limit of tolerance in terms of the stability of the areas of the mass center location.

The availability of flippers changes the nature of the deformation of caterpillars. Let us assume that the flipper is tightly connected to the duplex chassis and moves together with the chassis as a whole.

The displacement of the masses center of the complex in the direction of the flipper leads to the rotation of the chassis at a certain angle. The maximum rotation angle occurs when the mass center is set above the extreme supporting point of the flippers (Fig. 8, c). The angle of rotation of the chassis is

$$\beta = \frac{Mg}{c_F L_F}, \tag{8}$$

where L_F is the distance between the supporting points of the flipper and the caterpillar; i; c_F is flipper rigidity. The vertical line passing through point F and restricting the permissible range of the masses center rotates at the angle β (Fig. 7). Thus, the presence of transverse angular movements of the chassis as a result of the deformation of caterpillars narrows the range of the mass center. It is limited to the $QKNF$ contour.

The contour in the CN section corresponds to the arc of the circle as for the equation given above (6). The contour in sections FN and QK is characterized by straight lines with equations

$$z_{FN} = \left(x_0 + \frac{L_0}{2} - x_A + L_F\right) \cdot \mathrm{tg}\left(\frac{\pi}{2} - \beta\right);$$

$$z_{QK} = -\left(x_0 - \frac{L_0}{2} - x_A\right) \cdot \mathrm{tg}\left(\frac{\pi}{2} - \alpha\right).$$

After trigonometric functions transformation and substitution of angles from (7, 8), we will get

$$z_{FN} = \left(x_0 + \frac{L_0}{2} - x_A + L_F\right) \cdot \frac{c_F L_F}{M \cdot g}; \tag{9}$$

$$z_{QK} = -\left(x_0 - \frac{L_0}{2} - x_A\right) \cdot \frac{C_Q L_0}{M \cdot g}. \tag{10}$$

It was assumed that the angles α and β in these formulas are small and, respectively, $\mathrm{tg}\,\alpha \approx \alpha$, $\mathrm{tg}\,\beta \approx \beta$.

The abovementioned terms of static equilibrium were proposed for a complex positioned on a horizontal surface. Incline or lifting of the supporting surface of the chassis change the equilibrium conditions of the complex.

If there is a slope with an angle θ of point F the flipper support and the caterpillar Q are shifted from the position FQ (at $\theta = 0$) and supporting surface of the chassis moves to the position $F'Q'$ (Fig. 8).

The horizontal axis x_0 of the local coordinate system of the complex passes into the axis x'_0 and the axis z_0 — into the axis z'_0. The range of the mass center of the complex shifts at the same time to the right and is limited to the contour $Q'K'N'F'$. This contour in the local coordinate system of a complex is described by dependencies similar to (9, 10) used to determine the area of a complex located on a horizontal plane. Simultaneously, the contour on the section $K'N'$ corresponds to the arc of the circle as for the equation (6). The equations of straight line $K'N'$ and $Q'K'$ referred to (9, 10) will be modified after modification of the straight lines angular coefficients.

The angle β in the equations converts into the angle $\theta + \beta$, and the angle α into $\alpha - \theta$. Accordingly, the equations of the straight lines $F'N'$ and $Q'K'$ in the local coordinate system are determined by the dependencies

