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PARALLEL COMPUTATIONAL ALGORITHMS IN THERMAL PROCESSES IN METALLURGY AND MINING

Purpose. Formation of parallel algorithms in the thermal process simulation in metallurgy and mining. The proposed parallel form of the algorithms must be maximal, and, therefore, have the minimum possible implementation time in parallel computing systems. Elimination of recurrent computation structure of the desired decision vectors, which, as a rule, leads to the rounding errors accumulation. Such class simulation problems are realized by multiprocessor computing systems.

Methodology. Implementation of parallelizing process of mathematical problem definition is realized by an approach based on the "odd-even" reduction algorithm. The essence of this approach lies in exclusion of simulation coefficients of the process under research by realizing elementary rows transformations of the constructed equations system. A directly parallel form of the algorithm for solving problems is realized by a numerical-analytical approach. It is shown that the compiled parallel form is the maximum, which, in turn, provides minimum time for solving the set problems by multiprocessor computing systems.

Findings. The presented research studies in this paper showed a high efficiency of parallelization of systems of tridiagonal structure linear algebraic equations by example of solving thermal problems. The proposed numerical-analytical method for parallelizing tridiagonal systems does not impose any restrictions on the grid nodes topology of the computational domain. With respect to parallel computations of arithmetic expressions, the original data error is separated from the rounding operations by the proposed method. This approach excludes the recurrent structure of the desired decision vectors computation, which, as a rule, leads to accumulation of rounding errors. The proposed parallel form of the algorithms must be maximal, and, therefore, have the minimum possible implementation time in parallel computing systems. Computational experiments conducted by a multiprocessor computer system showed high efficiency of the developed parallel algorithms.

Originality. Within decomposition algorithms, based on the "odd-even" reduction method, a new approach to the distributed solution of linear algebraic equation systems is proposed for the first time, which differs from the known methods in the closed parallel form with respect to the central grid node and with a high degree of vectorization. There was proposed, analyzed and implemented a new approach to the solution of metallurgical production problems, which allows increasing economy, productivity and speed of computations. It is proved that this approach provides the highest computation vectorization degree, predetermines the maximum parallel algorithmic form and, as a consequence, the minimum possible time for implementing algorithms on parallel computing systems.

Practical value. By using a high-performance multiprocessor system, the developed approach allows processing and interpreting the thermal experiments results, and achieving a high accuracy degree, a significant reduction in the processing time of experimental data.

Keywords: *computational vectorization, parallelization algorithms, multiprocessor systems, thermal problems*

Introduction. The mining and metallurgical industry faces a lot of diverse and interrelated processes. Thus, the thermal regime is one of the main factors that determines physical and mechanical properties of rocks. At the same time, the physical-technical justification of the thermal regime of excavations ensures safe mining of deposits. In the metallurgical industry, these are technologies for smelting and casting iron-carbon alloys, pro-

cesses of heating and cooling the metal, and thermal treatment of metal products.

The recent years practice shows that neither the mining and metallurgical production processes intensification, nor the constructive improvement of corresponding equipment is possible without the study and analysis of heat and mass transfer by mathematical simulation methods. Mathematical simulation is based on computer technology. Its importance is growing due to the fact that modern science and technology constantly requires data on such processes, which experimental study both in laboratory and field conditions is very difficult and expensive, and in some cases merely impossible. The classical approach to the thermal problem solution of metallurgy and mining is not always suitable for multiprocessor computing systems because of the low speed of solving the problem, low accuracy, and large amount of memory of computing resources. In this connection, the tools of computational mathematics stimulate development and realization of new ideas. And then, as a consequence, a new approach for constructing numerical experiments is being created based on a more advanced mathematical apparatus aimed towards the use of multiprocessor computing systems.

Increasing the capabilities of the designing computer technology has always influenced development of computational mathematics, especially with regard to numerical methods and corresponding software. Until recently, technological progress has not caused a radical change in the computing system structure. Therefore, increase in the speed of single-processor computers and even advent of machine and system complexes with a relatively small number of processors working simultaneously did not require a thorough research on either numerical methods or software developed before. Now the situation has changed dramatically.

At present, new computing systems are characterized by the possibility of simultaneous and parallel use of a large number of processors for processing relevant information. The creation of such systems has identified one of the most important areas for increasing the speed of solving complex and time-consuming problems. The operation experience of the first parallel systems has shown that for their effective use it is necessary to change the structure of numerical methods radically.

Obviously, the class of problems that can be solved by parallel algorithms is not only of a diverse type, such tasks are very important for industry and economy. However, it should be borne in mind that the problem parallelism depends not only on its physical meaning, but also on the chosen numerical algorithm. This paper shows possibility of creating the maximum parallel form of computational algorithms by example of solving thermal problems of mathematical physics.

