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SYNERGETIC MODEL OF THE WAVE ABRASIVE-FATIGUE WEAR OF RUBBER LINING IN THE BALL-TUBE MILLS

Purpose. To develop a synergetic model and, on its basis, a theory of wave abrasive-fatigue wear of rubber lining in the ball mills; to confirm fidelity of the analytical model by industrial test results.

Methodology. A comprehensive approach was used, which included analytical studies and extensive industrial tests of different ball mills.

Findings. On the basis of the designed synergetic model and the Polya-Velikanov equations, mathematical equations were formulated, with the help of which it is possible to describe a pattern of wave abrasive-fatigue wear of rubber lining; on the basis of the theory of deterministic chaos, key regularities were identified in the wave motion of inputs inside the tube of the mill; correctness of the analytical model was confirmed by results of industrial tests, which are presented herein.

Originality. An original synergetic model was developed and, on its basis, a theory of the wave abrasive-fatigue wear of rubber lining in the ball mills was created. Key regularities of wave motion of inputs inside the tube of the mill were identified.

Practical value. On the basis of analytical calculations, the Wave-type rubber linings were designed and implemented, which are currently competitive at the world market. On the basis of the lining, a new energy- and resource-saving technology of mineral disintegration in the ball-tube ore-grinding mills was created and implemented into industry.

Keywords: *ball mills, synergetic model, abrasive-fatigue wear, wave theory*

Formulation of the problem and analysis of the recent research. Some of the issues of this rather complex and insufficiently explored problem are set forth in the literature cited below.

Behind this written work is generality of dynamic processes and morphometric features experimentally identified during the studying of saturated streamflows and interaction between the inputs and protective lining in the ball mills. Both cases are characterized by a powerful turbulence, and, consequently, pulsation of velocities and pressures; in both cases, characteristic morphometric features in the form of the “river bed” or “river pattern” are observed. In the first case, effect of the “river bed” is caused by wave-like washing-in and washing-out of solids (sand, in most cases), and in the second case – by the wave abrasion-fatigue wear of the lining.

There are a lot of works concerning streamflow dynamics, among which is classical monograph of M. A. Velikanov “Dynamics of Channel Flows”, which was published in 1949; concerning ball-mill dynamics, there are publications [1, 2], in which some problems of wave wear of rubber lining are considered.

As it is known from hydrodynamics, mass motion of solid particles along the stream bottom at low speeds always occurs in the form of wave motion. These waves move at a rate lower than the stream velocity. There are a number of theories (for example, the Exner theory, the Polya theory and others), which consider deformation of eroding bottom and, in particular, motion of sand waves. The Exner and Polya theories explain both motion and deformation of the sand waves in the process of their moving. The experimental data received by these scientists on morphometric characteristics of the stream in the form of river pattern has much in common with similar features observed when a rubber lining in the ball-tube mill is destructed (i. e. abrasive-fatigue wear): the same wave migrations in the course of time and the same asymmetry along the tube length.

However, these theories do not explain causes of the wave occurrence since they do not take into account a turbulence factor. M. A. Velikanov made an attempt to consider the phenomena of turbulence and developed a model of the sand wave occurrence basing on the effect of the stream pulsing velocities and pressures on the sand grains.

The model is based on a number of postulates and assumptions adopted in hydrodynamics, the essence of which lies in the following.

1. For the rough bed (at presence of sharp ribs and pits), only turbulent flow of the stream should be considered. The turbulent flow features pulsation of velocities and pressures: velocity field and pressure field are irregular, and their values vary in the course of time within the range of some averaged value.

2. Turbulent stream is characterized by the structural patterns, geometry of which can feature certain regularity, and their dimensions are continuously changed according to the random laws; such structural patterns play a dominant role in the streamflows.

3. Turbulent flow of the stream consists of a number of pulsations: large-scale pulsation, which is usually associated with averaged pulsation, and small-scale pulsations of velocity field superimposed on this flow. Large-scale pulsations dominate in diffusion processes, and small-scale pulsations dominate in the processes of en-

ergy dissipation. Viscosity of the stream plays an important role as it transfers motion between the layers and forms a continuous velocity field of the stream; aeration – or saturation of the stream by air bubbles at its turbulent flow – increases dissipation of the energy. According to the principle of minimum dissipation introduced into the hydrodynamics by Rayleigh and Helmholtz, of all of the velocity fields, only those are realized at which dissipation is minimal.

4. A certain relationship exists between turbulence as a structural pattern of the streamflow and morphometric features of the stream bed. Formation of waves at the bottom of the stream is directly connected with pulsation of the velocities, which condition the scale of turbulence, and it is exactly large-scale pulsations that condition geometric dimensions of the waves, which are formed at the bottom of the stream.

5. Distribution of the pulsing velocity values in the turbulent stream follows the probability laws.

Certainly, such a hydrodynamic model constructed for the sand wave dynamics has some differences from the wave wear of the lining; however, a lot of elements in this model are also valid for the problem under consideration. It should be mentioned that, in view of complexity of studying the phenomena of turbulence, no adequate model is available today for the occurrence, motion, and destruction of the sand waves in maximally saturated streamflows; this is also true for occurrence of wave pattern in the protective lining wear in the ball-tube mills. At the same time (and it was confirmed by long practical history), it is the wave-like wearing of the rubber lining, on which service life of the lining and quality of mineral disintegration much depend [1, 3].