$$z'_{FN} = \left(x'_0 + \frac{L_0}{2} - x_A + L_F\right) \cdot \mathrm{ctg}\left(\theta + \frac{M \cdot g}{C_F L_F}\right); \tag{11}$$
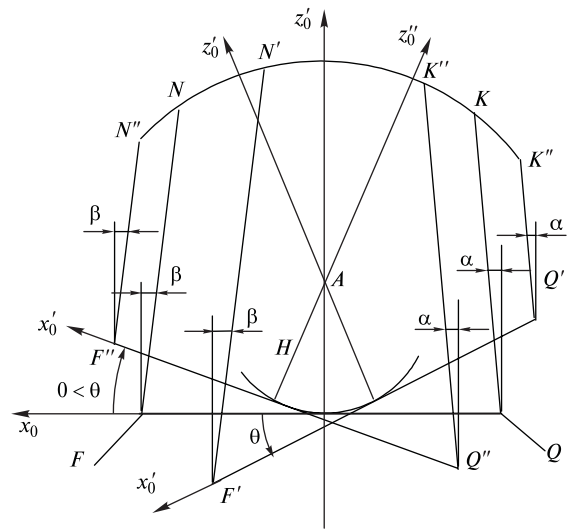

Fig. 8. Changes in the permissible area of location of the complex mass center are due to the presence of a slope ($\theta > 0$) or a rise ($\theta < 0$) of the supporting surface

$$z'_{QK} = \left(x'_0 - \frac{L_0}{2} - x_A\right) \cdot \mathrm{ctg}\left(\theta - \frac{M \cdot g}{C L_0}\right). \tag{12}$$

The oddness of the trigonometric function of the cotangent is shown in the last dependence.

When moving the complex up (ascends $\theta < 0$), the position of the local coordinate system is moved into $x''_0 z''_0$. The range of standard conditions within static steadiness of the complex mass center positions is restricted by the contour $Q''K''N''F''$. The contour at the section $K''N''$ corresponds to the circle arc, and it will be described by the dependences (11, 12) at the sections $F''N''$ and $Q''K''$ where $\theta < 0$.

Moving the complex down or up the slope significantly changes the permissible mass center location.

When moving down the slope, the area is shifted towards the rear part of the complex and when rising – to the front part of it (Fig. 9).

The analysis of admissible areas of the mass centers location demonstrates that the performance with the objects of increased mass should be carried out in the front part of the chassis between the flippers when the complex is situated on the slope and is directed towards the peak.

It is recommended to turn up the gear with the flippers under a negative chassis angle (shown in dotted lines in Fig. 9, b) in order to increase the resilience reserve when working with heavy objects.

The presence of flippers in the complex raises static stability in case the manipulator average plane is close to the longitudinal axis of the complex. Therefore, the optimal area of contacting with hazardous objects in the plan is the S_0 section between the flippers.

An experimental model of mobile terrestrial robotic complex was developed on the basis of conducted research. It is grounded on a modular circuit and includes autonomous modules with a wheeled or caterpillar mover. Each module has an autonomous power supply and a control system. If necessary, the module is used independently. It is equipped with an additional footing (Fig. 10).

A similar design has a caterpillar-mounted module (Fig. 11).

Each module is provided with necessary equipment, manipulators of different types in particular. The modules have junction units to be connected with each other.

The best variant is a robotic complex which is the result of the combination of wheel and caterpillar modules (Fig. 12).
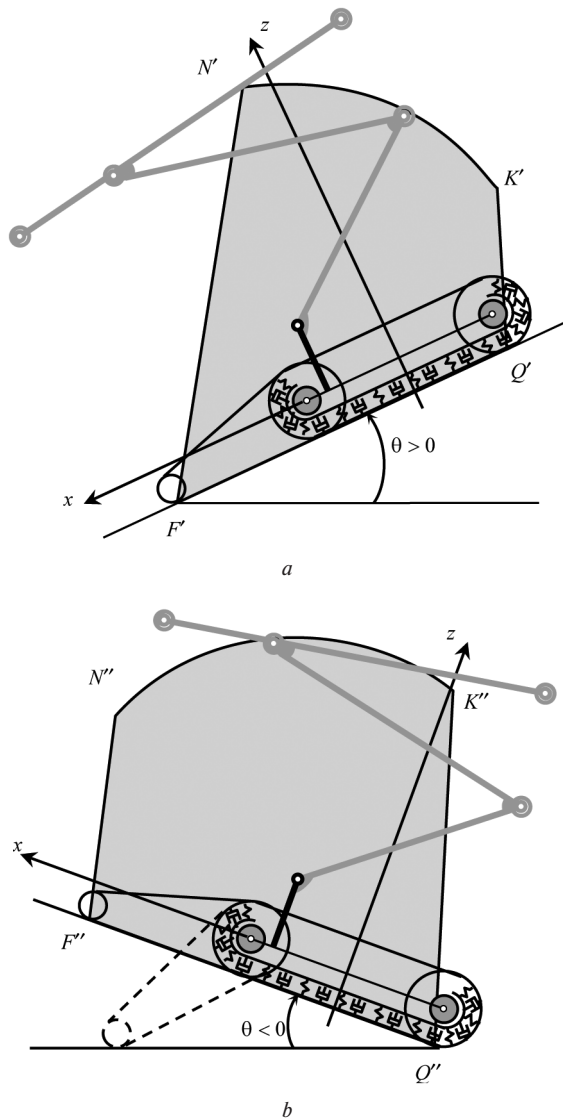
a