Problem statement and latest achievements analysis in this field. The problem of developing numerical methods for studying the thermal processes of mining and metallurgical complex is beyond any doubt today. Thus, the need to take into account the heat transfer processes in ventilation computations is discussed in papers [1, 2], a rather promising approach to the description of thermal processes in metallurgy is presented in papers [3, 4].

At the same time, it should be noted that the service packs used to study thermal processes are based on the processing of data sets, ordered relatively to grid nodes. Herewith, in the practice of using finite-difference approximation methods, there is a stable opinion about the change uncertainty in the unknown function in the intervals between the nodal points of the grid area. Thus, according to the authors of papers [5, 6], the simple ideas that underlie the primitive replacement of derivatives by finite differences, without analysis and ignoring specific properties of the particular class of problem solutions, cannot be successful. When compiling a computational algorithm, it is necessary to use a priori information about the problem, and first of all about its membership in this or that class of smoothness of functions that describe corresponding processes. This approach became the basis for the distributed simulation of decision vectors for applied problems of mining and metallurgical complex.

Significant computation acceleration of finite-difference programs is achieved due to the parallelization effect [7]. Special attention should be paid to numerical and analytical algorithms for solving the problems stated [8]. The high acceleration of computations in comparison with the finite-difference approach can be explained by analytical solutions that allow computing simultaneously and in parallel across all time layers without using combined memory. Consequently, one of the most promising approaches to mathematical simulation of problems that is being researched in this paper is the one based on numerical-analytical solutions.

In this regard, an effective tool for studying heat and mass transfer in mining and metallurgical production may be considered the application of parallel computing technologies on distributed cluster-type systems [9, 10], which have a relatively low cost and can be easily scaled by both the number of processors and by the amount of RAM. In this paper on the example of solving the heat conduction problem, we present algorithms for solving such problems, which differ by the greatest parallelism degree.

Research purpose and objectives. Classical methods for solving thermal problems of mining and metallurgical complex when using multiprocessor computing systems are solved no faster, and sometimes much more slowly, than when using single-processor computers. This circumstance is explained by the recurrent approach, which is the basis of classical methods. In this connection, this research purpose is to construct the most parallel forms in the simulation of thermal problems. In this regard, this research purpose is to construct the most parallel structures of algorithms for solving thermal problems and their mapping to the architecture of multiprocessor systems.

To achieve this goal, it is necessary to solve the following problems:

- the algorithm of "odd-even" reduction is the basis of the method for parallelizing a mathematical problem, since it contains many possibilities for parallelization;

- parallelizing the systems of the tridiagonal structure with a numerical-analytical approach, which predetermines their maximally parallel algorithmic form;

- take into account the necessary smoothness of original solution and input data for a particular problem, which allows adapting the computational algorithm to the numerical solution singularities.

Materials and research methods. In this paper, computational experiments were carried out with a multiprocessor system [11]. Let us note that recently the process of creating the high-performance computing systems has been developed, for the most part in one direction - the unification of many processors for parallel execution of one large and complex problem. For this reason, the concept of a supercomputer and a parallel (multiprocessor) computer system is currently identified. At the same time, for building supercomputers, serial microprocessors are used, provided with their own local memory, which are connected by means of some communication environment. This architecture has several advantages: to increase the performance of such a system, one can add the necessary number of processors. On the other hand, knowing the required processing power, one can easily select the necessary system configuration. The name of such systems emphasizes the theoretically unlimited scalability of devices of this class.

Analysis of the ways of development of high-performance installations shows that a real breakthrough in mastering parallel computing technologies can be achieved by developing an additional (actually, basic) level in the hierarchy of hardware capacities of the MPP multiprocessor computer systems – architecture – personal computing clusters. Thus, it is proposed to create the foundation of hardware pyramid of parallel computing technology in the form of personal computing clusters by analogy to the existing foundation of hardware pyramid of traditional sequential computing technologies in the form of a PC.

Computational experiments, which were carried out on the basis of the proposed approach, showed high efficiency in solving a wide range of applied problems.

Exposition of the main research material. For the heat equation of the form

$$\frac{\partial Y}{\partial t}=\alpha\frac{\partial^2 Y}{\partial \delta^2},\quad x\in\left[x_0,x_L\right],\quad t\in\left[t_0,T\right], \tag{1}$$

we consider solution of the Dirichlet boundary value problem with the initial

$$Y_{\left|t=t_0\right.}=\varphi(x), \tag{2}$$

and boundary conditions

$$Y_{\left|x=x_0\right.}=YW(t);\quad Y_{\left|x=x_L\right.}=YL(t). \tag{3}$$

The definition domains of the desired function $Y(t, x)$ in the problem (1–3) are comparable to the grid area

$$\left.\begin{aligned} &t_j=J\times Dt1,\quad j=\overline{1,M},\\ &Dt1=T/M,\quad M\in Z\\ &x_p=p\times Dx1,\quad p=\overline{0,2m}\\ &Dx1=(x_L-x_0)/2m,\quad m\in Z \end{aligned}\right\}, \tag{4}$$

where m is an integer parameter in the mesh node topology with respect along the spatial variable x.