Objectives of the article. To develop a synergetic model and, on its basis, a theory of wave abrasive-fatigue wear of rubber lining in the ore ball-tube mills.

Presentation of the main research. Let us use some postulates from Velikanov's model in order to explain a wave-like character of the rubber lining elements wearing in the ball-tube mills. As it is known, rubber refers to the hereditary-viscoelastic media, or, in other words, to the media with great dissipation and ability for large reversible deformations.

The following assumptions are accepted, which do not distort general ideas concerning the mechanism of rubber abrasive-fatigue wearing and are in line with the adopted synergetic model:

- particles (counterbodies) in the stream are of the same size;
- balance equation is accepted in the form given by Polya and Velikanov (the stream is assumed uniform and in steady-state, i.e. two-dimensional problem)

$$\frac{\partial z}{\partial t}+\frac{\partial p}{\partial x}=0,$$

where z is the bottom height; x is a coordinate; $\partial p/\partial x$ is flood of solid matters; if $\partial p/\partial x > 0$ it is the bottom depression; if $\partial p/\partial x < 0$ it is the bottom elevation.

Let us take a segment along the length of the stream between the values of the abscissa x and $x + \delta x$; during a certain time interval δt, volume $p\delta t$ of the flood will enter this segment, and volume $\left(p+\frac{\partial p}{\partial x}x\right)t$ will exit from it.

In this segment, difference $\frac{\partial p}{\partial x}xt$ causes depression of the bottom at $\partial p/\partial x > 0$, i.e. washing-out of the lining plate together with mass transfer of rubber particles; this equation is valid for any time interval with the only condition – it is necessary to formulate a corresponding expression for the solid matter discharge p:

- hydrodynamic conditions are assumed to be unchanged, i.e. flow velocity $\upsilon = \text{const}$;
- as pulp is flowing as a maximally saturated stream consisting of particles of worn balls and ore particles, abrasive-fatigue wear of rubber will occur; when these particles (counterbodies) collide with the lining, mass transfer of rubber will occur, i.e. fragments of material will be torn away; let us assume that tear of one element away from the fragment depends on pulsation of the pressure and counterbody in the stream in accordance with the random law;
- let us accept expression for the flood of solid matters in the form of

$$p=\int_0^\infty Dm(\upsilon)\upsilon d\upsilon,$$

where D is average particle diameter; m is dynamic coefficient of continuity related to the aggregate of particles (counterbodies) moving with velocities within the interval from υ to $\upsilon + \delta\upsilon$; the coefficient m is used as a probability function or a curve of particle distribution by velocities of their motion; the function $m(\upsilon)$, by its physical sense, is proportional to the number of counterbodies moving at a speed υ and, therefore, it must be proportional to the frequency of those values of the pulsing velocities, which move these counterbodies at precisely these velocities;

- distribution of the pulsing velocity values in the turbulent flow is assumed as the normal Gaussian distribution

$$\varphi(u)=\frac{1}{\sqrt{2\pi}\sigma_u}\exp\left[-\frac{(u-\bar{u})^2}{2\sigma_u^2}\right],$$

where u is the flow velocity; $\bar{u}$ is average value of the flow velocity; is root-mean-square deviation.

Having made some transformations, we receive the following equation

$$\varphi(u)=\frac{1}{\sqrt{2\pi\sigma}}\exp\left[-\frac{\nu^2}{2}\right]\frac{1}{\sqrt{2\pi\sigma}\sqrt{1-r^2}}\exp\left(-\frac{\mu^2}{2}\right),$$

where $r=\left(1-\frac{x^2}{\ell^2}\right)^2$; $\mu=\frac{u'-ru_0}{\sigma\sqrt{1-r^2}}=\frac{\upsilon+u_{(0)}-u-ru_0}{\sigma\sqrt{1-r^2}}$;

$\nu=\frac{u_0'}{\sigma}$; $u_0'=u-\bar{u}$; $\bar{u}_{(0)}=\bar{u}$; ℓ is a length of the stream bottom (lining plates) represented in dimensionless designation from 0 to 1; υ is velocity of the counterbody motion; $u_{(0)}$ is flow velocity at the point $x = 0$; u_0 is initial value of the flow velocity.

With these assumptions accepted, we obtain the following expressions for the flood of the solid matters p

$$p = \frac{D}{\sqrt{2\pi}}\left\{\alpha \exp\left(-\frac{\mu_0^2}{2}\right) - \beta \int_{\mu_0}^{+\infty} \exp\left(-\frac{\mu_0^2}{2}\right) d\mu\right\}.$$

By applying this equation to the protective lining, it is possible to obtain probable rate of the lining plate z height change in the course of time

$$\frac{\partial z}{\partial t} = \frac{\partial p}{\partial x} = \frac{kD}{\ell}\psi\left(\xi, \varepsilon, \nu\right), \tag{1}$$

where $\xi = \frac{x}{\ell}$; $\varepsilon = \frac{u_{(0)} - \overline{u}}{\sigma}$; $r = \left(1 - \xi^2\right)^2$; $\mu_0 = \frac{\varepsilon - r\nu}{\sqrt{1 - r^2}}$;

$$\psi = \frac{dr}{d\xi}\left\{\frac{1}{\sqrt{1 - r^2}}\frac{1}{\sqrt{2\pi}}e^{\frac{-\mu_0^2}{2}} - \nu\frac{1}{\sqrt{2\pi}}\int_{\mu_0}^{+\infty} e^{-\frac{\mu_0^2}{2}} d\mu\right\}.$$

Further solution of this complex equation is possible only by numerical methods.