b

Fig. 9. Longitudinal sections of the permissible mass center location of the complex when placed on a slope down (a) and on a rise up (b)

The experimental examination of the developed experimental model of the terrestrial robotic complex confirmed the reliability of static stability determination under different conditions of the chassis location on the supporting surface.
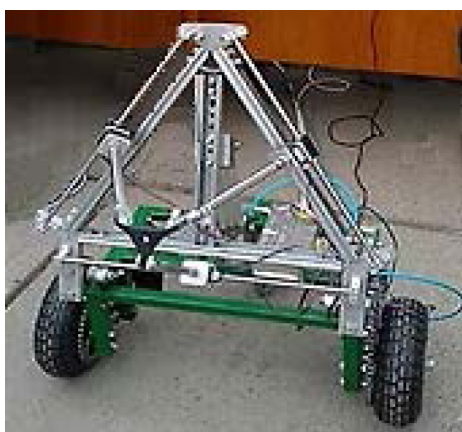

Fig. 10. The wheeled module of the developed experimental model of the terrestrial robotic complex
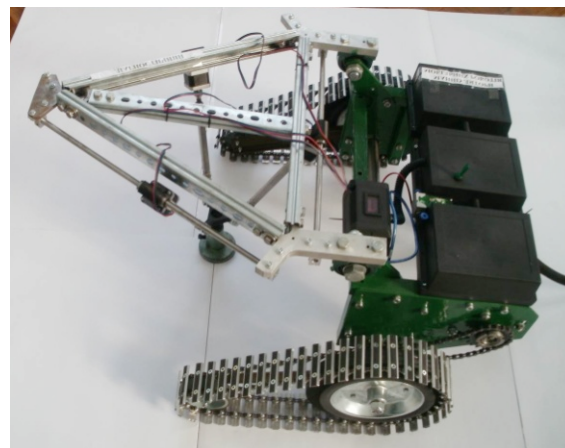

Fig. 11. Module of terrestrial robotic complex with a caterpillar mover
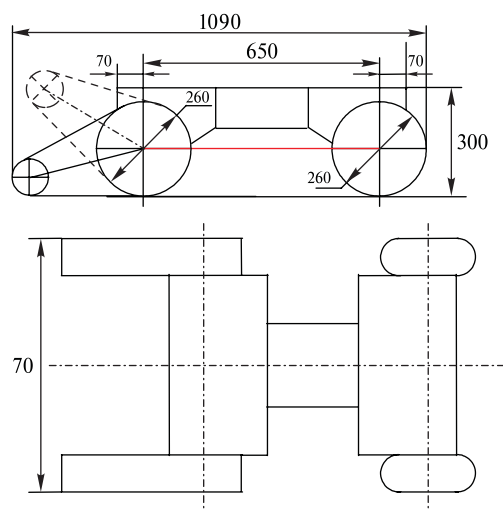

Fig. 12. Scheme of the chassis of the terrestrial robotic complex which is formed by combination of wheeled and caterpillar modules

Conclusions. It was established that the condition of the static equilibrium of a mobile terrestrial robotic complex is the zero sum of the moments of the gravitational forces acting on the units of the complex relative to the axes passing through the extreme chassis supporting points of the complex.