The finite difference method. Having used the implicit scheme for time variable and central differences apparatus along the variable x, one arrives at the following system of equations

$$C_pY_{p+1,1}-Y_{p,1}+D_pY_{p-1,1}=f_{p,1};\quad p=\overline{1,2m-1}, \tag{5}$$

where

$$\left.\begin{aligned} &C_p=D_p=\frac{A}{(1+2A)},\quad A=\frac{\alpha}{Dx1^2}Dt1\\ &f_{p_1}=-\frac{YO_{p,1}}{(1+2A)} \end{aligned}\right\}. \tag{6}$$

Here the grid functions $Y_{0,1}=fW(t_j)$, $Y_{2m,1}=fL(t_j)$ define the boundary conditions (3), and the right-hand sides $f_{p,1}$ are initial conditions, since the grid functions $YO_{p,1}$ are taken from the previous $j-1^{st}$ time layer. Consequently, the numerical algorithm (5, 6) is evolutionary and consists of transition acts from one time moment t_{j-1} to another $t_j=t_{j-1}+Dt1$.

The scheme of method of lines. Applying the time-discretization procedure with respect to the equation (1), we arrive at a system of ordinary differential equations of the form

$$Y''_{p+\varepsilon_{x,1}}(\varepsilon_x)-\frac{1}{A}Y_{p+\varepsilon_{x,1}}(\varepsilon_x)=-\frac{1}{A}YO_{p+\varepsilon_{x,1}}(x), \tag{7}$$

where $YO_{p+\varepsilon_{x,1}}(x)$ is a function determined by initial conditions; $\varepsilon_x=\dfrac{(x-x_p)}{(x_{p+1}-x_p)}\in\left[-1,+1\right]$ is a normalized spatial variable.

In its final form, the solution of equation (7) is the following

$$Y_{p+\varepsilon_{x,1}}(\varepsilon_x)=Y^*_{p+\varepsilon_x,1}(\varepsilon_x)+C_pC\eta\beta(\varepsilon_x)+D_pS\eta\beta(\varepsilon_x),$$

where C_p, D_p are integration constants; $Y^*_{p+\varepsilon_x,1}(x)$ a particular solution of inhomogeneous equation (7); $\beta=\sqrt{\dfrac{1}{\mathrm{A}}}$ are eigenvalues of the characteristic equation.

Integration constants C_p, D_p are determined on the basis of the following conditions $\varepsilon_x=\pm1$

$$Y_{p+\varepsilon_{x,1}}(\varepsilon_x)_{\left|\varepsilon_x\pm1\right.}=Y_{p\pm1,1},$$

then a solution of equation (7) is obtained in the form of the following relationship

$$\begin{aligned} \mathrm{Y}_{p+\varepsilon_{x,1}}(\varepsilon_x)=&\left\{Y^*{}_{p+\varepsilon_{x,1}}(\varepsilon_x)+\right.\\ &+\frac{S\eta\beta(1+\varepsilon_x)}{S\eta\beta(\varepsilon_x)}\left[Y_{p+1,1}-Y^*{}_{p+1,1}\right]+\\ &\left.+\frac{S\eta\beta(1-\varepsilon_x)}{S\eta\beta(\varepsilon_x)}\left[Y_{p-1,1}-Y^*{}_{p+1,1}\right]\right\}. \end{aligned} \tag{8}$$

Putting $\varepsilon_x=0$ in expression (8), we move from distributed solution form to its discrete analogue in the

form of a system of linear algebraic equations (5), but with different functional content

$$\left.\begin{aligned} C_p = D_p = \frac{S\eta\beta(\varepsilon_x)}{S\eta\beta(\varepsilon_x)} = \frac{1}{2C\eta\beta(\varepsilon_x)} \\ f_{p,1} = C_p Y^*_{p+1,1} - Y^*_{p,1} + D_p Y^*_{p-1,1} \end{aligned}\right\}, \quad p = \overline{1,2m-1}, \quad (9)$$

which differs from the considered finite-difference approach, which has the form (6).

Parallelizing a mathematical simulation by permutation method. The parallelizing procedure of mathematical problem simulation, defined by a system of equations (5) with a functional content (6) or (9) is based on application of the "odd-even" reduction algorithm [9]. The peculiarity of such an algorithm is the elimination of the corresponding coefficients of algebraic equation system (5) in accordance with the elementary transformations of system rows. We show that this procedure is possible if and only if the integer value of parameter m in the topology of the grid nodes along the coordinate x in (4) is, firstly, an even number, and, secondly, meets the algebraic relation

$$m = 2^{k_*},$$

where k_* is a natural number. The countability topology of the grid area interior nodes along the coordinate x (4) for different values of the parameter m is as follows

$$\begin{aligned} k_* = 1, \quad m = 2, \quad p = \overline{1,3}; \\ k_* = 2, \quad m = 4, \quad p = \overline{1,7}; \\ k_* = 3, \quad m = 8, \quad p = \overline{1,15}; \\ k_* = 4, \quad m = 16, \quad p = \overline{1,31}, \end{aligned}$$

and so on.