For ξ with values from 0 to 1, values of r, $\frac{dr}{d\xi}$, and $\frac{r}{\sqrt{1 - r^2}}$ were calculated with intervals of 0.05, and diagram (Fig. 1) of ψ dependence on the ξ for $\varepsilon = 3$ and $\nu = 3$ was constructed.

It should be stated that to date, there is no experimental information (especially in terms of statistics), which could help to solve the equation and obtain the rate of the lining plate height change in the course of time. Difficulties associated with this procedure are obvious: turbulent flow of the solid saturated stream, nonlinear nature of interaction between the lining and inputs, and others.

Nevertheless, the described synergetic model and theory make it possible to show approximately the wave-like character of the lining destruction.

Following Velikanov's model, and in order to analyze the results, let us proceed from physical nature of the dimensionless values of ε and ν.

The positive value of ε corresponds to the case when average velocity is less than that, which is required for the counterbody for tearing a rubber fragment away from the lining or, in other words, the fragment tearing-away (at positive v) is possible only beginning from the moment when the inequality

$$\overline{u} + u_0' > u_{(0)} \quad \text{or} \quad v \geq 3$$

is satisfied.

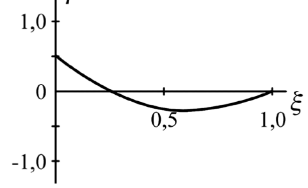

Fig. 1. Dependence $\psi(\xi)$

If value of ε is negative (when $\overline{u} > u_{(0)}$), the rubber fragment tearing-away starts at average velocity (even when $v = 0$) and terminates only at negative values satisfying the inequality $v \leq \varepsilon$.

As it can be seen from the diagram (Fig. 1), the following change in the lining height occurs in the segment from 0 to 1: at $\varepsilon = +3$ and $v = 3$, there is an intensive wear of the plate, which becomes less intensive when ξ approaches 1. This phenomenon is explained by the fact that influence of the higher velocity at the initial point decreases along the length, and the lower pulsating velocities, which are insufficient for tearing a rubber fragment away from the matrix, are more probable. In the sequel, we obtain a clear wave of the lining wearing, which is also observed in practice [1, 3].

It is exactly due to turbulence and pulsations of pressures and velocities, in particular, that surface of even a completely flat rubber lining becomes wave-like in the course of time.

Therefore, the equation (1) allows describing the nature of wave abrasive-fatigue wear of the rubber lining.

It should be stressed that effect of the wave-like wear is characteristic for all types of the lining (Plate-Wave, Plate-Lifter-Wave, and others) and, practically, for all types of the tube mills. This fact confirms generality of dynamic processes occurred in the tube mills at disintegration of mineral raw materials and offers the challenge for creating optimal designs of the linings.

Let us consider some cases when the theory under consideration is applied to the dynamics of the tube mills.

Peculiarities of the rubber lining destruction. The process of the lining destruction has a space form and lasts for a certain period of time – from several hours to several years. The destruction process itself is a part of a more general process – interaction between the lining and technological material – and, therefore, is a function of many variables: geometric dimensions of the tube, velocity of its rotation, properties of material being processed (abrasiveness, lump size, presence of solid matter in the pulp, and so forth), geometry of the rubber lining, physical and mechanical characteristics of rubber, and others. Therefore, destruction can be considered as a multi-vector, stochastic and non-linear process. Changing of any of the system parameters, for example, the diameter of the tube or velocity of its rotation, can lead to change in the lining destruction character. There is another critical problem: due to the specific operation of the mills, modeling of processes occurring in them faces a number of experimental difficulties, sometimes insurmountable. Therefore, these processes can be studied indirectly, through their consequences: for example, the mechanism of wearing can be studied by morphology of surface destruction and degree of the lining element wear, and so forth.

As it is noted in [1, 3], key processes of technological material disintegration occur in the charging area. However, processes relating directly to the lining and, above all, its service life and specificity of its destruction, occur at the boundary between the pulp-lining surface phases.

Phenomenon of deterministic chaos. It is known that motion of inputs inside the tube of the mill features a complex hierarchical structure with turbulent flow of pulp and vortex-like pulsating flow of the charged inputs; generally, this process of flow is stochastic and nonlinear. Turbulence as an irregular behavior of the nonlinear system is directly connected with deterministic multidimensional chaos and is characterized by complex space-time behavior.

In mechanics, deterministic chaos is understood as an irregular or chaotic motion caused by nonlinearity of the medium, for which dynamic laws of motion uniquely determine temporal evolution of the system state [4]. Shear turbulence of the pulp is observed in the mill tube, especially at the boundary between the pulp-lining phases, while deterministic chaos can occur at the local points of the charging area.