The masses center location of the robotic complex with a lever-type manipulator, whose levers move in parallel planes under different configurations of the manipulator, is limited by a curve which is close to the arc of a circle whose radius is determined by the mass of the object of manipulation.

The presence of chassis static deformations and the location of the complex on an inclined surface narrows the circular area of the allowed total mass center location of the complex under condition of static stability. Placement of the complex on the rise and manipulation by the object located in front part of the complex is a positive feature in terms of providing static stability.

References.
1. Jun Qian, Bin Zi, Daoming Wang, Yangang Ma, & Dan Zhang (2017). The design and development of an omni-directional mobile robot orientated to an intelligent manufacturing system. *Sensors*, *17*, 2073. DOI: 10.3390/s17092073.
2. Strutynskyi, S. V., & Hurzhii, A. A. (2017). Definition of vibro displacements of drive systems with laser triangulation meters and setting their integral characteristics via hyper-spectral analysis methods. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 1*, 43-51.

3. Ho-Seung Jeong, & Jong-Rae Cho (2016). Optimal design of head expander for a lightweight and high frequency vibration shaker. *International Journal of Precision Engineering and Manufacturing*, *17*(7), 909-916. DOI:10.1007/s12541-016-0111-z .

4. Kot, T., & Novak, P. (2018). Application of virtual reality in teleoperation of the military mobile robotic system TAROS. *International journal of advanced robotic systems*, (January-February), 1-6. DOI: 10.1177%2F1729881417751545.

5. Korayem, M. H., & Dehkordi, S. F. (2018). Derivation of motion equation for mobile manipulator with viscoelastic links and revolute−prismatic flexible joints via recursive Gibbs−Appell formulations. *Robotics and Autonomous Systems*, *103*, 175-198. DOI: 10.1016/j.robot.2018.02.013.

6. Polishchuk, L. K., Kozlov, L. G., Piontkevych, O. V., Gromaszek, K., & Mussabekova, A. (2018). Study of the dynamic stability of the conveyor belt adaptive drive. *Proc. SPIE 10808, Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2018*, 1080862 (1 October 2018). DOI: 10.1117/12.2501535.

7. Hyun-Min Joe & Jun-Ho Oh (2018). Balance recovery through model predictive control based on capture point dynamics for biped walking robot. *Robotics and Autonomous Systems*, *105*, 1-10.

8. Xiaogeng Jiang & Robert J. Cripps (2015). A method of testing position independent geometric errors in rotary axes of a five-axis machine tool using a double ball bar. *International Journal of Machine Tools and Manufacture*, *89*, 151-158. DOI: 10.1016/j.ijmachtools.2014.10.010.

9. Min Wang, Tao Zan, Xiangsheng Gao, & Songwei Li (2016). Suppression of the time-varying vibration of ball screws induced from the continuous movement of the nut using multiple tuned mass dampers. *International Journal of Machine Tools and Manufacture*, *107*, 41-49. DOI: 10.1016/j.ijmachtools.2016.05.003.

10. Marlow, K., Isaksson, M., Dai, J. S., & Nahavandi, S. (2016). Motion Force Transmission Analysis of Parallel Mechanisms with Planar Closed-Loop Subchains. *Journal of Mechanical Design*, *138*(6), 21-32. DOI: 10.1115/1.4033338.

11. Tao Liang, Dun Lu, Xiaojun Yang, Jun Zhang, Xiaobo Ma, & Wanhua Zhao (2016). Feed fluctuation of ball screw feed systems and its effects on partsur face quality. *International Journal of Machine Tools and Manufacture*, *101*, 1-9. DOI: 10.1016/j.ijmachtools.2015.11.002.