The reduction process of the equation system is realized as follows. In the system of equations (5), we decrease and increase the index p by one. Then, the algorithm of the "odd-even" reduction is compared with a set of points $p = \overline{1,2m-1}$ of the computational domain. Under such circumstances, a system of equations is obtained

$$\left.\begin{aligned} Y_{p-1,1} = -f_{p-1,1} + C_{p-1}Y_{p,1} + D_{p-1}Y_{p-2,1} \\ Y_{p+1,1} = -f_{p+1,1} + C_{p+1}Y_{p+2,1} + D_{p+1}Y_{p,1} \end{aligned}\right\}. \quad (10)$$

Then, after substituting $Y_{p\pm1,1}$ from equations (10) into equation system (5), we obtain an equations of the same structure, but with respect to the grid functions with even numbers $Y_{2,1}$, $Y_{4,1}$, $Y_{6,1}$, ..., $Y_{2m-2,1}$

$$C^{(1)}_p Y_{p+2,1} - Y_{p,1} + D^{(1)}_p Y_{p-2,1} = f^{(1)}_{p,1}.$$

Wherein

$$\left.\begin{aligned} C^{(1)}_p = \frac{C^{(0)}_p C^{(0)}_{p+1}}{Det_1}, \quad D^{(1)}_p = \frac{D^{(0)}_p D^{(0)}_{p-1}}{Det_1} \\ f^{(1)}_{p,1} = \left(f^{(0)}_{pu} + C^{(0)}_p f^{(0)}_{p+1,1} + D^{(0)}_p f^{(0)}_{p-1,1}\right) \Big/ Det_1 \\ Det_1 = (1 - C^{(0)}_p D^{(0)}_{p+1} - D^{(0)}_p C^{(0)}_{p-1}) \end{aligned}\right\}, \quad (11)$$
$$p = 2,4,\ldots,2m-2.$$

In the relations (11) the sequences $C^{(0)}_p$, $D^{(0)}_p$, $f^{(0)}_{p,1}$ correspond to the coefficients and the right-hand sides of the original SLAE (5) in the form (6) or (9). Therefore, by excluding variables with odd numbers, we obtain the SLAE with two orders lower. When solving it, for example, by the sweep method, it could be followed by finding the values of variables with odd numbers by recounting from formulas (10).

Obviously, after the re-application of the reduction process, in the final analysis, one can come to one relationship with respect to some function of the central node $Y_{m,1}$. This operation can be performed in three stages of the corresponding transformations.

On the basis of the studies carried out, it is already possible to discuss the most important properties of parallel forms of the algorithm for the SLAE (5) processing by the "odd-even" reduction method. Firstly, it is obvious that application of graphs to describe and research the general problem of computational mathematics problems mapping of the algorithm described above to the architecture of parallel computing systems makes this procedure clear. Secondly, for instance, from the operations sequence analysis of the SLAE (5) processing with $m = 8$ it follows that, once the value of the central node grid function $Y_{8,1}$, for the zero level is computed, the variables $Y_{4,1}$, $Y_{12,1}$ can be computed simultaneously for the first level. After computation of the values of variables $Y_{4,1}$, $Y_{12,1}$ the values of grid functions $Y_{2,4}$, $Y_{6,1}$, $Y_{10,1}$, $Y_{14,1}$ for the second level can be computed simultaneously. After computation of the values $Y_{2,4}$, $Y_{6,1}$, $Y_{10,1}$, $Y_{14,1}$ the values of the grid functions $Y_{4,1}$, $Y_{3,1}$, $Y_{5,1}$, $Y_{7,1}$, $Y_{9,1}$, $Y_{11,1}$, $Y_{13,1}$, $Y_{15,1}$ for the third level can be computed simultaneously. Thus, it is the third level that completes the procedure for SLAE processing of this topology. Let us pay attention to the following circumstance. The graph of the algorithm introduced into consideration is parametrized with respect to the parameter m and has an isomorphic branching structure of vertices. There is also a one-to-one correspondence between the numbers of vertices. The vertex number generated by two vertices of the lower tier is equal to arithmetic mean of the vertices numbers of the lower tier. Since at any reduction process level of the initial SLAE (5), the decomposition of all intermediate systems can be performed simultaneously, then the algorithm considered above has a sufficiently high degree of parallelism.

Parallelizing a mathematical simulation by a numerical-analytical method of straight lines. This research stage is aimed at implementing the process of the equation system (5) parallelizing, provided that its functional content corresponds to the form (9). Here, basic solution linear combinations of the boundary-value problem (7) are mapped to hyperbolic functions, and the right-hand sides represent the set of its particular solutions.