Therefore, the motion of inputs inside the mill can be considered as motion of an open system with great deviations of parameters from equilibrium state, nonlinearity of basic characteristics, and cooperative behavior of the subsystems, i.e. behavior of new space-time structures that constantly appear in the sector. Such structures appear due to the effect of some interrelated factors, such as: turbulent flow of pulp; slip of inputs relative to the lining and, as a result, occurrence of frictional oscillations; great energy dissipation in the charging area; asymmetric rotation of the tube caused by asymmetric distribution of inputs and structural defects, and others. Geometry of structural patterns in the mill's volume can feature a certain regularity due to the existence of some general trend of geometry formation (see below for more details), and their dimensions will vary according to the random laws. Of all the new structural patterns, the most stable are those in which principle of minimum dissipation acts. It is necessary to mention an important role of dissipation in dynamic behavior of inputs inside the tube: dissipation underlies hydrodynamic pulsations of inputs and the mechanism of geometry formation in the charging area; some part of the dissipated energy is converted into thermal energy and heats the inputs, and, thereby, reduces viscosity of the pulp, changes slip coefficient, and negatively affects the lining service life. In particular, one should note the great role of energy dissipation in the interaction between the charging area and rubber lining: mutual adaptation of the stream and bed of the lining contributes to determining hydromorphological dependences, which characterize both velocity field of the stream in local regions and morphometric parameters of the lining relief. Of all the possible options, only those are realized at which stream spends the least amount of energy, i.e. energy dissipation of the system must be minimal.

If motion of inputs is considered in the form of logarithmic spiral, then its trace is most clearly manifested at the boundary between the pulp-lining phases. This fact can serve as a very important argument for supporting the statement of D. K. Kryukov and his followers that working surface of the metal lining is worn precisely in the form of logarithmic spiral. For rubber lining, wear in the form of logarithmic spiral presents a special case.

Mechanics of input motion in the bottom zone of the back wave has one important feature, which was determined experimentally: at the junction between the inputs and the lining, a powerful structural pattern is observed, which is caused by turbulent flow of the pulp (Fig. 2). According to the hydrodynamics of motion of maximally saturated stream, a space with reduced pressure is always formed behind the moving body or compact system of solid particles, and it is exactly pressure difference that determines dynamic motion of this system. Besides, regime of turbulent motion is always accompanied by formation of vortices behind the body.

This structural pattern (earlier, in [3], it was called "monodispersoid") is characterized by instability of shape and sizes and instable motion (hunting). In the volume of monodispersoid, there is pulp, metal balls and inputs of various sizes to be ground; all these objects are in a quasi-liquefied (boiling) state due to turbulence. During a certain period of time, quasi-stability of the monodispersoid shape and sizes is dependent on velocity of the tube rotation, level of its filling, rate of input sliding relatively to the lining (coefficient of friction), viscosity of the pulp, and relief of the lining. This structural pattern is one of the key factors of rubber lining destruction because it is exactly its volume where metal balls or lumps of inputs to be ground directly touch surface of the lining; besides, motion of the balls is intensive and chaotic. Therefore, various types of impact on the rubber lining are observed in the contact zone: shock, impression and abrasive-fatigue wear. This statement does not contradict the results of investigations obtained by D. Kryukov, P. Malyarov [2, 5], and V. Kopchenkov [6].

Phenomenon of deterministic chaos, together with other effects (energy dissipation in the system, turbulent flow of pulp, the tube pulsating rotation, and others) is the basis for wave-like motion of inputs inside the mill and local destruction of the lining elements. In massive rubber elements (plates, lifters) with an inhomogeneous stress field, accumulation of damage on their surface and in their volume will also be inhomogeneous. Therefore, with all other conditions being equal (impact mode, external environment, and so forth), place and time of occurrence of their destruction is of probabilistic nature. In practice, this phenomenon is manifested in different rate of wear of plates or lifters located near to each other, different morphometric features of their surface destruction and their different service life before failure.

Regularity of wave-like motion of the inputs inside the tube of the mill. According to the laws of hydrodynamics, just turbulence of the stream flow is quite enough for the flat bed to be transformed into wavy-shaped bed. If we consider a rubber lining as a hydrodynamic form (Fig. 3), then, according to the Boussinesq theory, peak velocity in the spillway with a sharp edge will be ob-

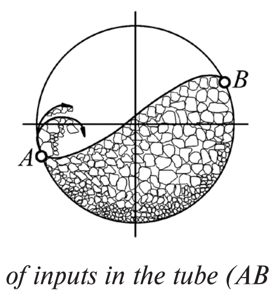

Fig. 2. Motion of inputs in the tube (AB is trace of compact zone)

served near the edge. In this case, isotachs (the stream guides) will start from the bottom and curve upward; bottom velocity decreases, though ascending flow of the pulp increases drastically. Maximum of the bottom velocity is observed in the roller of the shallow, i. e. at the edge of the lifter or plate. Precisely this distribution of velocities is behind the mechanism of abrasive-fatigue wear of the plates and lifters in rubber lining (Figs. 3, 4).

At the same time, morphometric relief of the lining is also explained by transverse circulation, which occurs in the stream and is caused by centrifugal force. This important fact, namely, that many phenomena associated with the stream flow are connected with intensity of mixing due to the pulsation of velocities, and this is exactly what is observed in the mill tube, was stated in early classical works of Boussinesq.

Long-term industrial testing of mills with rubber lining has shown that there is the closest connection between the structural shape of the stream turbulence and morphometric features of the lining relief. Both the stream and the lining are in a certain interaction and represent some unity of two mutually opposite aspects of the same phenomenon or, in other words, mechanical essence of the process of input motion can be reduced to the interaction between the stream and the lining: the stream forms relief of the lining, and the lining, in its turn, forms the stream pattern. Such interaction occurs in a certain space-time continuum: effect of the lining relief is directly and very quickly transferred to the velocity field and to geometry and sizes of structural patterns of the stream. Reverse effect – impact of the stream on the pattern of lining relief – is realized for a relatively long period of time, duration of which depends mainly on structural features of the stream and mechanical characteristics of the lining. Besides, this effect remains constant only for a short period of time: due to wear, both the rubber lining relief and geometric dimensions and, consequently, structural features of the stream will change.