12. Baoquan Li, Yongchun Fang, Guoqiang Hu, & Xuebo Zhang (2016). Model-Free Unified Tracking and Regulation Visual Servoing of Wheeled Mobile Robots. *Journal Sensors and Actuators A: Physical, IEEE Transactions on Control Systems Technology*, *24*(4), 1328-1339.

13. Meoni, F., & Carricato, M. (2016). Design of Nonovercon strained Energy-Efficient Multi-Axis Servo Presses for Deep-Drawing Applications. *Journal of Mechanical Design*, *138*(6). DOI: 10.1115/1.4033085.

14. Yongjie Zhao, KeQiu, Shuangxi Wang, & Ziqiang Zhang (2015). Inverse Kinematics and rigid-body dynamics for a three rotational degrees of freedom parallel manipulator. *Robotics and Computer-Integrated Manufacturing*, (31), 40-50.

15. Paul Ritzen, Erik Roebroek, Nathan van de Wouw, & Zhong-Ping Jiang (2016). Trailer Steering Control of a Tractor−Trailer Robot. *IEEE Transactions on Control Systems Technology*, *24*(4), 1240-1252.

16. Altin, B., & Barton, K. (2017). Exponential stability of nonlinear deferential repetitive processes with applications to iterative learning control. *Automatica*, *81*, 369-376. DOI: 10.1016/j.automatica.2017.04.004.

17. Mansoor Alghooneh, Christine Qiong Wu, & Masoumeh Esfandiari (2016). A Passive-Based Physical Bipedal Robot with a Dynamic and Energy-Efficient Gait on the Flat Ground. *Journal Transactions on Mechatronics*, *21*(4), 1977-1984.

18. Blanken, L., Boeren, F., Bruijnen, D., & Oomen, T. (2017). Batch-to-batch rational feedforward control: from iterative learning to identification approaches, with application to a wafer stage. *IEEE-ASME Transactions on mechatronics*, *22*(2), 826-837. DOI: 10.1109/TMECH.2016.2625309.

19. Strutynskyi, S., Kravchuk, V., & Semenchuk, R. (2018). Mathematical modelling of a specialized vehicle caterpillar mover dynamic processes under condition of the distributing the parameters of the caterpillar. *International Journal of Engineering & Techology*, 7(4/3), 40-46.

20. Strutynsky, V. B., Hurzhi, A. A., Kolot, O. V., & Polunichev, V. E. (2016). Determination of development grounds and characteristics of mobile multi-coordinate robotic machines for materials machining in field conditions. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 5*(155), 43-51.

Визначення умов статичної рівноваги мобільного наземного роботизованого комплексу

В. Б. Струтинський[1], *А. М. Гуржій*[2], *Л. Г. Козлов*[3]

1 − Національний технічний університет України „Київський політехнічний інститут імені Ігоря Сікорського“, м. Київ, Україна, e-mail: kvm_mmi@ukr.net

2 − Національна академія педагогічних наук України, м. Київ, Україна

3 − Вінницький національний технічний університет, м. Вінниця, Україна, e-mail: osna2030@gmail.com

Мета. Визначення область стійкого розташування роботизованого комплексу та об'єкта маніпулювання.

Методика. Застосовані теоретичні методи на основі розв'язку задач статики механічних систем, методів Монте-Карло та аналітичної геометрії з апробацією одержаних результатів на дослідному зразку роботизованого комплексу.