The algorithm being developed is constructed on the basis of piecewise-analytical solutions. It is quite obvious that this circumstance must be taken into account when implementing the process of parallelizing of the equation system (5). Such an idea can be developed on the basis of the sweep method application [8]. In this case, the direct sweep process is aimed at determining

the coefficients $E_p, G_p, (p = \overline{1,2m-1})$ that are determined on the basis of the relations of the form

$$E_p = \frac{C_p}{1 - D_p E_{p-1}}; \quad G_p = \frac{D_p G_{p-1} - f_{p,1}}{1 - D_p E_{p-1}}, \qquad (12)$$

wherein the beginning of such a problem solution is provided by the following parameters

$$E_0 = 0; \quad G_0 = Y_{0,1} = fW(t_j).$$

The direct sweep method proceeds in the direction of index p increasing to a $p = 2m - 1$ value. At the same time, the reverse sweep method is conducted by the following recurrent formula

$$Y_{p,1} = E_p Y_{p+1,1} + G_p. \qquad (13)$$

At the same time, the change direction is provided for index p from $p = 2m - 1$ to $p = 1$. The computation starts based on the condition $Y_{2m,1} = fL(t_j)$, which sets the right boundary condition (3) in the sweep algorithm.

It turns out that the use of functional content for coefficients C_p, D_p and right-hand parts $f_{p,1}$, in the form (9), allows performing direct sweep method computations and forming formulae for calculating coefficients E_p, G_p in accordance with formulas (12) as functions of the number of grid nodes

$$\left.\begin{aligned} E_p &= \frac{s\eta\beta(p)}{s\eta\beta(p+1)} \\ G_p &= \frac{1}{s\eta\beta(p+1)}\left[Y_{0,1} - \sum_{i=1}^{p} f_{i,1} s\eta\beta(i)\right] \end{aligned}\right\}, \quad p = \overline{1,2m-1}. \qquad (14)$$

This circumstance is further directed to the fact that, on the basis of substituting expressions (14) in the recurrent relation for the reverse sweep (13), it would be possible to determine solution of the equation system (5) for any node of the grid area (4)

$$Y_{2m-v,1}(v) = \frac{1}{s\eta\beta(2m)} \times \left\{ \begin{aligned} & s\eta\beta(2m-v)\left[Y_{2m,1} - \sum_{i=1}^{v} \frac{s\eta\beta(i)}{s\eta\beta(i+1)} f_{2m-i,1}\right] + \\ & + s\eta\beta(v)\left[Y_{0,1} - \sum_{i=1}^{2m-1-v} \frac{s\eta\beta(i)}{s\eta\beta(i+1)} f_{i,1}\right] \end{aligned} \right\},$$
$$v = \overline{1,2m-1},$$

where hyperbolic functions are the solution of homogeneous equation systems (7), and complexes $f_{p,1} = Y^*_{p+1,1} - 2c\eta\beta(p)Y^*_{p,1} + Y^*_{p-1,1}$ are the set of its particular solutions.

The developed approach to parallelizing a mathematical simulation is stable for a different type of input data. In addition, it has the most parallel form and is characterized by a minimum time for solving the problem with respect to multiprocessor computing systems. This is explained as follows. If it is hypothetically suggested that you can designate one processor for one node in the computed mesh area, you can assign the same processor in parallel and simultaneously at the same time for all nodes in the calculated mesh area.

Experimental data and its processing. The proposed approach to the solution of heat conduction problems is realized in the form of a procedure for mathematical simulation by a multiprocessor computing system. In this case, let us consider one of the options for its implementation.

Test problem. Let us suppose that boundary conditions of the first kind are given on the surface of the plate. For definiteness, the boundary function $TW(t)$ will be considered as known, but $TL(t)$ will be desired function in solving the heat conduction problem. In the quasilinear approximation, equation (1) in the grid area is associated with a system of its differential effects, which take the form of the System of Homogeneous Differential Equations, that is

$$\begin{aligned} T'_{p,n+1}(\varepsilon_t) = \{ & A_p(n+4)(n+2)T_{p,n+3}(\varepsilon_t) + \\ & + B_p(n+1)T_{p,n+2}(\varepsilon_t)\}, \end{aligned} \qquad (15)$$

for $n = 0,1,2,\ldots$, wherein

$$A_p = Dt1\frac{\lambda_{p,1}}{CV_{p,1}Dx1^2}, \quad \text{if} \quad p = \overline{1,2m-1}; \qquad (16)$$

$$B_p = Dt1\frac{\lambda_{p,2}}{CV_{p,1}Dx1^2}.$$

Assuming an implicit-in-time scheme of the first order as approximated and by defining the closing connections, we obtain central differences of the following form

$$\left.\begin{aligned} T_{p,2} &= \frac{1}{2}T(T_{p+1,1} - T_{p-1,1}) \\ T_{p,3} &= \frac{1}{2}(T_{p+1,1} + T_{p-1,1} - 2T_{p,1}) \end{aligned}\right\}. \qquad (17)$$