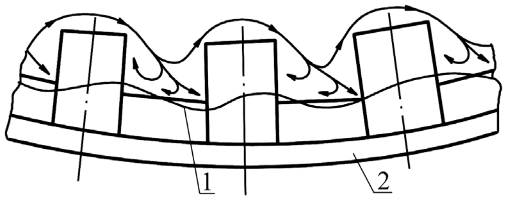

Fig. 3. The stream guides in the rubber lining "plate-lifter":
1 – curve of natural wear; 2 – tube

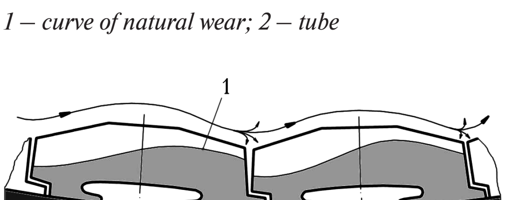

Fig. 4. The stream guides in the rubber lining "plate-plate":
1 – curve of natural wear; 2 – tube

Thereby, the lining relief is in a certain dependence on the stream pattern and is, in a sense, a trace of structural patterns of the stream turbulence. New patterns of the lining surface formed in this case are directly related to the pulsation of velocities and to linear correlation between instantaneous velocities, which conditions scale of this turbulence.

In practice, such mutual adaptability between the stream and lining occurs during some period of time, sometimes during several hundred hours. During this period, productivity of the mills decreases; afterwards, when a certain balance is established between the lining and the stream, the mill comes to its optimal operating mode.

Finally, such hydromorphological characteristics of the stream and morphometric relief of the lining are formed, at which the stream, in order to overcome all resistances, spends minimum energy or minimum dissipation. In other words, of all possible structural patterns of the stream, only those are realized, at which energy dissipation is minimal. At the steady motion of the mill, exactly this structural pattern of the stream is observed, and it corresponds to this principle formulated by Rayleigh-Helmholtz.

Here occurs an immediate task for the designers: it is necessary to choose such geometrical shape of the rubber lining, which is generated over time as a result of certain harmonic equilibrium between the stream and relief of the lining.

Apparently, energy dissipation plays an important role both in formation of morphometric relief of the lining and in the formation of the stream bed; it does not only condition the size and symmetry of the lining waves but also forms rhythm of the river pattern. Let us consider such formation on the example of the mill МШЦ 5.5 × 6.5.

In the first case, a single-wave metal lining with a ribbed surface was used. In the process of operation, working surface of the lining gradually took a wave pattern (Fig. 5); in this case, a clear asymmetry of the waves was observed (distance between the protrusions of neighboring waves differed by 10 % and more) with practically no river pattern.

In the second case, rubber lining "Plate-Wave" had a trapezoidal shape. During the operation, working surface of the lining took a wave pattern (Fig. 6); in this case, some asymmetry of the waves and river pattern of the bed were also observed. It is characteristic that the wave asymmetry (with less distance between the protrusions by about 15 %) was rhythmically repeated with every 36°, or approximately every four plates of lining. Principally, this is an evidence of at least two pulsations of the flow: amplitude of the first pulsation was about 12°, and amplitude of the second pulsation was about 36°.

As it is seen, the metal lining interacted with the stream as an elastic body with insignificant dissipation of energy, hence, featuring clear geometrical symmetry of the waves (asymmetry was observed in length difference between the protuberances of the waves) and practically with no river pattern. The rubber lining, being a viscoelastic and highly dissipative medium, reacted more flexibly to the turbulent flow; hence, there is not only asymmetry of the waves, but also a certain rhythm of the river pattern. All this confirms the presence of effect of hydraulic cush-

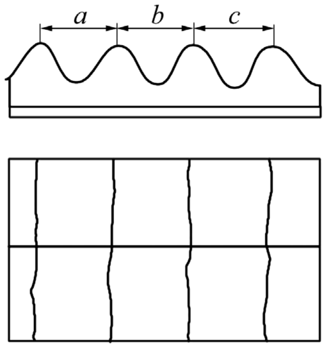

Fig. 5. Wave character of the metal lining wear with the effect of asymmetric wavelengths ($a > b > c$) and practically symmetric waves along the bed

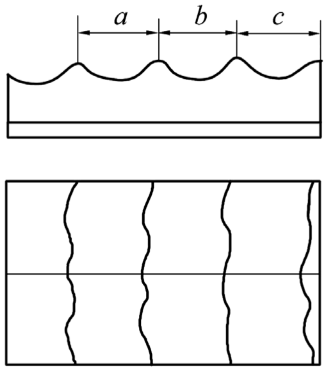

Fig. 6. Wave-like character of the rubber lining wear with the effect of asymmetric wavelengths ($a > b > c$) and river pattern of the bed

ion on the boundary between the input-lining phases with a strong turbulence of the stream and with a vortex-like spiral flow of the pulp. Despite the probabilistic nature of generation and decay of pulsation and origin and destruction of structural patterns, motion of the segment with the charged inputs, nevertheless, follows some strict regularity and certain dynamic asymmetry, which is manifested in a certain pattern-formation trend. This trend exists in the space-time continuum, and its manifestation, of course, must be expressed through certain universal constants. It is difficult to specify parameters of such a volumetric-spatial process experimentally as they manifest themselves only in the form of secondary structural patterns, i.e. traces of waves, river patterns and other morphometric features on the lining surface.