Результати. Встановлені умови статичної рівноваги мобільного наземного роботизованого комплексу з маніпулятором важільного типу. Вони базуються на аналітичному співвідношенні у вигляді суми моментів гравітаційних сил, що діють на вузли комплексу відносно вісей, які проходять через крайні опорні точки шасі. Визначена область розташування центра мас роботизованого комплексу з маніпулятором важільного типу. Область описана дугою кола, радіус якого залежить від маси об'єкта, що знаходиться у схваті маніпулятора. Встановлено вплив статичних деформацій шасі на допустиму область розташування загального центра мас комплексу. Визначено, як впливає розташування комплексу на похилій поверхні на допустиму з точки зору стійкості область розташування центра мас. Встановлені раціональні умови розташування об'єкта маніпулювання великої маси.

Наукова новизна. Уперше визначені необхідні й достатні умови знаходження наземного роботизованого комплексу у статичній рівновазі при роботі з об'єктами значної маси. На основі методів Монте-Карло, адаптованих для розв'язку задачі статики наземного роботизованого комплексу з маніпулятором важільного типу, встановлена область розташування загального центра мас комплексу при всіх можливих положеннях маніпулятора та при різній масі об'єкта маніпулювання. При цьому врахована деформативність несучої системи шасі та особливості положення комплексу при його розташуванні на похилій площині.

Практична значимість. Встановлені умови статичної стійкості роботизованого комплексу є основою вибору його геометричних параметрів при забезпеченні максимальної вантажопідйомності та працездатності в різних умовах експлуатації.

Ключові слова: *мобільний робот, важільний маніпулятор, статична рівновага, кінематична схема, положення*

центра мас, деформації шасі, похила площина, раціональні умови роботи маніпулятора

Определение условий статического равновесия мобильного наземного роботизированного комплекса

В. Б. Струтинский[1], *А. Н. Гуржий*[2], *Л. Г. Козлов*[3]

1 – Национальный технический университет Украины „Киевский политехнический институт имени Игоря Сикорского“, г. Киев, Украина, e-mail: kvm_mmi@ukr.net
2 – Национальная академия педагогических наук Украины, г. Киев, Украина
3 – Винницкий национальный технический университет, г. Винница, Украина, e-mail: osna2030@gmail.com

Цель. Определить область устойчивого расположения роботизированного комплекса и объекта манипулирования.

Методика. Применены теоретические методы на основе решении задач статики механических систем, методов Монте-Карло и аналитической геометрии с апробацией полученных результатов на опытном образце роботизированного комплекса.

Результаты. Установлены условия статического равновесия мобильного наземного роботизированного комплекса с манипулятором рычажного типа. Они базируются на аналитическом соотношении в виде суммы моментов гравитационных сил, действующих на узлы комплекса относительно осей, проходящих через крайние опорные точки шасси. Определена область расположения центра масс роботизированного комплекса с манипулятором рычажного типа. Область описана дугой окружности, радиус которого зависит от массы объекта, находящегося в схвате манипулятора. Установлено влияние статических деформаций шасси на допустимую область расположения общего центра масс комплекса. Определено, как влияет расположение комплекса на наклонной поверхности на допустимую, с точки зрения устойчивости, область расположения центра масс. Установлены рациональные условия расположения объекта манипулирования большой массы.

Научная новизна. Впервые определены необходимые и достаточные условия нахождения наземного роботизированного комплекса в статическом равновесии при работе с объектами значительной массы. На основе методов Монте-Карло, адаптированных для решения задачи статики наземного роботизированного комплекса с манипулятором рычажного типа, установлена область расположения общего центра масс комплекса при всех возможных положениях манипулятора и при различной массе объекта манипулирования. При этом учтена деформативность несущей системы шасси и особенности положения комплекса при его расположении на наклонной плоскости.

Практическая значимость. Установленные условия статической устойчивости роботизированного комплекса являются основой выбора его геометрических параметров при обеспечении максимальной грузоподъемности и работоспособности в различных условиях эксплуатации.

Ключевые слова: *мобильный робот, рычажный манипулятор, статическое равновесие, кинематическая схема, положение центра масс, деформации шасси, наклонная плоскость, рациональные условия работы манипулятора*