Next, we obtain an equation system of the tridiagonal structure, i. e.

$$C_p T_{p+1,1} - T_{p,1} + D_p T_{p-1,1} = f_{p,1}, \qquad (18)$$

wherein

$$C_p = \frac{(A_p + B_p/2)}{(1 + 2A_p)}; \quad D_p = \frac{(A_p + B_p/2)}{(1 + 2A_p)};$$

$$f_{p,1} = -\frac{TO_{p,1}}{(1 + 2A_p)}, \quad \text{if} \quad p = \overline{1,2m-1}. \qquad (19)$$

The solution of the system of equations (18) is realized by the sweep method. To solve a direct problem in the system of equations (18, 19), we must assume that $T_{0,1} = TW(t_j)$ and $T_{2m,1} = TL(t_j)$. When $TW(t_j)$ is a given, $TL(t_j)$ is the required boundary function, then, by solving the heat conduction problem, it is necessary to select the function $TL(t_j)$ in such a way that in the solutions of the equation systems (18, 19) the functional with the mean-square discrepancy should become minimal. The entire time interval where the desired function is deter-

mined by setting conditions (2, 3), is divided into time intervals $Dt1$ according to the sampling circuit. The scanning step during the solution of the boundary heat conduction problem does not have to coincide with the sampling step $Dt1$, adopted in the controlled system of equations (18, 19). The algorithm for solving the boundary heat conduction problem begins with the first scanning step. Let us assume that as a trivial approximation a certain value is taken, when $R = TL$, it is known from a priori information and is clearly greater than unknown boundary function. This circumstance is verified experimentally by the mathematical simulation method. After substituting the value of R into the solution of the controlled mathematical simulation (18, 19), the computed value of temperature function $T_p(t_j)$ is obtained at the same location along the coordinate where the function $T_e(t_j)$ is given, which is known from the experiment. This makes it possible to find the numerical value of the residual $(T_p(t_j) - T_e(t_j))$. If the residual $(T_p(t_j) - T_e(t_j)) > 0$, then the value of R decreases by some amount ΔT. In other cases, it increases by the same amount (ΔT). Further, this process develops until the residual changes its sign to the opposite. The establishment of this fact means that the required approximate solution of the heat conduction problem lies in the interval $[a, b]$, where a and b denote numbers where the functions values at these points have different signs. By computing the residual value at midpoint, which lies between a and b, we can further refine the unknown boundary function value by an interpolation formula. Repeating this iterative procedure as many times as necessary, the value of the control parameter is computed with any predetermined accuracy. Naturally, it should be borne in mind here that the function $T_e(t_j)$ is known by the results of the experiment and has some error ε. Thus, the required solution of the heat conduction problem occurs through scanning steps, and assuming a linear approximation in the time layers, we write that

$$TL_{v+\varepsilon_t,1}\left(\varepsilon_t\right) = TL_{v,1} + \varepsilon_t TL_{v,2},$$

wherein

$$\left.\begin{aligned} TL_{v,2} &= \left(TL_{v+1,1} - TL_{v,1}\right) \\ \varepsilon_t &= \frac{t - t_v}{t_{v+1} - t_v} \in \left[0,1\right] \end{aligned}\right\}, \quad \text{if} \quad v = 1,2,\ldots.$$

Then it is possible to solve the heat conduction problem on a full time interval, when $t \in [t_0, t_k]$ along v^{th} time layers. This possibility arises, firstly, because the sweep algorithm realizes the desired nonstationary solution on any time layer. The transition in the systems of equations (15–19) to the time variable ε_t unifies the algorithm with respect to any time interval by automatic accounting of the initial values. The Figure shows the test problem solution results of thermal conductivity for exact values of the experimental function.

The simulation process was implemented on a personal computing cluster. From simulation results analysis it follows that the developed method for solving the heat conduction problem sufficiently minimizes the residuals effectively. It should also be noted that the algorithm developed in this formulation is absolutely stable.