The presented synergetic model and theory of wave abrasive-fatigue wear of rubber as a hereditary-viscoelastic medium helps, at least in the first approximation, to explain the key effects and morphometric features of the lining destruction: occurrence of the wave-like wear, displacement of waves in the course of time, asymmetry of waves, river patterns of wear, and others.

As an example, it is possible to consider wear of the rubber lining of the "Plate-Wave" type (Fig. 6) in the mill with disintegration of mineral raw materials. Studies showed that the lining became wave-like immediately after the first several hours of the mill operation: asymmetry of waves and their displacement in the course of time were observed. As it is shown above, these effects are associated with a complex mechanism of abrasive-fatigue wear of rubber. And vibrational motion of the charged inputs in combination with turbulent nature of highly saturated two-phase stream flow substantially complicates general nature of interaction between the charged inputs and lining, and makes it difficult to construct an adequate mathematical model.

According to the authors, this finding confirms the previously expressed opinion [3] that an integral criterion is needed for computing the rubber lining. To this end, energy of abrasive-fatigue wear was proposed as such criterion, which is determined during direct experimental studies. Practice has confirmed correctness of this concept: the newly designed rubber lining of the "Wave" type together with the developed theory have confirmed their competitiveness in the world market.

The new resource- and energy-saving technology. Use of the "Wave" lining allowed creating a new energy-saving technology (the ES Technology) for grinding ores in the ball mills [1, 3]. Thanks to this technology, the following results were obtained with the mills МШЦ 3.6×5.5 designed for the second and third stages of iron-ore grinding: volume of finished class increased by 17–29 %; consumption of the grinding bodies decreased by 10 %; specific power consumption by the whole technological section decreased by 10–12 %. For example, in comparison with metal lining, the rubber lining "Plate-Wave" in the ball mills of the 2nd and 3rd stages of grinding allowed: reducing weight of the lining set by more than 3–5 times and, thereby, increasing service life of the supporting bearings; cutting costs of assembling-disassembling works on replacing the worn linings and reducing risk of accidents; reducing noise level by 2–3 times; increasing duty factor of the mills by 3–5 % (in comparison with metal lining, rubber lining is much thinner); providing operation of the mill with designed capacity starting from the first hours of its work; reducing consumption of grinding bodies by 6–10 %; reducing electricity consumption by 7–9 % (including 10–12 % reduce of consumption by the whole technological section); increasing service life by 80–150 %; double duration of interrepair cycle; reducing the mill downtime during schedule and off-schedule repairs by 25–30 %; increasing the volume of the finished class –0.056 mm by 17–29 % (with metal lining, increase of the finished class is 10–12 %).

Some more touches in addition: with rubber lining, there is no leakage of pulp; metal lining requires regular inspection, bolt-tightening and repair though with rubber lining, bolts are not subject to regular tightening.

The next step in developing the mill lining of the "Wave" type was creation of lining for the first stage of grinding by balls with diameter of 100 mm (in future, with diameter of 120 mm). To this end, two concepts are available.

The first concept assumed usage of rubber-metal lining: in order to increase bearing capacity, metal inserts (studs) made of high-manganese steel (G.M-Wave lining) were vulcanized into the rubber elements. In the SevGOK ore-dressing and processing enterprise, the rubber-metal lining of the G.M-Wave type was installed in the МШР 3.6 × 4.0 mill for the first stage of grinding with a ball diameter of 100 mm; it was for the first time in the world practice of the hard iron-ore disintegration. Test results were as follows: electricity consumption decreased by 5 %; specific consumption of grinding bodies decreased by 5 %; volume of the finished class increased by 10–12 %; service life before failure was 9044 hours.

The second concept assumed usage of combination of original rubber brand (is manufactured by nanotechnology and features higher dissipation of energy) and high-profile plates of the "H-Wave Plate" type for lining. In 2013, the SevGOK Company initiated comparative industrial tests of such lining and standard metal lining produced by the KZGO Company. The lining was installed in the ball mills of the МШР 3.6 × 4.0 type for the first stage of grinding with unloading through the grate. The ore-dressing factory No. 1 processes quartzite from the Pervomaiske and Annovske deposits in the Kryvyi Rih basin. In the cycle of ore preparation of the first stage of grinding, steel balls with diameter of 100 mm of the third group of hardness were used.

The rubber lining of the "H-Wave Plate" type (the first three rings from the side of feeding sector were with the plates with thickness of 270 mm, the rest rings had thickness of 240 mm) was installed at the mill No. 121, and the metal lining was installed at the mill No. 111. The both МШР 3.6 × 4.0 mills were installed in the cycle of the first stage of ore preparation in the same technical section No. 11–12 and operated in the closed cycle with a special classifier of the 2КСН 12.5/2.4 type.

The technological section No. 11-12 is equipped with automated process control system (APCS) for working in the "Optimization" and "Stability" modes. When the mill No. 121 with the rubber lining was put into service, it was not possible to adjust the first stage of grinding with the help of the APCS because of significant reduction of the mill noise characteristics: the noise sensor practically did not react to the operation of the mill.