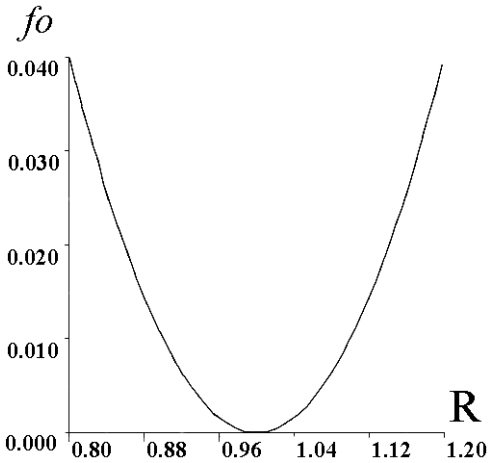

Fig. Results graph for restoring external boundary conditions

Results. At present, the multiprocessor computing systems are developing quite dynamically. For this reason, parallel computations have become available to many researchers. At the same time, a serious development in the field of parallel computing stimulated the design of computing clusters. For their construction, mass-production processors, standard network technologies and freely distributed software are used. These circumstances also explain why the problem of developing effective software turned out to be one of the central problems of parallel computing in general. The creation of parallel computing systems required the development of a new approach for the making of appropriate computational algorithms. It is quite obvious that such an approach is oriented to the implementation of vectorization procedure for computations. Also, under such conditions the description of the corresponding simulations leads to rather complicated dependences. For their comprehension and description, one has to turn to the newest achievements of various fields of applied mathematics. The methods for solving applied problems, which have already become classical, are not suitable from the point of view of algorithmic structure. In addition, they do not always meet the accuracy and speed of computation requirements. Thus, it is quite obvious that the discretization of problems is distinguished by those systems of equations that have a large number of unknowns. For this reason, new ideas and approaches in the field of computational mathematics get emerging, which are aimed at development of new methods for realizing numerical experiments. This paper shows the formation procedure for parallel computational algorithms by various methods on the example of solving thermal problems of mining and metallurgical complex.

Among the known algorithms for recursive decomposition of equations systems solutions, it is obvious that only the algorithm of cyclic reduction of permutations allows the highest degree of its vectorization. It is shown that in the algorithm of such an "odd-even" reduction there are many possibilities for parallelization. With this respect, the

mathematical problem parallelizing method is based on this algorithm. Let us note that its application is a very promising direction of applied mathematics, because the problems of linear algebra represent a significant part of all problems of numerical analysis (according to some estimates, about 75 %). It is obvious that the application of graphs for describing and research general mapping problem of the above algorithm on the architecture of parallel computing systems makes this procedure vivid.

The parallelization of systems of the tridiagonal structure was realized by a numerical-analytical approach, which predetermined their maximally parallel algorithmic form. This approach contributes to the minimum possible time for implementation of the developed algorithm on parallel computing systems. In addition, during the parallel computation of arithmetic expressions, in the developed algorithm, the output data error gets separated from the rounding operations. Such a procedure became possible, because with this approach the recursive structure of solution vectors computation is excluded, which usually predetermines the accumulation of rounding errors.

In addition, the necessary smoothness of the original solution and input data for a particular problem was taken into account, which allowed adapting computational algorithm to the singularities of numerical solution. This procedure was carried out by additional nodes located between the nodes of a given grid area. This allows displaying the computational algorithm accurately. In this case, the computations in the construction of graphs or isolines can be performed in parallel and simultaneously. The computational experiments carried out showed high efficiency of the proposed approach.

Conclusions. This paper presents the algorithms for solving thermal problems of metallurgy and mining by maximally parallel computational forms. It is shown that in the "odd-even" reduction algorithm many possibilities of parallelization are unrevealed. This is because the algorithm of permutations solves the system of equations simultaneously from both ends. Therefore, its use for solving tridiagonal systems on parallel systems can be very promising.

Parallelizing tridiagonal systems based on numerical-analytical methods of discretization, firstly, does not impose any restrictions on the topology of the grid nodes of the computational domain. And, secondly, with respect to the parallel computation of arithmetic expressions, the error in initial data is separated from the rounding operations common to a real PC. With this approach, the computation recurrent structure of the desired decision vectors is excluded, which, as a rule, leads to the accumulation of rounding errors. The parallel form of the algorithm constructed in this way is maximal, and, therefore, has the minimum possible time for implementing the algorithm on parallel computing systems.

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Паралельні обчислювальні алгоритми в теплових процесах металургії та гірничої справи

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Мета. Формування паралельних алгоритмів при моделюванні теплових процесів металургії та гірничої справи. Побудована паралельна форма алгоритмів повинна бути максимальною, і, відтак, мати мінімально можливий час реалізації на паралельних обчислювальних системах. Виключити рекурентну структуру обчислення шуканих векторів рішень, що, як правило, призводить до накопичення помилок округлення. Проблеми моделювання по-

дібного класу задач реалізувати за допомогою багатопроцесорних обчислювальних систем.

Методика. Для реалізації процесу розпаралелювання математичної постановки задачі реалізовано підхід на основі алгоритму „непарної-парної“ редукції. Суть такого підходу полягає у виключенні коефіцієнтів моделі досліджуваного процесу із реалізацією елементарних перетворень рядків складеної системи рівнянь. Безпосередньо паралельна форма алгоритму рішення задач реалізована за допомогою чисельно-аналітичного підходу. Показано, що складена паралельна форма є максимальною, що, у свою чергу, забезпечує мінімальний час вирішення поставлених завдань на основі використання багатопроцесорних обчислювальних систем.

Результати. Дослідження, наведені в даній роботі, показали високу ефективність розпаралелювання систем лінійних алгебраїчних рівнянь тридіагональної структури на прикладі рішення теплових задач. Запропонований чисельно-аналітичний метод розпаралелювання тридіагональної системи не накладає будь-яких обмежень на топологію сіткових вузлів розрахункової області. Відповідно до паралельних обчислень арифметичних виразів, у запропонованій методиці відокремлена похибка вихідних даних від операцій округлення. За такого підходу виключена рекуррентна структура обчислення шуканих векторів рішень, що, як правило, і призводить до накопичення помилок округлення. Побудована паралельна форма запропонованих алгоритмів є максимальною, і, відтак, має мінімально можливий час реалізації алгоритму на паралельних обчислювальних системах. Обчислювальні експерименти, проведені за допомогою багатопроцесорної обчислювальної системи, показали високу ефективність розроблених паралельних алгоритмів.

Наукова новизна. У рамках декомпозиційних алгоритмів на підставі методу „непарної-парної“ редукції вперше запропоновано новий підхід до розподіленого вирішення систем лінійних алгебраїчних рівнянь, що, у порівнянні з відомими методами, відрізняється замкнутою паралельною формою щодо центрального вузла сітки й високим ступенем векторизації. Запропоновано, проаналізовано й реалізовано новий підхід до вирішення завдань металургійного виробництва, що дозволяє підвищити економічність, продуктивність і швидкодію обчислень. Доведено, що такий підхід передбачає найвищий ступінь векторизації обчислень, зумовлює максимально паралельну їх алгоритмічну форму та, як наслідок, мінімально можливий час реалізації алгоритмів на паралельних обчислювальних системах.

Практична значимість. Шляхом застосування високопродуктивної багатопроцесорної системи розроблений підхід дозволяє обробляти та інтерпретувати результати теплових експериментів, домагаючись високого ступеня точності, істотного скорочення часу обробки експериментальних даних.

Ключові слова: *векторизація обчислень, алгоритми розпаралелювання, багатопроцесорні системи, теплові завдання*

Параллельные вычислительные алгоритмы в тепловых процессах металлургии и горного дела

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Цель. Формирование параллельных алгоритмов при моделировании тепловых процессов металлургии и горного дела. Построенная параллельная форма алгоритмов должна быть максимальной, и, следовательно, иметь минимально возможное время реализации на параллельных вычислительных системах. Исключить рекуррентную структуру вычисления искомых векторов решений, которая, как правило, приводит к накапливанию ошибок округления. Проблемы моделирования подобного класса задач реализовать при помощи многопроцессорных вычислительных систем.

Методика. Для реализации процесса распараллеливания математической постановки задачи реализован подход на основе алгоритма „нечетной-четной“ редукции. Суть такого подхода состоит в исключении коэффициентов модели изучаемого процесса с реализацией элементарных преобразований строк составленной системы уравнений. Непосредственно параллельная форма алгоритма решения задач реализована при помощи численно-аналитического подхода. Показано, что составленная параллельная форма является максимальной, что, в свою очередь, обеспечивает минимальное время решения поставленных задач на основе использования многопроцессорных вычислительных систем.

Результаты. Исследования, приведенные в данной работе, показали высокую эффективность распараллеливания систем линейных алгебраических уравнений трехдиагональной структуры на примере решения тепловых задач. Предложенный численно-аналитический метод распараллеливания трехдиагональных систем не накладывает каких-либо ограничений на топологию сеточных узлов расчетной области. Применительно к параллельным вычислениям арифметических выражений, в предложенной методике отделена погрешность исходных данных от операций округления. При таком подходе исключена рекуррентная структура вычисления искомых векторов решений, которая, как правило, и приводит к накапливанию ошибок округления. Построенная параллельная форма предложенных алгоритмов является максимальной, и, следовательно, имеет минимально возможное время реализации алгоритма на параллельных вычислительных системах. Вычислительные эксперименты, проведенные при помощи многопроцессорной вычислительной системы, показали высокую эффективность разработанных параллельных алгоритмов.

Научная новизна. В рамках декомпозиционных алгоритмов на основании метода „нечетной-четной“ редукции впервые предложен новый подход к распределенному решению систем линейных алгебраических уравнений, который, по сравнению с известными методами, отличается замкнутой параллельной формой относительно центрального узла сетки и высокой степенью векторизации. Предложен, проанализирован и реализован новый подход к решению задач металлургического производства, который позволяет повысить экономичность, производительность и быстродействие вычислений. Доказано, что такой подход предусматривает наиболее высокую степень векторизирования вычислений, предопределяет максимально параллельную их алгоритмическую форму и, как следствие, минимально возможное время реализации алгоритмов на параллельных вычислительных системах.

Практическая значимость. Путем применения высокопроизводительной многопроцессорной системы разработанный подход позволяет обрабатывать и интерпретировать результаты тепловых экспериментов, добиваясь высокой степени точности, существенного сокращения времени обработки экспериментальных данных.

Ключевые слова: *векторизация вычислений, алгоритмы распараллеливания, многопроцессорные системы, тепловые задачи*

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