Thereby, due to changed noise characteristics of the mill with the rubber lining "H-Wave Plate", the cycle of the first stage of grinding of section No. 11 worked in the "Stabilization" mode.

During the entire test period, the ore-dressing plant РОФ-1 received crushed ore containing 12 % of size grade +20 mm, and specific work on the ore-burden disintegration was 8.8 kgm/cm^3; feed of ore with size of 50–200 mm (3–5 %) was random.

During the industrial tests, the main technological parameters were monitored; results of comparative tests are presented in Table.

Comparative analysis of the operation of the mill with a rubber lining "Wave" and mill with metal lining shows that usage of rubber lining in the cycle of ore preparation of the first stage of grinding results in increased volume of finished class in the samples discharged from the mill and increased specific load of the mill due to the additionally formed new class. Because of the changed motion of inputs inside the mill with rubber lining "H-Wave Plate", minerals are better opened and, as a consequence, iron content in the concentrate of the first stage of magnetic separation is higher.

Table

Results of comparative industrial tests

	Mill No. 121 (rubber lining)	Mill No. 111 (metal lining)
Average hourly load by initial ore, t/h	107	108
Content of finished class –0.056 mm samples discharged from the mill, %	39.9	37.1
Specific load by additionally formed new class 0.056 mm	1.486	1.451
Content of Fe in concentrate the 1st stage of magnetic separation	51.4	50.8

The test results:

1. The mill МШР 3.6 × 4.0 with rubber lining and with rubber-metal grate manufactured by the Valsa-GTV Company successfully passed industrial tests and reached the end of its guaranteed service life: operating time was 6850 hours.

2. The wave profile of the rubber lining had positive effect on technological parameters of grinding: content of finished class in the samples discharged from the mill increased by 2.8 %, specific load of the mill was increased by additionally formed new class, and iron content in the concentrate of the first stage of magnetic separation increased by 0.6 %.

3. In the ore-dressing plant РОФ-1 of the SevGOK Company, at the first stage of grinding, steel balls with a diameter of 100 mm of the 3rd hardness group produced by the Azovstal Company were used as grinding bodies. During the whole period of operation, specific consumption of the balls was 0.586 kg/t of ore in the mill No. 121, and 0.645 kg/t of ore in the mill No. 111; it means that specific consumption of the balls decreased by 10.0 %.

4. Analysis of electric power consumed by the mill with rubber lining during the period under consideration showed that specific power consumption per one ton of processed ore decreased by 5.0 % if to compare with mill with metal lining.

5. One should make a special emphasis on high survivability of the rubber lining of the "H-Wave Plate" type and its good resistance to variable size of technological material. For certain technical reasons, during the whole period of the rubber lining operation, inputs included 3–5 % of lump size 50–200 mm, which were not typical for the first stage of grinding. Nevertheless, the mill productivity was not essentially changed, grinding fineness increased from 30 to 48 %, and finished class -0.056 mm in the samples discharged from the mill contained greater amount of fine fraction including nanoparticles (less than 30–40 μ).

Correct choice of the method for grinding hard iron ores by the ES-Technology and using rubber lining of the H-Wave Plate type was confirmed by technological tests conducted by the Poltava GOK Company at the mill МШР 4.0 × 5.0 for the first stage of grinding with balls with diameter of 100 mm.

The comparative tests were conducted from March 2014 to June 2015 at two mills МШР 4.0 × 5.0: mill No. 72 with rubber lining of the H-Wave Plate type (first two rows were 270 mm thick, the rest were 240 mm thick) and mill No. 52 with metal lining (ЛМЦ steel grade); during the whole period of testing, crushed ore of 6–8 mm class was fed to the sections.

6. The mill No. 72 with rubber lining of the H-Wave Plate type in the tube and rubber-metal grate successfully passed industrial tests: with warranty period of 8.000 hours, it factually worked for 8891 hours. The tests were terminated due to wear of the grate; generally, minimum residual thickness of the lining plates in the tube was 110 mm, and 45 mm only in a few areas.

7. During the test period, overall productivity of the ore processing sections in the mills No. 72 and No. 52 was approximately the same; deviation did not exceed 0.34 %.

8. Specific consumption of grinding bodies for processing one ton of ore in the mill No. 72 with rubber lining was 0.528 kg/t mainly due to failure of the rubber-metal grate; in the mill No. 52 with metal lining, this consumption was 0.531 kg/t (0.6 % deviation is within permissible error).

9. Specific electric power consumption per one ton of additionally formed finished new class of size –0.071 mm in the mill No. 72 with rubber lining was 20.905 and 23.415 kWh/t in the mill No. 52 with metal lining; reduction of specific power consumption in the mill with rubber lining was 2.511 kWh/t (10.72 %); reduction of specific power consumption per one ton of processed ore was 7.67 %.

Fig. 7 shows rubber lining of the "H-Wave Plate" type manufactured by the Valsa-GTV Company: wave pattern of abrasive-fatigue wear of the plates is clearly observed, hence, confirming correctness of calculations for the plate layout in the tube. Fig. 7 also shows a profilogram of the rubber plate wear, which is satisfactorily described by algebraic curve of the witch of the Agnesi type, which is the witness of the following: in the tube of the mill, such interaction is established between the inputs and lining, at which minimum energy is consumed, and principle of minimum entropy production (or minimum chaos in the system, in other words) is realized. Such harmonic interaction in the complex multiparameter "input-lining" system is achieved primarily through the use of the new original high-profile rubber linings of the "H-Wave Plate" type. This wave profile of the lining working surface appeared during the first one-two weeks and lasted for about 8 months of operation, and ensured high-quality ore grinding and optimal service life of the plates during 9000 hours.

Fig. 7. Rubber lining of the "H-Wave Plate" type in the mill МШР 4.0 × 5.0 (the Poltava GOK Company, the first stage of grinding, iron ore, balls ∅100 mm, t = 8891 h)

Some conclusions made on the basis the industrial test results.

1. Taking into account general deterioration of mineral quality and, therefore, the need for finer grinding of mineral raw materials (up to class 40 μm and finer), it is very important for the mining and dressing enterprises to improve productivity of grinding sectors and reduce expenditures per unit of the processed products. The main components of this problem – improvement of mill designs and use of rational technological schemes – have reached, to some extent, their end point.

2. Practice has shown [7–9] that power intensity of the grinding process can be reduced, and productivity of the mills (ball mills, autogenous mills and semi-autogenous mills), both in terms of energy consumption and finished class, can be improved mainly through the use of rubber and rubber-metal linings.

3. The Valsa-GTV Company designed new models of the rubber linings; seven-year experience of their application by more than 20 enterprises in different countries (approximately 180 mills for different technological functions) showed clear advantages of the rubber lining over the metal lining.

4. Use of new rubber-lining designs at all stages of grinding process allows obtaining additional energy reserve for the mills, increasing the volume of products ground by the ore-dressing and processing enterprise on the whole by 10–15 %, and significantly reducing capital and operating costs.

5. It is for the first time in the world practice when rubber lining of high profile ("H-Wave Plate") has been used at the first stage of hard iron-ore grinding by balls with a diameter of 100 mm. Test results were as follows: service life before failure was 7000–9000 hours; saving of ball consumption was 10 %, and saving of power consumption was up to 5–8 %.

6. It is also for the first time when new designs of rubber lining have allowed creating a new resource- and energy-saving technology (the ES-Technology) for grinding mineral raw materials in ball mills.

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Синергетична модель хвильового абразивно-втомного зносу гумової футерівки в барабанних кульових млинах

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Мета. Розробка синергетичної моделі та, на її основі, теорії хвильового абразивно-втомного зносу гумової футерівки кульових млинів; підтвердження аналітичної моделі результатами промислових випробувань.

Методика. Використаний комплексний підхід, що включає аналітичні дослідження й результати значних промислових випробувань кульових млинів різного типу.

Результати. На основі розробленої синергетичної моделі й рівнянь Поліа-Веліканова отримані математичні рівняння, що дозволяють описати природу хвильового абразивно-втомного зносу гумової футерівки; на основі теорії детермінованого хаосу встановлені основні закономірності хвильового руху, внутрішньомлинового завантаження в барабані млина; наведені результати промислових випробувань, що підтверджують правильність аналітичної моделі.

Наукова новизна. Розроблена оригінальна синергетична модель і, на її основі, створена теорія хвильового абразивно-втомного зносу гумової футерівки кульових млинів. Встановлені основні закономірності хвильового руху внутрішньомлинового завантаження в барабані млина.

Практична значимість. На основі аналітичних розрахунків створені та впроваджені конструкції гумової футерівки типу „Хвиля", конкурентоспроможні на світовому ринку. На основі футеровок створена та впроваджена у промисловість нова енерго- й ресурсозберігаюча технологія дезінтеграції мінеральної сировини в барабанних кульових рудоподрібнюючих млинах.

Ключові слова: *кульові млини, синергетична модель, абразивно-втомний знос, хвильова теорія*

Синергетическая модель волнового абразивно-усталостного износа резиновой футеровки в барабанных шаровых мельницах

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Цель. Разработка синергетической модели и, на её основе, теории волнового абразивно-усталостного износа резиновой футеровки шаровых мельниц; подтверждение аналитической модели результатами промышленных испытаний.

Методика. Использован комплексный подход, включающий аналитические исследования и результаты обширных промышленных испытаний шаровых мельниц различного типа.

Результаты. На основе разработанной синергетической модели и уравнений Полиа-Великанова получены математические уравнения, позволяющие описать природу волнового абразивно-усталостного износа резиновой футеровки; на основе теории детерминированного хаоса установлены основные закономерности волнового движения внутримельничной загрузки в барабане мельницы; приведены результаты промышленных испытаний, подтверждающие правильность аналитической модели.

Научная новизна. Разработана оригинальная синергетическая модель и, на её основе, создана теория волнового абразивно-усталостного износа резиновой футеровки шаровых мельниц. Установлены основные закономерности волнового движения внутримельничной загрузки в барабане мельницы.

Практическая значимость. На основе аналитических расчётов созданы и внедрены конструкции резиновой футеровки типа „Волна", конкурентоспособные на мировом рынке. На основе футеровок создана и внедрена в промышленность новая энерго- и ресурсосберегающая технология дезинтеграции минерального сырья в барабанных шаровых рудоизмельчительных мельницах.

Ключевые слова: *шаровые мельницы, синергетическая модель, абразивно-усталостный износ, волновая теория*

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